AOSC 431 D. Kleist 12/8/16

Fall '16 SOLUTION SET

HW # 6

---

(1)

a) curvature effect from surface energy or surface tension. energy of system (ΔE) must increase to form drop of larger size if $e \le e_s$. Small drops evaporate more easily relative to that from flat surface. Or... "e_s (drops)" > "e_s (flat)"

b) this is related to heterogeneous nucleation and Raoult's. The salt reduces the saturation vapor pressure and increases likelihood of condensation, similar to cloud condensation nuclei.

c) related to (a) above; even when air is saturated with respect to a flat surface ($e_s(\infty)$), it is unsaturated with respect to a curved droplet of pure water. There for air must be supersaturated (WRT flat sfc) for equilibrium or growth.

d) The growth equation by WH (6.20) shows that $\frac{dr}{dt}$ is inversely proportional to r. Therefore, drops grow quickly at initial time (if larger than critical radius), but asymptotes to some value around ~20-30 μm (much to small to fall out as rain and not evaporate)!

e) Because of lack of CCN and different types of CCN in marine environments (salt) v. continental (misc.), the drop sizes in marine clouds are larger and cover a broader spectrum (WH Fig. 6.7). This results in higher likelihood of forming "collector drops" and accelerating the collision/coalescence processes. Continental clouds →

do not form drops > ~ 20 μm. See discussion in page 230 (W-H).

f.) because of break-up processes and collisions.
(aerodynamic effects)

g) Lightning is associated with convection/updrafts. Solar heating impacts land more than ocean ~ heating up more quickly, reaching convective temperature (more instability), etc.

(2)

$$r = \frac{2\sigma}{nkT\ln\left(\frac{e}{e_s}\right)} \quad \text{so} \quad \ln\frac{e}{e_s} = \frac{2\sigma}{nkTr}$$

$n = 3.3\times10^{28}\,m^{-3}$ $k = 1.38\times10^{-23}$ $T = 273K$

$\sigma = 0.076\,J/m^2$ $r = 0.5\times10^{-6}\,m$

$$\ln\left(\frac{e}{e_s}\right) = \frac{2(0.076\,J/m^2)}{(3.3\times10^{28}\,m^{-3})(1.38\times10^{-23})(273K)(0.5\times10^{-6}\,m)}$$

$$\ln\left(\frac{e}{e_s}\right) \approx 2.4452\times10^{-3}$$

∴ RH → 1.00245 or **100.245%**

(3)

a) SS ~ 0.1%, $m = 10^{-18}$ kg, NaCl $r \cong 0.75\,\mu m$
(curve #3)

b) curve #5, $r = 0.05\,\mu m$, RH ≅ 97%

c) SS_c, curve # 5, $SS_c \cong 0.45\%$

(4)

$$\frac{dm}{dt} = \pi r^2 V E w_c$$ w_c - cloud water content.

(variation of WH 6-26)

$m = \frac{4}{3}\pi r^3 \rho$ plug into 6.26 to get in terms of $\frac{dr}{dt}$

$$\frac{4}{3}\pi \rho_L (3r^2)\frac{dr}{dt} = \pi r^2 V E w_c$$

$$4\pi \rho_L r^2 \frac{dr}{dt} = \pi r^2 V E w_c$$

$$4\rho_L \frac{dr}{dt} = V E w_c$$

$$\frac{dr}{dt} = \frac{V E w_c}{4\rho_L}$$ here $\rho_L = 1000 \frac{kg}{m^3}$

now, $V = 6\times10^3 r$ plug in

$$\frac{dr}{dt} = \frac{(6\times10^3) r E w_c}{4\rho_L}$$

$$\frac{1}{r}\frac{dr}{dt} = \frac{(6\times10^3) E w_c}{4\rho_L}$$

Compute w_c: $100 \frac{drops}{cm^3} \frac{10^6 cm^3}{1 m^3}$

so $w_c = \frac{4}{3}\pi (r_2)^3 \rho_L \left(100\times10^6 \text{ drops}/m^3\right)$

4 cont.:

$$\frac{1}{r}\frac{dr}{dt} = \frac{(6\times10^3)(0.8)\left(\frac{4}{3}\pi(r_2)^3\rho_L\right)(100\times10^6\ \text{drops}/m^3)}{4\rho_L}$$

smaller $r_2 = 10\,\mu m = 10^{-5}\,m$

$$\frac{1}{r}\frac{dr}{dt} = \frac{(6\times10^3)(0.8)(\pi)(10^{-5}m)^3(100\times10^6\ \text{drops}/m^3)}{3}$$

• integrate for r, R from $100\,\mu m\ (10^{-4}m)$ to $1mm\ (10^{-3}m)$

dt to other side, integrate

$$\int_{10^{-4}}^{10^{-3}}\frac{1}{r}\,dr = \int_0^t \frac{(6\times10^3)(0.8)(\pi)(10^{-15}m^3)(100\times10^6\ \frac{\text{drops}}{m^3})}{3}\,dt$$

$$\ln(10^{-3}m) - \ln(10^{-4}m) = (0.00050265)\cdot t$$

so

$$t = \frac{\ln(10^{-3}) - \ln(10^{-4})}{0.00050265}$$

$t \cong 4580.9s \approx 76$ minutes