AOSC 470/600
Homework #5
DUE: 25 October 2016

1) [25 points] Starting from the quasi-geostrophic vorticity and thermodynamic energy equations:

$$\frac{\partial \zeta_g}{\partial t} = -\vec{V}_g \cdot \nabla\left(\zeta_g + f\right) + f_o \frac{\partial \omega}{\partial p}$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial p}\right) = -\vec{V}_g \cdot \nabla\left(-\frac{\partial \phi}{\partial p}\right) + \sigma\omega$$

Defining χ as the geopoential tendency

$$\chi = \frac{\partial \phi}{\partial t}$$

and recalling that the geostrophic vorticity can be expressed as:

$$\zeta_g = \frac{1}{f_o}\nabla^2\phi$$

- ***First, show that the QG equations can be written in terms of geopotential tendency as follows***:

$$\nabla^2\chi = -f_o\vec{V}_g \cdot \nabla\left(\frac{1}{f_o}\nabla^2\phi + f\right) + f_o^2\frac{\partial \omega}{\partial p}$$

$$\frac{\partial \chi}{\partial p} = -\vec{V}_g \cdot \nabla\left(\frac{\partial \phi}{\partial p}\right) - \sigma\omega$$

- ***Then, differentiate the thermodynamic energy equation with respect to pressure and multiply by f_o^2/σ (yellow box below) and combine it with the vorticity equation (red box above) to eliminate omega***:

$$\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\left[\frac{\partial \chi}{\partial p} = -\vec{V}_g \cdot \nabla\left(\frac{\partial \phi}{\partial p}\right) - \sigma\omega\right]$$

Similar to the QG omega equation, you should end up with a single expression for geopotential tendency that involves something like the 3D Laplacian on one side and two forcing terms on the other. Please carefully show your work.

2) [25 points] The Lagrangian rate of change of the horizontal potential temperature gradient following $\vec{V}_g$ is given by $\frac{d}{dt_g}(\nabla\theta)$, where $\frac{d}{dt_g} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$.

a) Expand the expression and assume that $\frac{d\theta}{dt_g} = 0$. Recall that the horizontal potential temperature gradient is given by $\nabla\theta = \frac{\partial\theta}{\partial x}\hat{i} + \frac{\partial\theta}{\partial y}\hat{j}$.

b) The final expression should look familiar. Describe in a sentence or less what the final expression looks like or represents.

[HINTS]

- For terms that look like the following (where a is some arbitrary scalar):

$$u\frac{\partial}{\partial x}\left(\frac{\partial a}{\partial x}\right)\hat{i}$$

Use the Chain Rule, i.e.:

$$\frac{\partial}{\partial x}\left(u\frac{\partial a}{\partial x}\right)\hat{i} = \frac{\partial u}{\partial x}\frac{\partial a}{\partial x}\hat{i} + u\frac{\partial}{\partial x}\frac{\partial a}{\partial x}\hat{i}$$

To substitute and rewrite as:

$$u\frac{\partial}{\partial x}\left(\frac{\partial a}{\partial x}\right)\hat{i} = \frac{\partial}{\partial x}\left(u\frac{\partial a}{\partial x}\right)\hat{i} - \frac{\partial u}{\partial x}\frac{\partial a}{\partial x}\hat{i}$$

- Do a rearrangement in vector form to combine into something that involves a combination of derivatives of temperature and geostrophic wind