Introduction to microwave radiative transfer May 2002

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Abstract

The following text gives a brief overview of microwave radiative transfer in atmospheres covering the principles of radiative transfer in an absorbing and scattering atmosphere including surface reflection and emission. A few approaches for radiative transfer modelling are presented. Models for atmospheric absorption, particle absorption and scattering as well as surface emission are introduced. Please note that the reference frequency range was chosen to be 1-200 GHz which covers all current spaceborne microwave radiometer channels in operation for tropospheric remote sensing. As a description of current microwave radiative transfer problems, the COST-712 Project 1 report is recommended.

Table of contents

1. Radiative transfer

The radiative transfer equation can be expressed as the differential change of radiance L_ν along path s:

$$\mathrm{d}L_\nu = -L_\nu k_{\mathrm{ext}} \mathrm{d}s \tag{1}$$

In vertical coordinates, and including slant paths, the path coordinate is modified to optical depth $\mathrm{d}\delta/\mu = k_{\mathrm{ext}} \mathrm{d}z/\mu$ with zenith angle $\theta = \cos^{-1}\mu$. $k_{\mathrm{ext}} = k_{\mathrm{abs}} + k_{\mathrm{sct}}$ denotes the volume extinction coefficient comprising absorption and scattering by all relevant media in the atmosphere. All optical quantities are frequency dependent, thus the subscript 'ν' will be omitted hereafter. Including source terms (1) translates to:

$$\begin{aligned} \mu \frac{\mathrm{d}L(\delta;\mu,\phi)}{\mathrm{d}\delta} &= L(\delta;\mu,\phi) - J(\delta;\mu,\phi) && \text{(upward)} \\ -\mu \frac{\mathrm{d}L(\delta;-\mu,\phi)}{\mathrm{d}\delta} &= L(\delta;-\mu,\phi) - J(\delta;-\mu,\phi) && \text{(downward).} \end{aligned} \tag{2}$$

ϕ denotes the azimuth angle so that the angular dependence may be combined to $\Omega = (\mu, \phi)$, $\mathrm{d}\Omega = \mathrm{d}\mu\mathrm{d}\phi$. The source term covers contributions from scattering (hydrometeors) and emission (oxygen, water vapour, dry air, hydrometeors):

$$J(\delta;\mu,\phi) = \frac{\omega_0}{4\pi}\int_0^{2\pi}\int_{-1}^{1} L(\delta;\mu',\phi')P(\mu,\phi;\mu',\phi')\mathrm{d}\mu'\mathrm{d}\phi' + (1-\omega_0)B(T) \tag{3}$$

Scattering of radiance is expressed in terms of a normalized scattering phase function:

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1} P(\mu,\phi;\mu',\phi')\mathrm{d}\mu'\mathrm{d}\phi' = 1 \tag{4}$$

describing the distribution of incident radiance (μ', ϕ') to observation direction (μ, ϕ). $\omega_0 = k_{\mathrm{sct}}/k_{\mathrm{ext}}$ denotes the single scattering albedo and provides a measure for the fraction of scattered radiation while $(1-\omega_0)$ is the fraction of absorbed / emitted radiation. $B(T)$ is the blackbody equivalent radiance according to temperature.

The differential form of the radiative transfer equation can be integrated from the surface ($\delta = \delta^*$) to the top of the atmosphere, $\delta = 0$:

$$L(\delta=0;\mu,\phi) = \varepsilon_0 B(T_0)\tau(\delta^*;\mu,\phi) + \\ + \int_{\delta^*}^{0} J(\delta';\mu,\phi)\exp[-\delta'/\mu]\ \mathrm{d}\delta'/\mu + \\ + (1-\varepsilon_0)\tau(\delta^*;\mu,\phi)\int_{0}^{\delta^*} J(\delta';\mu,\phi)\exp[-(\delta^*-\delta')/\mu]\ \mathrm{d}\delta'/\mu \tag{5}$$

with total atmospheric transmission:

$$\tau(\delta^*;\mu,\phi) = \int_{\delta^*}^{0} \exp[-\delta'/\mu]\ \mathrm{d}\delta'/\mu \tag{6}$$

and surface emissivity, ε_0, which is a function of temperature, roughness, foam coverage, and salinity in the case of sea water and a function of temperature, moisture, soil type, vegetation, and roughness (among others) for land surfaces.

1.1 Emission

In the case of a purely absorbing medium, $\omega_0 = 0$. Neglecting the azimuth angle dependence (6) becomes:

$$L(\delta=0,\mu) = \varepsilon_0 B(T_0)\tau(\delta^*,\mu) + \int_{\delta^*}^{0} B[T(\delta')]\exp[-\delta'/\mu]\,\mathrm{d}\delta'/\mu \\ + (1-\varepsilon_0)\tau(\delta^*,\mu)\int_{\delta^*}^{0} B[T(\delta')]\exp[-(\delta^*-\delta')/\mu]\,\mathrm{d}\delta'/\mu \tag{7}$$

The term $\exp[-\delta'/\mu]$ defines the atmospheric transmission from level δ' to the top of the atmosphere, i.e.:

$$\tau(\delta,\mu) = \exp[-\delta/\mu], \qquad \frac{\partial\tau(\delta,\mu)}{\partial\delta} = -\frac{1}{\mu}\tau(\delta,\mu) \tag{8}$$

which by insertion into (7) (neglecting surface contributions) provides the basic radiative transfer equation for vertical atmospheric sounding (note the exchange of the integration limits):

$$L(\delta=0,\mu) = \int_{0}^{\delta^*} B[T(\delta')]\frac{\partial\tau(\delta',\mu)}{\partial\delta}\,\mathrm{d}\delta' \tag{9}$$

In the microwave spectrum and in cloudfree areas, optical depth is mainly a function of atmospheric temperature (50–60 GHz, 118 GHz) and water vapour (22 GHz, 183 GHz) due to line absorption; however, continuum contributions by dry air and water vapour increase slowly with frequency and are non-negligible though difficult to parameterize.

The term $\partial\tau(\delta,\mu)/\partial\delta$ is called the 'weighting function' since it provides weights of the contribution of $B[T(\delta)]$ to $I(\delta=0,\mu)$. Thus only as much radiance from a certain layer can reach the top of the atmosphere as is transmitted through the overlying layers. Typically, the weighting functions have 'Gaussian' shapes with a maximum at the level where τ has a maximum gradient. This level defines the layer to which a sounding has a maximum sensitivity; however the width of the weighting function indicates that the sounding level is not discrete but rather blurred and depends on the local conditions to be retrieved.

For remote sensing applications, it is important to notice that the integrand in (9) comprises two profile variables, i.e. $T(\delta)$ and $\tau(\delta, \mu)$. An inversion of (9) is more accurate for temperature profile retrievals near or at oxygen absorption lines because there, absorption is a function of temperature as is $B[T(\delta)]$. Since oxygen is a well mixed gas in the troposphere, absorption does not depend on gas concentration but only temperature.

In the case of water vapour profile retrievals, $\tau(\delta, \mu)$ is a function of water vapour content while $B[T(\delta)]$ is a function of temperature. Thus, both absorber density and temperature profiles are convolved and cause less accurate retrievals of e.g. water vapour contents at specific altitudes. Here, a combination of temperature profile retrievals at, say 50–60 GHz, and water vapour profile retrievals at 183 GHz are of advantage. In terms of water vapour content (e.g. mixing ratio), the integrand in (9) may be expressed as:

$$B[T(\delta)]\frac{\partial\tau(\delta,\mu)}{\partial\delta}\,\mathrm{d}\delta = B[T(z)]\frac{\partial\tau}{\partial\delta}\frac{\partial\delta}{\partial w}\frac{\partial w}{\partial z}\,\mathrm{d}z\,. \tag{10}$$

Here, $\partial\tau/\partial\delta$ and $\partial\delta/\partial w$ are known so that (10) is as complex as for temperature sounding but expressed in terms of the desired quantity.

Of some importance is the way the integration in (9) is carried out. Considering a single layer, it may be assumed (1) that $B(\overline{T}) = \mathrm{const}$ with layer average temperature $\overline{T}$, or (2) that B changes linearly within the layer, i.e., $B[T(\delta)] = B[T_0] + B_1\delta$ with T_0 being the temperature at the top of the layer (where $\delta = 0$) and $B_1 = \Delta B/\Delta\delta$ being the lapse rate. Then:

$$\int_0^{\delta} B[T(\delta')]\frac{\partial\tau(\delta',\mu)}{\partial\delta}\,\mathrm{d}\delta' = \begin{cases} B(T)[1-\exp(-\delta/\mu)] & \text{: const} \\ B(T_0) + B_1\mu - \tau[B(T_0) + B_1(\mu+\delta)] & \text{: linear} \end{cases} \tag{11}$$

Depending on the optical depth of the layer, the difference between the two options may be several K. At microwave wavelengths, this becomes important in clouds and precipitation but may be neglected in optically thin cloud-free atmospheres.

1.2 Scattering

In the microwave part of the electromagnetic spectrum, the dependence of scattered radiation on azimuth angle can be neglected in most cases because multiple scattering of diffuse radiation is much less anisotropic than that of e.g. solar radiation. In this case, the phase function in (4) becomes:

$$P(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} P(\mu,\phi;\mu',\phi')\,\mathrm{d}\phi \tag{12}$$

and (2) reduces to:

$$\pm\mu\frac{\mathrm{d}L(\delta,\pm\mu)}{\mathrm{d}\delta} = L(\delta,\pm\mu) - \frac{\omega_0}{2}\int_{-1}^{1} L(\delta,\mu')P(\pm\mu,\mu')\,\mathrm{d}\mu' - (1-\omega_0)B(T) \tag{13}$$

A rather accurate approximation to the phase function in (12) is given by the Henyey–Greenstein function which is only applicable in scalar radiative transfer:

$$P(\mu, \phi;\mu', \phi') = \frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}}$$
$$\cos\theta = \pm\mu\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\phi-\phi') \tag{14}$$

g denotes the asymmetry parameter which represents the angle-averaged phase function, $g \in [-1, 1]$:

$$g = \frac{1}{2}\int_{-1}^{1} P(\cos\Theta)\cos\Theta \;\, d\cos\Theta = \overline{\cos\Theta} \tag{15}$$

This parameter is >0 / <0 if more radiation is scattered in forward/backward than backward/forward direction. For Rayleigh scattering $g = 0$.

1.3 Polarization

Another important source of information is polarization, because surfaces polarize incoming unpolarized radiation by reflection and particles polarize by scattering. In both cases, a strong dependence of illumination vs. observation geometry exists, and the degree and angular distribution of the polarized radiation is determined by surface reflectivity and roughness, and particle scattering efficiency and shape. Polarization calculations require the expansion of radiances into vertically and horizontally polarized components which are elements of the Stokes vector $\mathbf{L} = (I, Q, U, V)$:

$$\begin{aligned} I &= I_h + I_v \\ Q &= I_h - I_v \\ U &= I\cos(2\alpha)\sin(2\beta) \\ V &= I\sin(2\beta) \end{aligned} \tag{16}$$

The (v, h) are defined by a plane between the incoming and scattered/reflected radiation beams. ‘v’ represents the vertical component and ‘h’ the parallel component to this plane. Angle β defines the orientation of the vector $\mathbf{L}$ with respect to the ‘h’-direction while α stands for the ellipticity of the polarization. The sign of α describes the sense of rotation, i.e., the sign of the phase (Φ) difference between I_h and I_v. Elliptically polarized radiation represents the most general definition form for polarized radiation with special cases of unpolarized, linearly polarized, and circularly polarized radiation: thus $I_h = I_v$, $\beta = 0$; $I_h = I_v$, $\Phi_h = \Phi_v$; and $I_h/I_v = 1$, $\Phi_h = \Phi_v \pm \pi/2$, respectively.

The consequence for the radiative transfer equation (13) is that all radiance terms become (Stokes) vectors, scattering and extinction coefficients become matrices, while the scattering phase function also becomes a matrix:

$$\pm\mu\frac{d\mathbf{L}(\delta, \pm\mu)}{d\delta} = \mathbf{L}(\delta, \pm\mu) - \frac{\omega_0}{2}\int_{-1}^{1}\mathbf{I}(\delta, \mu')\mathbf{M}(\pm\mu, \mu')\;d\mu' - (1-\omega_0)\mathbf{B}(\mathbf{T}) \tag{17}$$

$\mathbf{L}$ is the Stokes vector representation of the scalar intensity, $\mathbf{B}$ is still scalar because blackbody emission is unpolarized ($\mathbf{B} = (B, 0, 0, 0)$), $\mathbf{M}$ denotes the 4×4 scattering matrix (Mueller matrix), and ω_0 has also become a 4×4 matrix determining the amount of scattering per component. The latter expansion is only required if scattering by non-spherical particles is included, otherwise ω_0 remains a scalar.

Since scattering is always described in the local scattering geometry (as is the polarization), i.e., in reference to the plane determined by the incoming and scattered radiation beams with a particle at the center, a coordinate transformation has to be carried out before and after the scattering event with respect to $(\mu, \phi; \mu', \phi')$:

$$\mathbf{M}(\mu, \phi; \mu', \phi') = \mathbf{F}(i_2 - \pi)\mathbf{P}(\cos\Theta)\mathbf{F}(i_1) \tag{18}$$

$\mathbf{F}$ represents a rotation matrix:

$$\mathbf{F}(i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2i & -\sin 2i & 0 \\ 0 & \sin 2i & \cos 2i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The most general form of the scattering matrix $\mathbf{P}$ is:

$$\mathbf{P}(\cos\Theta) = \begin{bmatrix} \frac{M_2 + M_3 + M_4 + M_1}{2} & \frac{M_2 - M_3 + M_4 - M_1}{2} & S_{23} + S_{41} & -D_{23} - D_{41} \\ \frac{M_2 + M_3 - M_4 - M_1}{2} & \frac{M_2 - M_3 - M_4 + M_1}{2} & S_{23} - S_{41} & -D_{23} + D_{41} \\ S_{24} + S_{31} & S_{24} - S_{31} & S_{21} - S_{34} & -D_{21} + D_{34} \\ D_{24} + D_{31} & D_{24} - D_{31} & D_{21} + D_{34} & S_{21} - S_{34} \end{bmatrix} \tag{20}$$

which can be reduced to a more simple form for spherical particles:

$$\mathbf{P}(\cos\Theta) = \begin{bmatrix} \frac{M_2 + M_1}{2} & \frac{M_2 - M_1}{2} & 0 & 0 \\ \frac{M_2 - M_1}{2} & \frac{M_2 + M_1}{2} & 0 & 0 \\ 0 & 0 & S_{21} & -D_{21} \\ 0 & 0 & D_{21} & S_{21} \end{bmatrix} \tag{21}$$

Other simplifications apply for particles with less general symmetry than spheres.

1.4 Boundaries

Downwelling radiation from space is commonly approximated by B (2.7 K) for all incidence angles. Surfaces can be treated as reflectors which are specular for e.g. a calm water surface. In that case, the Fresnel equations apply which translate to a reflection matrix:

$$\mathbf{R}_0(\mu, \mu') = \begin{bmatrix} (|r_v|^2 + |r_h|^2)/2 & (|r_v|^2 - |r_h|^2)/2 & 0 & 0 \\ (|r_v|^2 - |r_h|^2)/2 & (|r_v|^2 + |r_h|^2)/2 & 0 & 0 \\ 0 & 0 & \mathrm{Re}(r_v r_h^*) & -\mathrm{Im}(r_v r_h^*) \\ 0 & 0 & \mathrm{Im}(r_v r_h^*) & \mathrm{Re}(r_v r_h^*) \end{bmatrix} \tag{22}$$

with reflection coefficients for a medium with complex permittivity $\varepsilon = \varepsilon' - i\varepsilon''$:

$$\begin{aligned} r_h(\mu) &= \frac{(p-\mu)^2 + q^2}{(p+\mu)^2 + q^2} \\ r_v(\mu) &= \frac{(\varepsilon'\mu - p)^2 + (\varepsilon''\mu - q)^2}{(\varepsilon'\mu + p)^2 + (\varepsilon''\mu + q)^2} \\ p &= \frac{1}{\sqrt{2}}\{[(\varepsilon' - \mu - 1)^2 + \varepsilon''^2]^{1/2} + (\varepsilon' - \mu - 1)^2\} \\ q &= \frac{1}{\sqrt{2}}\{[(\varepsilon' - \mu - 1)^2 + \varepsilon''^2]^{1/2} - (\varepsilon' - \mu - 1)^2\} \end{aligned} \tag{23}$$

Surface transmissivity is neglected in most cases assuming a penetration depth $d_p = \sqrt{\varepsilon'}\lambda/(2\pi\varepsilon'')$ of zero so that $\varepsilon_0 = 1 - \mathbf{R}$ becomes a matrix in (5). Most natural surfaces, however, are rough so that—at least theoretically—bistatic reflection coefficients have to be calculated giving the fraction of scattered radiation for any incidence and scattering angle combination. Another approximation to surface reflection is represented by a Lambertian reflector for which the distribution of reflected radiation is isotropic over all angles.

1.5 Antenna patterns

Microwave antennas on current satellites represent a compromise between desired spatial resolution at the surface and affordable antenna size. Since measurements at all frequencies are usually obtained with the same antenna, spatial resolution varies with frequency. Spatial resolution of the instantaneous field of view, *IFOV*, can roughly be estimated from (here at nadir):

$$IFOV(\mu = 0) = \frac{ah\lambda}{d} \tag{24}$$

with satellite altitude h, antenna diameter d, an antenna efficiency factor a (e.g. = 1.5) and wavelength λ. This increases for inclined observation geometry and distorts circular antenna patterns to ellipsoids. Since the antenna size is more or less of the dimension of several wavelengths, diffraction is important and leads to interference patterns. Most antennas have very efficient main lobes but side lobe effects are mostly non-negligible over scenes with strong horizontal gradients of radiance emission / scattering (near coastlines and over clouds and precipitation). An idealization of the antenna imaging is represented by the approximation of the main lobe by a Gauss function with a halfwidth according to the nominal 3 dB beamwidth given in technical documents (e.g. 40 km x 60 km for the SSM/I 19.35 GHz channel). Thus a spatially inhomogeneous radiance field has to be integrated over the radiometer field of view with the antenna pattern $G(\theta, \phi)$:

$$\hat{\mathbf{L}}(\delta = 0, \theta, \phi) = \frac{\int_\Omega \mathbf{L}(\delta = 0, \mu; \theta', \phi') G(\theta', \phi') \sin\theta' d\theta' d\phi'}{\int_\Omega G(\theta', \phi') \sin\theta' d\theta' d\phi'} \tag{25}$$

The angle Ω represents the effective field of view (EFOV) in coordinates of zenith and azimuth angles and is the instantaneous field of view including its distortion by the scanning motion of the antenna.

2. RADIATIVE TRANSFER MODELS

2.1 Principles

The major problem for radiative transfer modeling is the solution of (3) with (5) because layer interaction can not be solved analytically in the case of scattering. Here, simplifications and numerical approaches are required. So far, the radiative transfer equation was expressed by scalar radiances or Stokes vectors in dependence of e.g. zenith angle. Depending on the application and the required accuracy (in particular when multiple scattering is included), even plane-parallel radiative transfer—which assumes constant medium properties in horizontal directions - requires the integration of scattered radiances from the upper and lower half-space into the observation direction. This already appeared in (3).

The angle dimension is treated in the same way as the elements of the Stokes vector or the Mueller matrix so that the dimension of the vectors / matrices does not change. Then:

$$\mathbf{L} = \begin{bmatrix} \hat{\mathbf{L}}(\mu_1) \\ \dots \\ \hat{\mathbf{L}}(\mu_n) \end{bmatrix} \qquad \hat{\mathbf{L}}(\mu) = \begin{bmatrix} I(\mu) \\ Q(\mu) \\ U(\mu) \\ V(\mu) \end{bmatrix}$$
$$\mathbf{P}(\mu, \mu') = \begin{bmatrix} \mathbf{P}(\mu_1, \mu_1) & \dots & \mathbf{P}(\mu_n, \mu_1) \\ \dots & \dots & \dots \\ \mathbf{P}(\mu_1, \mu_n) & \dots & \mathbf{P}(\mu_n, \mu_n) \end{bmatrix} \tag{26}$$

This aims already at the numerical integration of incoming and scattered radiances into the observation direction by a simple matrix-vector multiplication. To replace the integration by a summation, the Gaussian quadrature formula provides an accurate means for integration in the interval of [-1, 1]:

$$\int_{-1}^{1} f(\mu) \mathrm{d}\mu \approx \sum_{j=-m}^{m} a_j f(\mu_j) \tag{27}$$

At microwave wavelengths, usually less than 16 discrete angles provide enough accuracy, at infrared / visible wavelength a higher number is required to resolve the stronger anisotropy of the radiance fields.

As mentioned above, the azimuthal dependence is mostly neglected due to the lesser degree of anisotropy. If included, it involves a separation of angle dependencies for the phase function and intensities:

$$P(\mu, \phi; \mu', \phi') = \sum_{m=0}^{N} \sum_{l=m}^{N} \tilde{\omega}_l P_l^m(\mu) P_l^m(\mu') \cos(\phi' - \phi)$$
$$L(\delta, \mu, \phi) = \sum_{m=0}^{N} L^m(\delta, \mu) \cos(\phi' - \phi) \tag{28}$$

where the P_l^m denote the associated Legendre polynominals and:

$$\begin{aligned}\tilde{\omega}_l^m &= (2-\delta_{0,m})\tilde{\omega}_l\frac{(l-m)!}{(l+m)!}, \qquad l = m, \ldots, N, \quad 0 \le m \le N \\ \delta_{0,m} &= \begin{cases} 1 & m = 0 \\ 0 & \text{otherwise} \end{cases}\end{aligned} \tag{29}$$

Eqs. (2) and (3) would be formulated for each mode I^m and summed according to (28) after integration in the μ-dimension.

The equations for the differential radiative transfer have to be formulated for discrete layers assuming that the optical properties do not change through the layer (following the structure of the input data or other checks). Most of the time the layer temperature is taken to be the average between the bounding levels; however, for optically thick media this assumption produces uncertainties (see below).

The basic formulation originates from the interaction principle, i.e., the linear superposition of contributions from adjacent layers:

$$\begin{aligned}\mathbf{L}_1^- &= \mathbf{T}^-\mathbf{L}_0^- + \mathbf{R}^- L_1^+ + \mathbf{S}^- \\ \mathbf{L}_0^+ &= \mathbf{T}^+\mathbf{L}_1^+ + \mathbf{R}^+ L_0^- + \mathbf{S}^+\end{aligned} \tag{30}$$

where '+' and '–' denote the upwelling and downwelling radiances, respectively, and $\mathbf{T}$, $\mathbf{R}$, and $\mathbf{S}$ denote the transmission, reflection, and emission operators. At bottom and top of the atmosphere, $L_0^-(\delta = 0) = B(2.7\mathrm{K})$ and at the surface $\mathbf{T}^- = 0$, $\mathbf{R}^-$ is taken from (22) while $\mathbf{S}^+ = \varepsilon_0[B(T_0, \mu_1), 0, 0, 0; \ldots; B(T_0, \mu_n), 0, 0, 0]$.

2.2 Emission

In the case of pure emission all reflection matrices are zero so that (neglecting polarization at this point):

$$\begin{aligned}\mathbf{L}_1^- &= \mathbf{T}^-\mathbf{L}_0^- + \mathbf{S}^- \\ \mathbf{L}_0^+ &= \mathbf{T}^+\mathbf{L}_1^+ + \mathbf{S}^+\end{aligned} \tag{31}$$

with:

$$\begin{aligned}\mathbf{T}^\pm &= \begin{bmatrix} \exp(\Delta\delta/\pm\mu_1) & \ldots & 0 \\ 0 & \ldots & 0 \\ 0 & \ldots & \exp(\Delta\delta/\pm\mu_n) \end{bmatrix} \\ \mathbf{S}^\pm &= \begin{bmatrix} [1-\exp(\Delta\delta/\pm\mu_1)]B(T) \\ \ldots \\ [1-\exp(\Delta\delta/\pm\mu_n)]B(T) \end{bmatrix}\end{aligned} \tag{32}$$

For optically thin layers $1-\exp(\Delta\delta/\pm\mu_i) \approx \Delta\delta/\pm\mu_i$. This can be further simplified if only one zenith angle is used, so that if only the first two elements of the Stokes vector are needed (e.g. to include polarization introduced by surface reflection) (31) has only two elements. This is the form used in current numerical prediction models.

2.3 Scattering—doubling/adding

In the case of scattering atmospheres, the reflection and transmission matrices include terms of the scattering phase function. The numerical procedure called ‘doubling / adding’ treats the multiple scattering in layers and the integration of radiances throughout the atmosphere along the following line:

1) Starting point is a layer with constant optical properties in the atmosphere (say the upmost layer). This layer has to be divided into sublayers for which single scattering can be assumed. This is usually the case for optical depths $< 10^{-5}$. For a sublayer initial reflection and transmission matrices are calculated:

$$
\begin{aligned}
\mathbf{R} &= \frac{\omega_0}{2}\mathbf{M}^{-1}\mathbf{P}^{+-}\mathbf{A}\Delta\delta \\
\mathbf{T} &= \mathbf{E} - \mathbf{M}^{-1}\Delta\delta + \frac{\omega_0}{2}\mathbf{M}^{-1}\mathbf{P}^{++}\mathbf{A}\Delta\delta \\
\mathbf{M} &= \begin{bmatrix} \mu_1 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & \mu_n \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} a_1 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & a_n \end{bmatrix}
\end{aligned}
\tag{33}
$$

where the a_i are the weighting coefficients of the Gauss quadrature as in (27). The notation of $\mathbf{P}^{\pm\pm}$ represents the phase function at discrete angles μ_i where the first sign stands for the direction of the incoming radiance and the second for the scattered radiance. Positive signs refer to the lower half space while negative signs refer to the upper half space. The initial operators are independent of viewing direction so that an exchange of viewing direction has no effect. The source term vectors remain as in (32). There are various initializations available which produce less different results in the microwave region than at shorter wavelengths.

2) If two adjacent layers are added (or doubled in the case of their optical properties being identical) the exiting radiances at top and bottom are:

$$
\begin{aligned}
\mathbf{L}_2^- &= \mathbf{T}_{12}\mathbf{L}_1^- + \mathbf{R}_{21}L_2^+ + \mathbf{S}_{12} \\
\mathbf{L}_0^+ &= \mathbf{T}_{10}\mathbf{L}_1^+ + \mathbf{R}_{01}L_0^- + \mathbf{S}_{10}
\end{aligned}
\tag{34}
$$

and those at the layer interface are:

$$
\begin{aligned}
\mathbf{L}_1^- &= \mathbf{T}_{01}\mathbf{L}_0^- + \mathbf{R}_{10}L_1^+ + \mathbf{S}_{01} \\
\mathbf{L}_1^+ &= \mathbf{T}_{21}\mathbf{L}_2^+ + \mathbf{R}_{12}L_1^- + \mathbf{S}_{10}
\end{aligned}
\tag{35}
$$

$\mathbf{L}_2^-$ and $\mathbf{L}_0^+$ can also be formulated using combined layer operators:

$$
\begin{aligned}
\mathbf{L}_2^- &= \mathbf{T}_{02}\mathbf{L}_0^- + \mathbf{R}_{20}L_2^+ + \mathbf{S}_{02} \\
\mathbf{L}_0^+ &= \mathbf{T}_{20}\mathbf{L}_2^+ + \mathbf{R}_{02}L_0^- + \mathbf{S}_{20}
\end{aligned}
\tag{36}
$$

By inserting (35) in (34) and rearrangement, the combined operators from (36) can be obtained from:

$$\begin{aligned} \mathbf{R}_{20} &= \mathbf{R}_{21} + \mathbf{T}_{12}\Gamma^{-}\mathbf{R}_{10}\mathbf{T}_{21} \\ \mathbf{T}_{02} &= \mathbf{T}_{12}\Gamma^{-}\mathbf{T}_{01} \\ \mathbf{S}_{02} &= \mathbf{S}_{12} + \mathbf{T}_{12}\Gamma^{-}(\mathbf{S}_{01} + \mathbf{R}_{10}\mathbf{S}_{21}) \\ \Gamma^{-} &= [\mathbf{E} - \mathbf{R}_{10}\mathbf{R}_{12}]^{-1} = \sum_{i=0}^{\infty}(\mathbf{R}_{10}\mathbf{R}_{12})^{i} \end{aligned} \tag{37}$$

and

$$\begin{aligned} \mathbf{R}_{02} &= \mathbf{R}_{01} + \mathbf{T}_{10}\Gamma^{+}\mathbf{R}_{12}\mathbf{T}_{01} \\ \mathbf{T}_{20} &= \mathbf{T}_{10}\Gamma^{+}\mathbf{T}_{21} \\ \mathbf{S}_{20} &= \mathbf{S}_{10} + \mathbf{T}_{10}\Gamma^{+}(\mathbf{S}_{21} + \mathbf{R}_{12}\mathbf{S}_{01}) \\ \Gamma^{+} &= [\mathbf{E} - \mathbf{R}_{12}\mathbf{R}_{10}]^{-1} = \sum_{i=0}^{\infty}(\mathbf{R}_{12}\mathbf{R}_{10})^{i} \end{aligned} \tag{38}$$

The factors $\Gamma^{\pm}$ can be interpreted as a multiple reflection / scattering factor.

3) This is carried out successively until the full layer operators are calculated. The sublayer combined operators are computed for optical depths which double the optical depth of the previous layers (starting with the initial layer with $\Delta\delta < 10^{-5}$), thus this procedure is called ‘Doubling’. All layers of the atmosphere can be treated in the same fashion. As mentioned above, for optically thick media the assumption of an average temperature throughout the doubling process produces errors. For a temperature that changes linearly with optical depth, the adaptation of $B(T) = B_0 + B_1\delta'$ with actual optical depth δ' at each doubling step gives much better results.

4) Layers with different optical properties for which the doubling procedure was carried out previously are then combined using the same formulae thus this step is called ‘Adding’.

2.4 Scattering—Approximations

2.4 (a) Eddington approximation. The Eddington approximation to radiative transfer represents an example for an approximative method. The approximation lies in the development of the radiance vector and phase function to the first order so that only one angle (i.e. the observation angle) is needed and the anisotropic radiance field is decomposed into an isotropic and anisotropic component, respectively:

$$\begin{aligned} L(\delta,\mu) &= L_0(\delta) + \mu L_1(\delta) \\ P(\cos\theta) &= 1 - 3g\cos\theta \end{aligned} \tag{39}$$

so that the source function translates to:

$$J(\delta,\mu) = [1 - \omega_0(\delta)]B[T(\delta)] + \omega_0(\delta)[L_0(\delta) + g(\delta)\mu L_1(\delta)] \tag{40}$$

for azimuthally averaged fields. g is the asymmetry parameter representing an average scattering angle calculated from the phase function averaged over the full hemisphere.

If these quantities are inserted into (3), two mixed equations are obtained:

$$\frac{\mathrm{d}L_0(\delta)}{\mathrm{d}\delta} = -[1-\omega_0(\delta)g(\delta)]L_1(\delta)$$
$$\frac{\mathrm{d}L_1(\delta)}{\mathrm{d}\delta} = -3[1-\omega_0(\delta)]\{L_0(\delta)-B[T(\delta)]\} \tag{41}$$

The second derivative of e.g. L_0 provides:

$$\frac{\mathrm{d}^2L_0(\delta)}{\mathrm{d}\delta^2} = \Lambda^2(\delta)\{L_0(\delta)-B[T(\delta)]\} \tag{42}$$

$$\Lambda^2(\delta) = 3[1-\omega_0(\delta)][1-\omega_0(\delta)g(\delta)] \tag{43}$$

with the general solution:

$$L_0(\delta) = D_+\exp(\Lambda\delta) + D_-\exp(-\Lambda\delta) + B[T_0] + B_1\mathrm{d}\delta \tag{44}$$

Again, a linear dependence of temperature with optical depth is assumed.

The coefficients $D_\pm$ are to be obtained from the boundary conditions, i.e., space background radiation, surface emission and reflection as well as with the requirement of flux continuity at the layer boundaries:

$$\left(L_0 - \frac{\partial L_0}{h\partial\delta}\right)_{\delta=0} = 2.7 \tag{45}$$

$$\left(L_0 + \frac{\partial L_0}{h\partial\delta}\right)_{\delta=\delta^*} = \varepsilon B(T) + (1-\varepsilon)\left(L_0 - \frac{\partial L_0}{h\partial\delta}\right)_{\delta=\delta^*}$$
$$\left(L_0 + \frac{\partial L_0}{h\partial\delta}\right)^j_{\delta=\delta_i} = \left(L_0 + \frac{\partial L_0}{h\partial\delta}\right)^{j+1}_{\delta=\delta_i} \tag{46}$$

where $h = 1.5(1-\omega_0 g)$ and *i* denotes the *i*-th layer interface between *j*-th and (*j* + 1)-th layer.

The Delta-approximation modifies k, ω_0, and g as a consequence of the approximation of the fractional forward peak of the phase function by a delta-function:

$$g' = \frac{g}{1+g}, \qquad \omega_0' = \frac{(1-g)^2\omega_0}{1-g^2\omega_0}, \qquad \delta' = (1-g^2\omega_0)\delta \tag{47}$$

which has proven to significantly improve the treatment of radiative transfer in two-stream-type models in strongly scattering media. Another modification can be introduced to adapt this model to three-dimensional problems. In that case the upward and downward directed radiances are calculated along the slant path of satellite observations providing a first order approximation to three-dimensional radiative transfer. For media with significant scattering, however, large contributions to the observation originate from outside this beam, i.e., from a larger volume than represented by a single path, so that the accuracy of this approach has limitations.

2.4 (b) Two-stream approximation. In the two-stream approximation, (27) is only applied for $m = \pm 1$, i.e., at $\mu = \pm 1/\sqrt{3} = \pm 0.57735$ or $\theta = \pm 54.7°$. Then (2) is written:

$$\pm\mu\frac{dL(\delta,\pm\mu)}{d\delta} = L(\delta,\pm\mu) \\ -\omega_0(1-b)L(\delta,\pm\mu_1)-\omega_0 bL(\delta,\mp\mu_1) \\ -(1-\omega_0)B(T) \tag{48}$$

where $b = (1-g)/2$. Again, a system of differential equations can be set up and solved by use of boundary conditions.

2.5 Scattering—Others

There are other ways to approach the multiple scattering problem such as the Discrete Ordinate Method, the Successive Order of Scattering Method or Monte Carlo techniques. The latter are widely used for the simulation of microwave radiative transfer in three-dimensional rainclouds where significant scattering occurs and represent the most accurate even though most computationally expensive methods. Some efficiency is gained if the scattering paths of photons are treated in the backward direction. Then, only those photons actually reaching the radiometer (here taken as the starting point of the calculation) are used. That way the total number of required photons to represent three-dimensional radiance fields is optimized.

An interesting approach is the Successive Order of Scattering technique because it allows the estimation of the order of multiple scattering required to accurately determine the radiance fields. Here, the radiative transfer equation is calculated for each scattering order individually. These terms have to be eventually added to obtain the total diffuse radiance:

$$L(\delta,\pm\mu) \geq \sum_{i=0}^{n} L_i(\delta,\pm\mu) \tag{49}$$

In the case of visible (or radar) radiative transfer modeling, a direct, forward propagated radiance has to be taken into account. The recursive formulae for the radiances (here for a finite layer with optical depth δ and in upward direction) and source terms are:

$$L_n(\delta=0,\mu) = \int_{\delta}^{0} J_n(\delta',\mu)\exp(-\delta/\mu)\, d\delta'/\mu$$

$$J_n(\delta',\mu) = \frac{\omega_0}{2}\int_{-1}^{1} L_{n-1}(\delta',\mu')P(\mu,\mu')\, d\mu' + \delta_{1n}(1-\omega_0)B(T)$$

with $\delta_{nm} = 1$ for $n = m$ and $\delta_{nm} = 0$ for $n \neq m$. The term $(1-\omega_0)B(T)$ represents the internal emission source which is only considered once (for calculations in the visible wavelength range, this would be replaced by the solar source term). With no incident radiances, $L_0 = 0$.

Assuming that temperature and single scattering albedo are constant in the layer, the first and subsequent order radiances can be expressed as:

$$L_1(\delta=0,\mu) = (1-\omega_0)B(T)[1-\exp(-\delta/\mu)\,] \\ L_n(\delta=0,\mu) = L_1(\delta=0,\mu)\omega_0[1-\exp(-\delta/\mu)\,]^{n-1} \tag{50}$$

Thus, by applying the summation in (49) with $\omega_0/\eta = \omega_0[1-\exp(-\delta/\mu)\,]$:

$$L(\delta = 0, \mu) = \frac{L_1(\delta = 0, \mu)}{1 - \omega_0\eta} \approx L_1(\delta = 0, \mu)\frac{(\omega_0\eta)^n - 1}{\omega_0\eta - 1} \tag{51}$$

The choice of summation limit n has to ensure convergence of (49), that is the summation on the right hand side of (51) has to be accurate within predefined error limits. This is the case, for example, if $\exp(-\delta/\mu) = 0.9$, $\omega_0 = 0.3 : n = 2$, or if $\exp(-\delta/\mu) = 0.3$, $\omega_0 = 0.9 : n = 13$, choosing an error limit of 0.1 K.

3. Atmospheric absorption

At microwave wavelengths, the main atmospheric absorbers are oxygen (O_2), through rotational line absorption between 50–60 GHz as well as 118 GHz; water vapour (H_2O), through rotational line absorption at 22.235 GHz and 183.31 GHz; and continuum absorption by water vapour and dry air. Between 1–200 GHz, stratospheric contributions are less important so that the total atmospheric absorption can be computed from profiles of temperature, pressure, and humidity. Models for the simulation of atmospheric absorption are based on laboratory and field measurements under various environmental conditions and have to cover the line intensity, width, and overlap for line absorption as a function of temperature, pressure, and humidity. There are several databases of these measurements available as well as so-called 'line-by-line' models which include contributions from all known absorption lines at the desired frequency. As an example, the Millimeter Propagation Model (MPM) computes local absorption, k_a (in km^{-1}), in terms of a complex atmospheric refractive index $m_a = m'_a - im''_a$ at frequency ν in GHz:

$$\begin{aligned} m_a &= m_0 + m'(\nu) + im''(\nu) \\ k_a &= 0.04197\nu m''(\nu) \end{aligned} \tag{52}$$

The above mentioned contributions are summed up in the imaginary part, $m''(\nu)$ over all absorption lines (in the case of MPM, 44 oxygen and 30 water vapour lines).

The largest unknown in this frequency range is the water vapour continuum which is rather a correction of a line absorption model. Since these fits are aimed at high accuracy at frequencies near the absorption lines, there may be larger uncertainties in the window regions where the continuum contribution dominates. Another problem is the compatibility of laboratory and field measurements when synthesized in unified models due to representativity of measurements and technical constraints.

4. Surface emission/reflection

4.1 Ocean

For a flat sea surface, the emission and reflection could be calculated from the permittivity of water as in (56) and from the Fresnel equations (22) and (23). Surface roughness in the case of non-zero windspeeds as well as foam generation contribute significantly to the modification of the equations. These are treated in different ways. For large gravity waves, the waves are approximated by probability distributions of facet orientations with respect to the viewing angle. For each facet, the Fresnel equations are applied. The distributions depend on wind speed and direction. Facet contributions are integrated to yield the total emission / reflection. Two-scale models also account for capillary waves and small gravity waves which are superimposed on the large gravity waves. Both components are treated independently where the latter maybe obtained from perturbation theory. Important is the choice of the cut-off frequency to distinguish between the two states. Foam contributions mainly increase ocean emissivity and are parameterized by their fractional coverage and the permittivity of the foam itself. Salinity plays only a role at frequencies below 5 GHz. Depending on the requirements of the atmospheric component of the radiative transfer model, either zenith angle dependent emissivities (as in (23)) or bistatic reflection coefficients have to be calculated:

$$\varepsilon_0 = 1 - \frac{1}{4\pi\mu}\int_0^{2\pi}\int_0^{\pi/2}\gamma_0(p;\mu,\phi;\mu',\phi')\,\mathrm{d}\Omega \tag{53}$$

The $\gamma_0(p)$ and $\gamma_0(q)$ represent the co- and cross-polarization contributions with respect to polarizations p and q:

$$\begin{aligned}\gamma_0(p;\mu,\phi;\mu',\phi') &= \gamma_0(pp;\mu,\phi;\mu',\phi') + \gamma_0(qp;\mu,\phi;\mu',\phi')\\ \gamma_0(q;\mu,\phi;\mu',\phi') &= \gamma_0(qq;\mu,\phi;\mu',\phi') + \gamma_0(pq;\mu,\phi;\mu',\phi')\end{aligned} \tag{54}$$

$$\gamma_0(pq;\mu,\phi;\mu',\phi') = \frac{(kQ|U_{pq}|)^2}{2Q_x^4\sigma_c\sigma_u}\exp\left[-\frac{1}{2Q^2}\left(\frac{Q_x^2}{\sigma_c^2}+\frac{Q_y^2}{\sigma_u^2}\right)\right] \tag{55}$$

where $k = 1/\lambda$ denotes the wavenumber, $\mathbf{Q} = (Q_x, Q_y Q_z)$ is the phase vector, U_{pq} is the scattered elctric field with polarization state pq, and σ_c, σ_u are the standard deviations of facet slopes orthogonal and parallel to wind direction. The latter are obtained from the above mentioned slope distributions as a function of surface stress. Important to notice is that multiple reflection and shadowing of facets at larger roughnesses may be significant.

4.2 Land

Over land surfaces, the main contributions to be accounted for are those from soil, vegetation and snow. The radiative transfer can be treated by regarding the land surface as a discrete boundary using the definition of bistatic reflection coefficients as in (53). Depending on penetration depth and targeted media, a full radiative transfer model with a layered structure, as already introduced for the atmosphere, is used. Vegetation, soil, or snow are then treated in layers of constant optical properties and methods such as the adding / doubling technique carry out the vertical and angular integration. Important, however, is the applicability of the far-field approximation in scattering media such as snow or vegetation. Here, radiative transfer theory for dense media are superior.

The determination of the optical properties at microwave wavelengths of soil, snow, and vegetation is difficult because the number of contributing parameters is much higher. For soil, mainly soil water content and texture as well as roughness drive its permittivity. For snow, permittivity is rather accurately known but snow depth, water content, and grain sizes are problematic. In the microwave spectral region, vegetation water content and vegetation coverage are of importance. Scattering by branches and leaves is small but generally non-negligible.

5. Hydrometeors

5.1 Permittivity

For conducting materials the complex permittivity, $\varepsilon = \varepsilon' + i\varepsilon''$, determines the effect of an external dielectric field on the internal distribution of charges (over time). It is connected to the complex refractive index of that material, $m = m' - im''$, through $m^2 = \varepsilon$ thus $\varepsilon' = m'^2 - m''^2$, $\varepsilon'' = 2m'm''$. Away from the relaxation frequency and for a medium in which friction effects dominate its polarizabilitythe Debye–equations apply:

$$\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2\tau_e^2}, \qquad \varepsilon'' = \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau_e}{1 + \omega^2\tau_e^2} + \frac{\sigma}{\omega\varepsilon_0} \tag{56}$$

with $\omega = 2\pi\nu$ and frequency ν, and where τ_e denotes the effective relaxation time, ε_s the static permittivity ($\nu \to 0$), ε_∞ the high-frequency permittivity ($\nu \to \infty$), ε_0 the permittivity in vacuum and σ the ionic conductivity, respectively. The main effect of ε' is the dispersion of the phase delay induced on an electromagnetic wave passing through a medium, while ε'' represents the loss of energy.

The available models for the permittivities of water and ice present parameterizations of the above equations with respect to the dependence of their ingredients on temperature and frequency. Another influence is the effect of soluble materials such as salt as expressed by the second term in (56) for ε'' which is only important at frequencies below 5 GHz.

5.2 Cloud absorption and scattering

The parameterization of the microwave properties of water and ice clouds is based upon two major assumptions: (1) maximum particle size is well below the wavelength thus the Rayleigh approximation to particle scattering and absorption applies and the shape of the size distribution has negligible effects on the derived properties; (2) scattering is negligible thus clouds are treated as pure emitters and transmission, τ, depends only on cloud absorption, thus $\tau = \exp(-\beta_a \Delta z \cos^{-1}\Theta)$ with absorption coefficient β_a, layer depth Δz, and zenith angle Θ.

For a monodisperse particle distribution, the liquid water content, w, is calculated from:

$$w = \rho_w \frac{4}{3}\pi \int_0^\infty r'^3 n(r')\,dr' = \rho_w \frac{4}{3}\pi r^3 N \tag{57}$$

where ρ_w denotes water density, r particle radius, and N total particle number obtained from particle number density, $n(r)$:

$$N = \int_0^\infty n(r')\,dr' \tag{58}$$

The absorption and scattering coefficients, $k_{abs,\,sct}$ are obtained from the integration of particle absorption and scattering cross sections, $Q_{abs,\,sct}$, over the same distribution:

$$k_{abs,\,sct} = \pi \int_0^\infty Q_{abs,\,sct}(r') r'^2 n(r')\,dr' = \pi r^2 N Q_{abs,\,sct}(r) = \frac{3w}{\rho_w 4r} Q_{abs,\,sct} \tag{59}$$

If the Rayleigh approximation holds the cross sections are given by:

$$Q_{abs} = -4x\,\mathrm{Im}\left[\frac{m^2-1}{m^2+2}\right], \qquad Q_{sct} = \frac{8}{3}x^4 \left|\frac{m^2-1}{m^2+2}\right|^2 \tag{60}$$

which are the first terms of the series expansion used in Mie calculations. $x = 2\pi r/\lambda$ denotes the size parameterwith wavelength $\lambda = c/\nu$, speed of light in vacuum c, and frequency ν.

After some manipulation, (60) for the absorption coefficient is simplified to:

$$k_{\mathrm{abs}} = A\left[-\mathrm{Im}\left(\frac{m^2-1}{m^2+2}\right)\right] \quad (61)$$

with $A = 0.06283813$ m²kg⁻¹GHz⁻¹.

With $m^2 = \varepsilon$, (61) is transferred to:

$$k_{\mathrm{abs}} = B\frac{w\nu\varepsilon''}{(\varepsilon'+2)^2+\varepsilon''^2} \quad (62)$$

and $B = 0.18851441$ m²kg⁻¹GHz⁻¹, which holds for both water and ice. Thus (62) requires liquid- or ice-water contents, frequency and temperature as predictors, as well as one of the above models, for the complex permittivity of water and ice as a function of frequency and temperature.

5.3 Precipitation

For precipitation, scattering can only be neglected at very low frequencies and rainrates. Eq. (59) can not be approximated so that scattering and extinction coefficients as well as the phase matrix elements have to be integrated over the size spectrum with cross sections computed by Mie routines for spherical particles. Usually, precipitation size distributions are given in terms of diameter, D_x, rather than radius and with exponential size distributions:

$$N_x(D_x) = N_{0(x)}\exp(-\Lambda_x D_x) \quad (63)$$

with diameters D_x and intercepts $N_{0(x)}$. The index 'x' may refer to rain (r) or snow / graupel / hail (s,g,h) particles. The slope, Λ_x, is a function of water/ice content, q_x, and particle density, ρ_x, (both in g m⁻³):

$$\Lambda_x = \left(\frac{\pi\rho_x N_{0(x)}}{q_x}\right) \quad (64)$$

The input parameters for radiative transfer computations, i.e., extinction coefficient k_{ext}, single scattering albedo ω_0, asymmetry parameter g, and backscattering coefficient k_{bs} integrated over size spectra, are given by:

$$\begin{aligned} k_{\mathrm{ext}}, \omega_0, g, k_{\mathrm{bs}}(T) &= \frac{\pi}{4}\int_0^\infty Q_{\mathrm{ext}}, \frac{Q_{\mathrm{sct}}}{Q_{\mathrm{ext}}}, \frac{\overline{\cos\theta}}{Q_{\mathrm{sct}}}, Q_{\mathrm{bs}}(D_x, T)N(D_x)D_x^2\,\mathrm{d}D_x \\ \hat{\mathbf{P}}(T) &= \frac{\pi}{4\beta_s}\int_0^\infty \mathbf{P}(D_x, T)N(D_x)D_x^2\,\mathrm{d}D_x \end{aligned} \quad (65)$$

with backscattering, scattering and extinction cross sections Q_{bs}, Q_{sct} and Q_{ext}, average scattering angle $\overline{\cos\theta}$, and temperature T (which determines particle permittivity at frequency ν).

The summation of these parameters over hydrometeor types to obtain cloud layer properties follows:

$$\begin{aligned}
k_{\text{ext}}(T, \mathbf{q}) &= \sum_j k_{\text{ext}}(T, q_j), \qquad j = r, s, g, h \\
\omega_0(T, \mathbf{q}) &= \frac{\sum_j \omega_0 k_{\text{ext}}(T, q_j)}{\sum_j k_{\text{ext}}(T, q_j)} \\
g(T, \mathbf{q}) &= \frac{\sum_j g \omega_0 k_{\text{ext}}(T, q_j)}{\sum_j \omega_0 k_{\text{ext}}(T, q_j)} \\
k_{\text{bs}}(T, \mathbf{q}) &= \frac{\sum_j k_{\text{bs}} k_{\text{ext}}(T, q_j)}{\sum_j k_{\text{ext}}(T, q_j)}
\end{aligned} \tag{66}$$

where (r, s, g, h) represent rain, snow, graupel, and hail.

6. FAST MODELS

Fast radiative transfer models are needed once large data volumes are to be treated, e.g. in numerical prediction.The RTTOV-package is a widely used optimized radiative transfer modeling software for cloudfree and cloud-covere-datmospheres applicable to all operational infrared and microwave sensor channel frequencies. There are certain ways to make models fast which—apart from efficient programming—can be summarized as follows:

- As least as possible layers and frequencies / wavelengths are taken. The latter implies that simulations representing satellite observations are monochromatic, i.e., a radiometer channel corresponds to a single wavelength (the filter function would be a delta-function). In the case of RTTOV, (7) is numerically evaluated in cloudfree situations as:

$$L = \tau_s \varepsilon_s B(T_s) + \sum_{j=1}^{N} \frac{1}{2}[B_i + B_{i-1}][\tau_{i-1} - \tau_i + (1 - \varepsilon_s)(\tau_i^* - \tau_{i-1}^*)] \tag{67}$$

  It is assumed that the effective layer temperature is the average of the temperatures at the layer boundaries which also implies that the integration of the source function is given by $B(\overline{T})(1 - \tau_j)$. τ_j denotes the layer transmittance while τ_i denotes the transmittance from the *i*-th layer to space and τ_i^* denotes the transmittance from the *i*-th layer to the surface and back to space.
- The radiative transfer is treated with the assumption of a plane-parallel atmosphere. Horizontal inhomogeneity is not included and pixels are treated independently. This assumption holds for atmospheres without or with only weak scattering and where the input fields have similar resolutions as the satellite observations to be simulated.
- The dependence of radiances on azimuth angle is neglected which is almost always true for microwaves.
- Scattering by hydrometeors is neglected thus cases where scattering is likely to occur are screened and not used. If treatment of scattering is desired, then methods based on the two-stream or Eddington approximation provide good results for wavelengths below 100 GHz.
- Atmospheric absorption is parameterized with as least predictors as possible. This is of less severity at microwave wavelengths than e.g. in the infrared spectral region because mostly continuum and rotational absorption by a small variety of gas molecules has to be modeled. The parameterizations therefore depend on the targeted frequencies, i.e., depend on the instrument to be simulated. In the RTTOV-package, layer optical depth is parameterized for each nominal channel centre frequency as the difference of the layer-to-space optical depths of adjacent layers:

$$\delta_i - \delta_{i-1} = Y_i \sum_{j=1}^{9} a_{i,j} X_{i,j} \tag{68}$$

The predictors X, Y contain linearizing functions of $\cos\theta$, and profiles of *p*, *T*, and *q* as well as their departures from reference profiles obtained from representative soundings.

- Surface reflection / emission is parameterized so that non-specular effects are included but not explicitly treated in form of bistatic reflection. Over land surfaces the number of free variables is kept small. The inclusion of non-specular reflection may be represented by usage of an effective reflection angle. This modifies (67) in the sense that $\log\tau^* = \log\tau \ \sec\theta^* / \sec\theta$.
- Cloud contributions are excluded avoiding the problem of the treatment of clear and cloudy contributions to total radiances even though non-precipitating clouds do not scatter at most microwave wavelengths in use. However, the use of (67) with a linear superposition of cloud covered and cloudfree grid portions by:

$$L = (1-C)L^{\mathrm{clr}} + CL^{\mathrm{cld}} \tag{69}$$

and the implementation of (62) by $\tau_j^{\mathrm{cld}} = \tau_j^{\mathrm{clr}} \exp(-\beta_a \Delta z_i / \mu)$ does not increase the computational effort significantly.
- If clouds or even precipitation is included, particles are assumed to be spherical and homogeneous with parameterized size distributions.
- If scattering is included and multi-stream models are employed, the number of required discrete angles is kept small.
- Polarization is assumed to be only introduced by surface reflection / emission even though also spherical particles polarize incoming unpolarized radiation.

Thus the numerically worst case is to use a three-dimensional fully polarized Monte-Carlo model for the treatment of precipitating clouds at a large number of wavelengths accounting for the antenna patterns of imaging radiometers.

7. OUTSTANDING ISSUES

In numerical prediction the accuracy and computation speed of radiative transfer simulations is of utmost importance:

- Sensors have to provide continuous data and need to be well calibrated (w.r.t. the model). Accuracy requirements depend somewhat on which parameters the sensor is most sensitive to.
- Radiative transfer models in use avoid scattering calculation and polarization is only included by surface reflection which allows the separate calculation of ‘v’ and ‘h’ contributions.
- Radiative transfer models only account for one zenith angle (= observation direction) and treat pixels independently (which are decorrelated by thinning).
- Surface emissivity models therefore have to correct radiation streams in zenith direction by effects from off-zenith directions.
- Even though atmospheric absorption is comparably simple, parameterizations of absorption avoid integration over large spectral intervals necessary for line-by-line calculations.

Near-future developments in numerical prediction should include:

- Inclusion of clouds for which scattering effects are negligible including fractional cloud cover effects.

- Better modeling of land surface emissivities because microwave data sensitive to surfaces are not used over land.
- Code unification for general radiance data assimilation which is open for current and future sensors.

Most major problems are solved in microwave radiative transfer of which some still require refinement involving considerable computational effort. The largest uncertainties arise from unknown input parameters such as cloud particle size distributions, profiles, shapes, and composition as well as surface emission. Here, mainly land surfaces are difficult because a vast amount of free parameters influence the signal on sub-pixel scales.

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