3.5: Lateral boundary conditions for regional models

AOSC614 class

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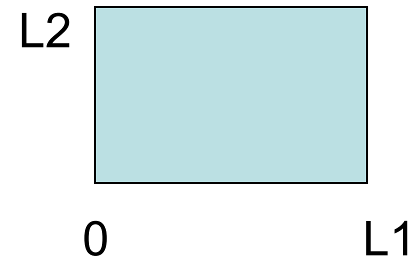
Recall: Boundary value problem

Chapter 3.1.2, page 70-71

1. Second-order Elliptic equations require one BC on each point of the spatial boundary

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

BC: u(x,0), u(x,L1), u(0,y), u(L2,y)

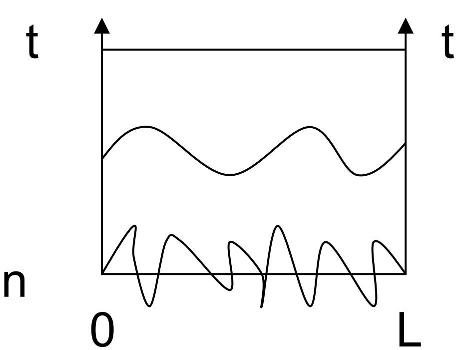


2. Parabolic (diffusion) equations require one boundary condition at every point in the boundary for each prognostic equation.

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}$$

BC: u(0,t), u(L,t)

The solution is smoothed as time goes on



3. for pure hyperbolic equations there should be as many BC imposed at a given boundary as the number of characteristics moving **into** the domain

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$

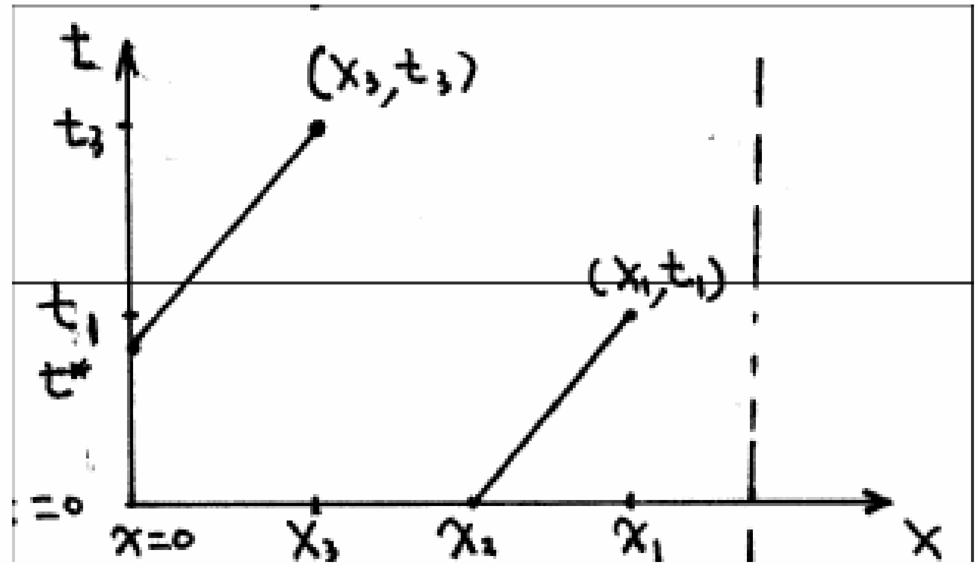
IC: u (x, 0)= f(x)

Solution: u(x,t)= f(x-ct)

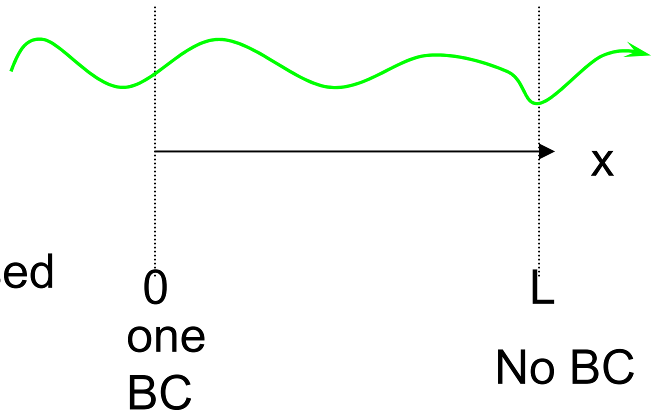
Assume c>0

Need BC at x=0: u (0, t)

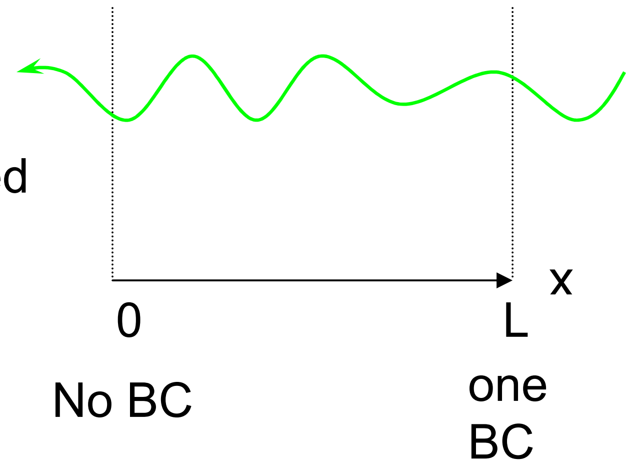
No need BC at x=L



$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$



If specify BC at x=L, cause ill-posed problem due to an overspecifying BC



If specify BC at x=0, cause ill-posed problem due to an overspecifying BC

Applications:

1, Charney (1950) barotropic vorticity equation

step 1: $$\frac{\partial \zeta}{\partial t} = -\mathbf{v} \bullet (\zeta + f)$$

Hyperbolic equations

one BC for vorticity at the inflow points

step 2: $$\nabla^2 \Psi = \zeta$$

Elliptic equations

need BC for streamfunction at all bountry points

2, SWE

$$u'_t + \bar{U}u'_x = -gh'_x$$
$$h'_t + \bar{U}h'_x = -Hu'_x$$

total number of characteristics is 2

How many characteristics moving **into** the domain?

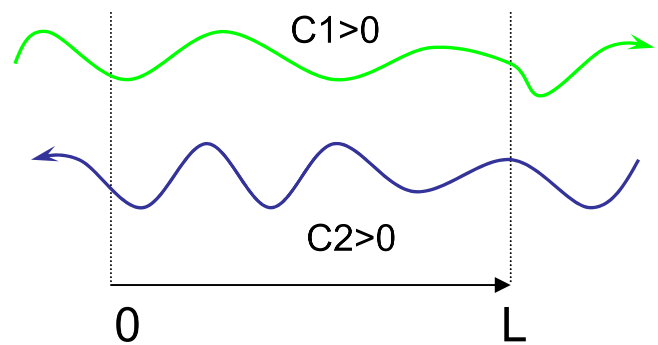
How many BC needed?

$$c_{1,2} = \overline{U} \pm (gH)^{1/2}$$

Assume $\overline{U} > 0$

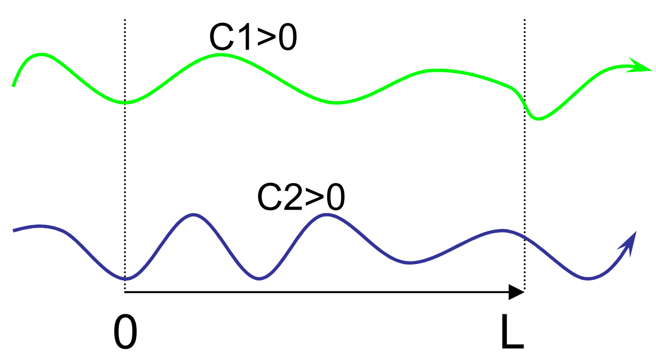
a) if $\overline{U} < \sqrt{gH}$

then c1>0, c2<0



b) if $\overline{U} > \sqrt{gH}$

then c1>0, c2>0



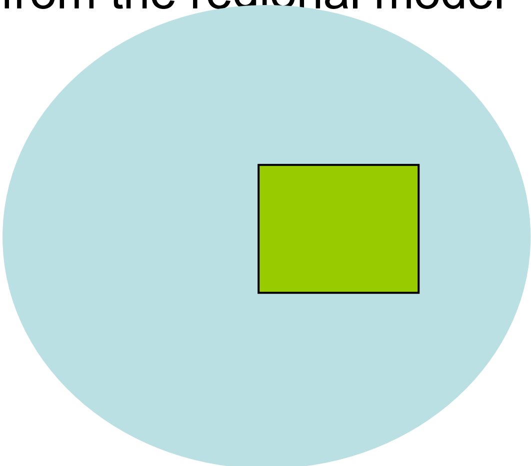
3.5.2 Lateral boundary conditions for one-way nested models

the host model, with coarser resolution, provides information about the boundary values to the nested regional model, but it is not affected by the regional model solution

Advantages:

a) it allows for independent development of the regional model

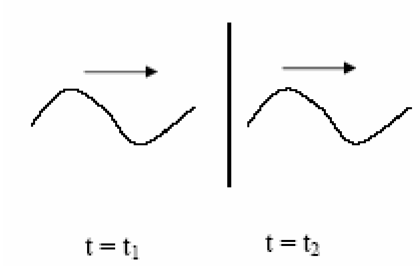
b) the host model can be run for long integrations without being "tainted" by problems associated with non-uniform resolution or from the regional model



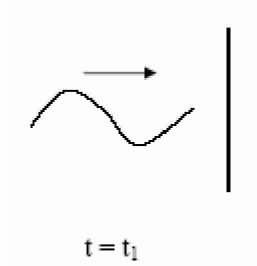
How to specify BC:

Two criteria that the boundary scheme should satisfy are:

a) it transmits incoming waves from the "host" model providing boundary information without appreciable change of phase or amplitude.



b) at the outflow boundaries, reflected waves do not reenter the domain of interest with appreciable amplitude.



Several Boundary schemes:

1. Pseudo-radiation boundary conditions:

In principle, only the subset of dynamic quantities that correspond to transfer of information into the domain should be specified at the boundary. Otherwise cause over-specification.

Therefore, we should first determine the flow moves into or out of the domain at the boundaries. If moves into, we specify the BC, if moves out, do not.

Several Boundary schemes:

1. Pseudo-radiation boundary conditions:

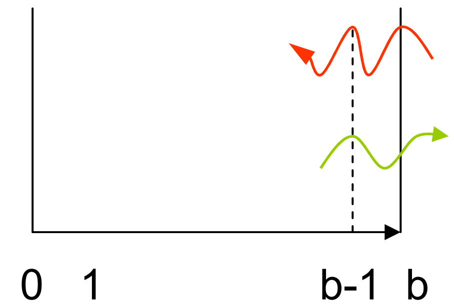
Assume the prognostic equations locally satisfy $\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$ then estimates the phase speed c at the points immediately inside the boundary

$$c' = -\frac{u_{b-1}^{n+1} - u_{b-1}^{n}}{\Delta t} \Big/ \frac{u_{b-1}^{n} - u_{b-2}^{n}}{\Delta x}$$

If c' point into the domain, need specify the boundary externally

If c' points out the domain, use upstream scheme to calculate boundary internally.

$$u_b^{n+1} = u_b^n - c'\Delta t/\Delta x(u_b^n - u_{b-1}^n)$$



Problem:

I) Estimation of c' is wrong

II) Overspecification

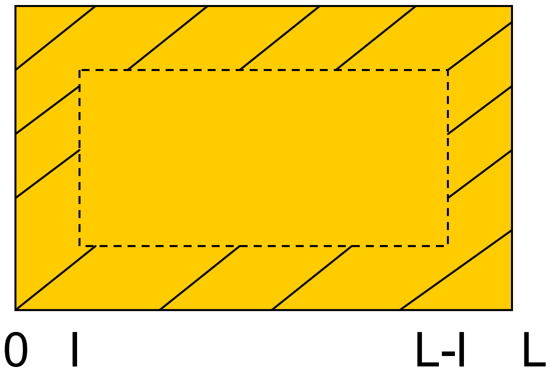
III) Values of BC are wrong

2. Boundary-zone (sponge layer) schemes

- Modify the flow-system in boundary-zone.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \Gamma_{bz}$$

- BC on both sides are specified which would overspecify the original system but is legitimate for the revised systems.



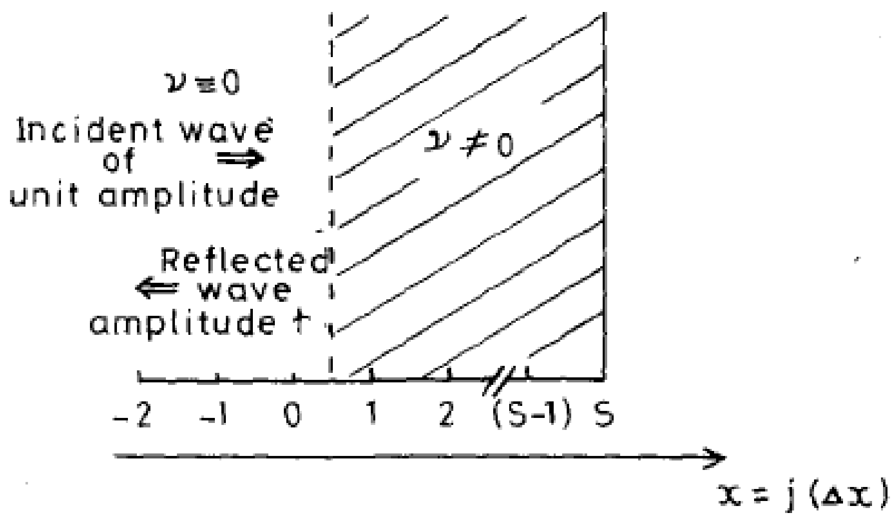
2.1) Diffusive damping scheme

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=(\frac{\partial}{\partial x}\nu\frac{\partial u}{\partial x})_{bz} \qquad \nu=\nu(x)$$

Advantage: alleviate the noise problem

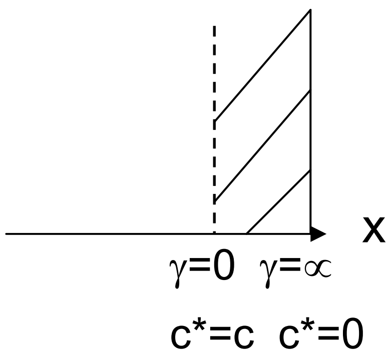
Disadvantage: 1) damps the incoming waves

2) produces spurious reflections of outgoing waves if ν increases abruptly (discontinuity of ν can act trigger a reflected wave)



2.2) Tendency modification scheme

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -\gamma \frac{\partial (u-\overline{u})}{\partial t})_{bz} \qquad \gamma = \gamma(x)$$



$$\frac{\partial \overline{u}}{\partial t} + c\frac{\partial \overline{u}}{\partial x} = 0$$ For global model

$$\frac{\partial u'}{\partial t} + c^*\frac{\partial u'}{\partial x} = 0$$ 'error'=u regional-u from global model

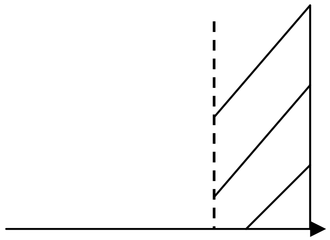
$$c^* = c/(1+\gamma)$$

Advantage: advects the error and slows it down to zero at the boundaries, avoids overspecification.

Disadvantage: produces spurious reflections of outgoing waves if γ increases abruptly

2.3) Flow relaxation scheme

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = -K(u - \bar{u})_{bz} \qquad K = K(x)$$



Force u to relax towards the externally specified field on a time scale 1/K

$$\frac{\partial \bar{u}}{\partial t} + c\frac{\partial \bar{u}}{\partial x} = 0$$ For external global flow

$$\frac{\partial u'}{\partial t} + c\frac{\partial u'}{\partial x} = -Ku'$$ 'error'=u regional-u from global model

Advantage: only damps u' without changing u, therefore

reduces the effects of overspecification at the outflow without changing inflow wave.

Disadvantage: produces spurious reflections of outgoing waves if K increases abruptly

A smoothly growing function for K

$$\frac{\partial u}{\partial t} = F - K(u - \overline{u})_{bz}$$

Discretize it using leapfrog scheme for the regular terms, and backward scheme for the relaxation term

$$\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F^n - K(u_i^{n+1} - \overline{u}^{n+1})_{bz}$$

In boundary-zone

$$u_i^{n+1} = u^{n+1} + 2\Delta t F^n$$

Regional solution before relaxing

Physical relationship between solution before and relaxing in the boundary-zone

$$u^{n+1} = (1-\alpha)u_i^{n+1} + \alpha\overline{u}^{n+1}$$

$$\alpha = 2\Delta t K$$

3.5.4 Two way interactive boundary conditions

the host model, with coarser resolution, provides information about the boundary values to the nested regional model. Regional solution, in turn, also affects the global solution.

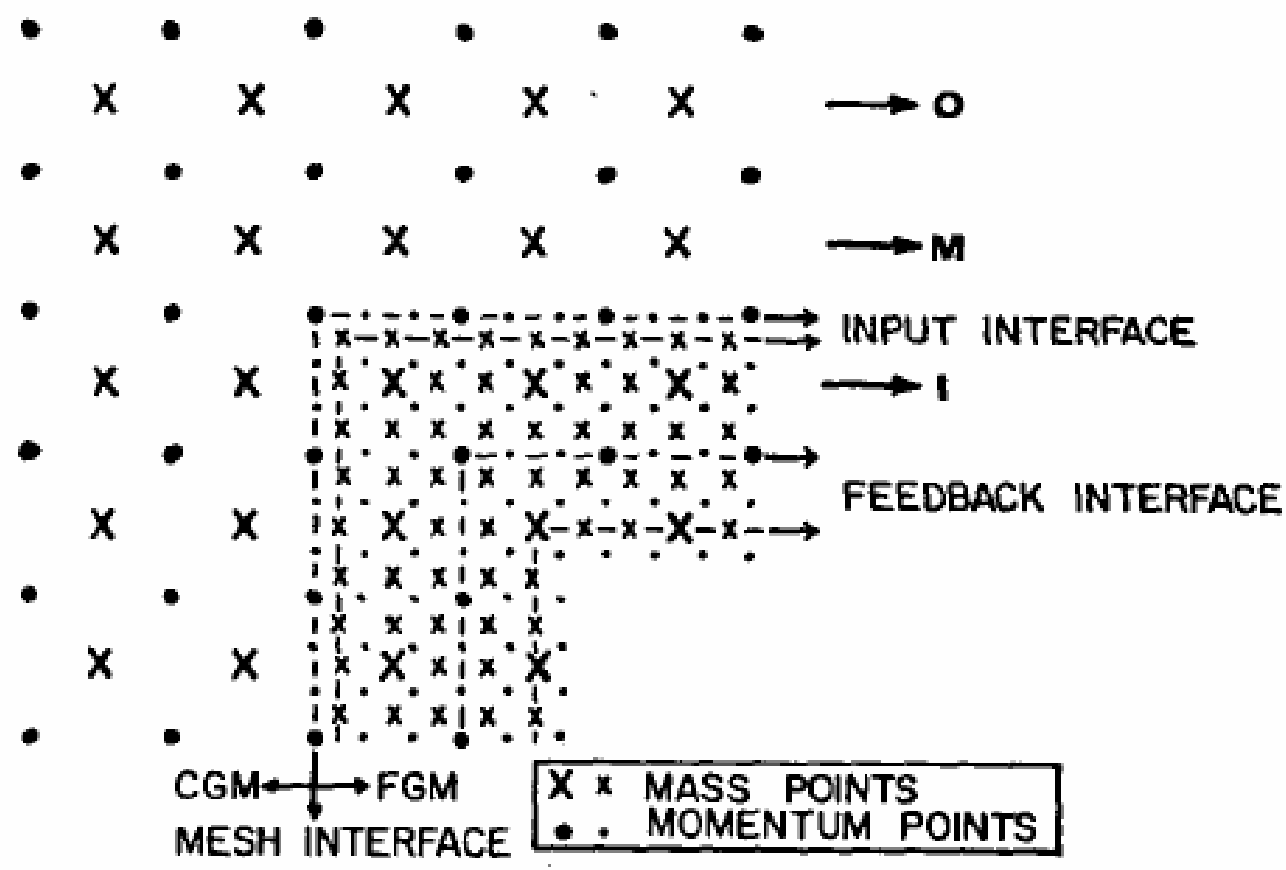
Advantages:

in principle this would seem a more accurate approach than the one-way BC

However, care has to be taken that the high-resolution information does not become distorted in the coarser resolution regions, which can result in worse overall results

1, truly nested model

Zhang et al, 1986



2, stretched horizontal coordinates

Benoit et al, 1989

- Still solve the whole Hemisphere, but only the region of interest is solved with high resolution
- No need for BC