Review for AMSC/CMSC 460 – Spring 2011

May 12, 2011

Expectations: Ability to

- Solve simple analytical problems.
- Apply the methods to simple problems.
- Write pseudo-codes and matlab codes (actual matlab execusion will not be in the exam).

1 Basics

- Describe digits, accuracy, and rounding.
- Perform Taylor series expansion and estimate error by rounding off at the n-th order.
- Identify source of errors.
- Computate absolute and relative errors.
- Define norms.

2 Polynomial interpolation

- Describe goal and approach of of polynomial interpolation, given a data set at N points.
- Describe major polynomial interpolation methods (ideas, basis functions, how to compute coefficients): Vandermonde approach; Lagrange form, Newton representation.
- Identify potential disadvantage of polynomial interpolation as N increases.
- Describe of major piecewise polynomial interpolation methods (ideas, how to evaluate the function) piecewise linear, piecewise cubic hermite, cubic spline.

3 Numerical Integration

- Describe goal and approach of numerical integration, given a data set at M points.
- Describe major numerical integration methods (ideas, no need to be able to derive general formulae): Newton-Cotes rules, Gaussian quadrature.
- Describe simple numerical interation: Trapezoidal rules, Simpsion's rule,
- Describe simple composite rule.

4 Matrix computation

- Take advantages of the structure of the matrix or formula to set up problem, i.e., Hornor's rule.
- Describe matrix-vector multiplication: row-oriented, column-oriented.
- Describe matrix-matrix multiplication: inner product, outer product, sappy method.

5 Solution for linear systems $\mathbf{Ax} = \mathbf{b}$., $\mathbf{A} \in \mathbb{R}^{N \times N}$ and $\mathbf{x}, \mathbf{b}. \in \mathbb{R}^N$

- Explain why we generally do not compute an inverse of a matrix.
- Describe goal and approach if Gaussian elimination: forward elimination & backward substitution, pivoting.
- Describe goal and apporach of LU factorization: how to use the LU factorization in solvin the linear system.

6 Least square estimation (LSE) to $\mathbf{Ax} = \mathbf{b}$, $\mathbf{A} \in \mathbb{R}^{L \times N}$, $\mathbf{x} \in \mathbb{R}^N$ and $\mathbf{b} \in \mathbb{R}^L$ $(L > N)$

- Describe goal and approach: idea of residual $\mathbf{x} \in \mathbb{R}^L$.
- Describe goal and apporach of QR factorization: orthogonality, rotation, and how to solve the linear system.
- Cholesky factorization: approach (how to solve the LSE problem), ability to solve the system for small N, L by hands, ability to write pseudo codes

7 Methods for solving nonlinear systems

- Solving for nonlinear systems:
  - Describe goal and approach for solving univariate nonlinear systems.
  - Describe the bisection, newton's method, secant method.
  - Describe extension to multivariate systems.
- Solving for optimization problem
  - Describe goal and approach for univariate & multivariate stsems.

8 Methods for solving initial value problem (IVP)

- Formulat the IVP in terms of standard form.
- Explain implicit vs explicit schemes, describe Predict-Evaluate-Correct-Evaluate methods.
- Explain ideas and approach of Euler forward/backward methods and compute local/global errors.
- Explain ideas and approach of Runge-Kutta method, and emonstrate for RK4.
- Explain ideas and approach of Adams family methods.
- Present geometrical representation of RK methods vs Euler methods.