Some Words About Myself

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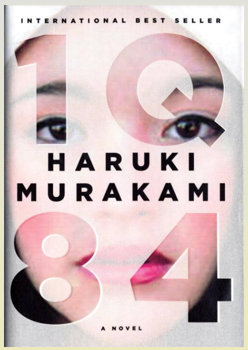
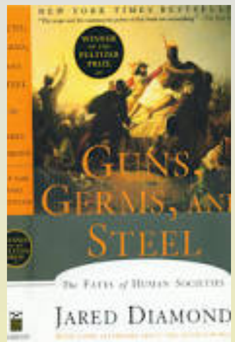
Some Background

- Born in New York City
- Undergraduate: Columbia University
Mathematics
- Graduate: Yale University
Computer Science (Numerical Analysis)
- University of Maryland since 1985
- Sabbaticals at Stanford University
University of Manchester
Oxford University
- Slowly adjusting to life outside NYC

Some Personal

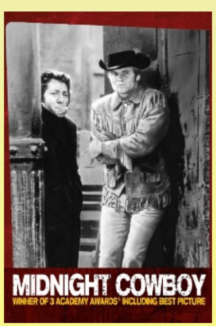
- Have a life (or try to)
  - Literature / History





  - Music

  - Movies et al.



  - Sports

Comment pertaining to this course

Requirement for me as a graduate student:

- One-year project in spring of 1st year – fall of 2nd year
- My project: Krylov subspace methods for nonsymmetric systems of linear equations

- Taught me how to concoct / test / explore ideas

- Led to my thesis. Main result, for $Au=f$, A nonsymmetric

$$\|f - Au_k\|_2 \le \rho^k \|f - Au_0\|_2$$

$$\rho = 1 - \frac{\lambda_{\min}((A + A^T)/2)}{\lambda_{\max}(A^T A)} < 1$$

Evolution of my work

- Started in numerical linear algebra: solve $Au = f$

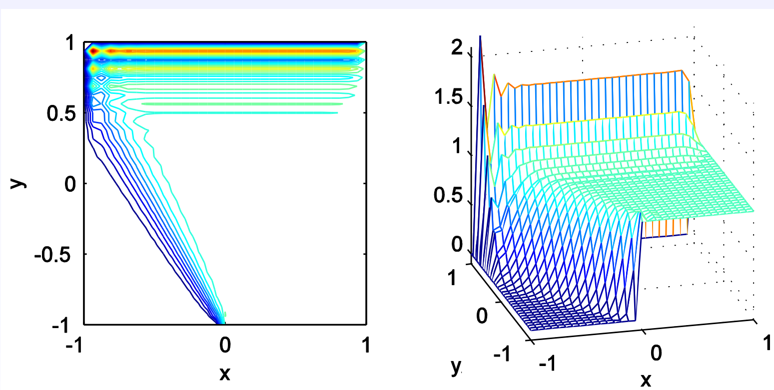
- Matrix A: from applications

$$\underbrace{-\nu\nabla^2 u}_{\text{Self-adjoint}} + \underbrace{w\cdot\nabla u}_{\text{Skew-self-adjoint}} = f$$

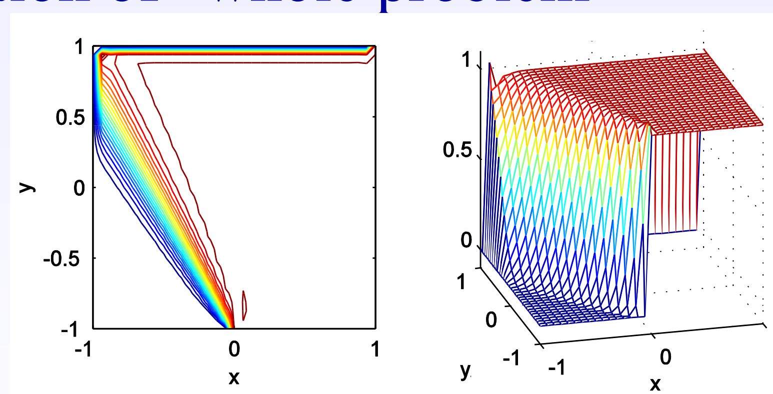
Convection-diffusion equation

A is nonsymmetric

- Over time: developed appreciation of "whole problem"



Unresolved layer



Adjusted discretization

The Navier-Stokes Equations

$$\boxed{-\nu \nabla^2 u + (u \cdot \text{grad})u} + \text{grad}\, p = f$$

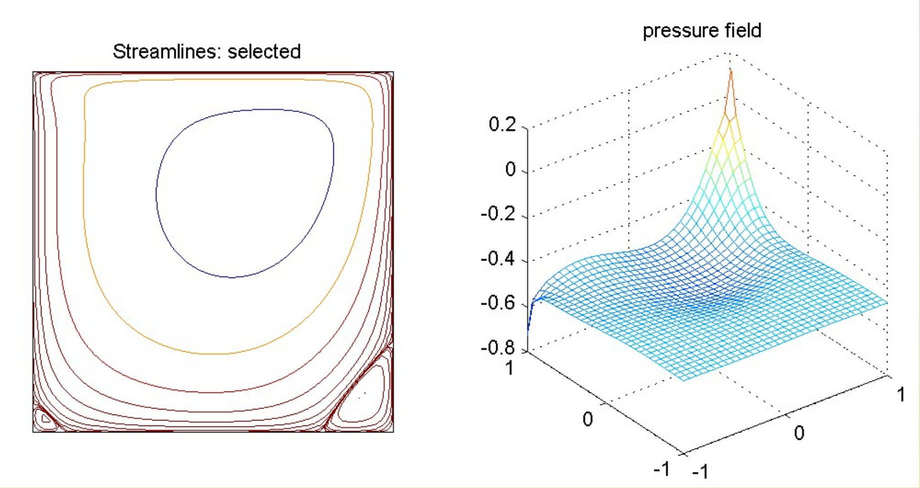
$$-\text{div}\, u = 0$$

Nonlinear convection-diffusion term

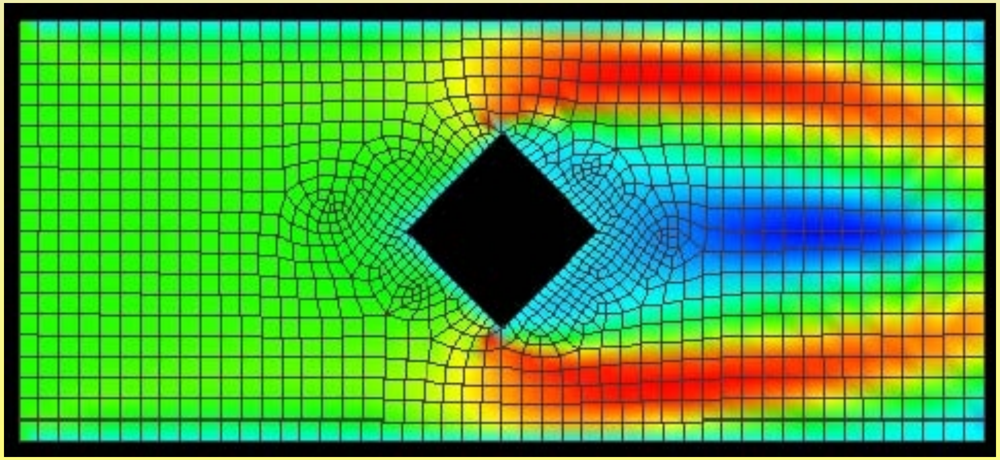
- Classic model of fluid dynamics, want efficient solution algorithms
- My main interest: efficient solution algorithms for nonlinear algebraic equations obtained from discretization
- Classic algorithmic approaches: based on 1960’s technology
- New approaches: developed in 1990’s – today
- Requires: knowledge of “whole problem”: fluid properties, discretization, *plus* matrix properties

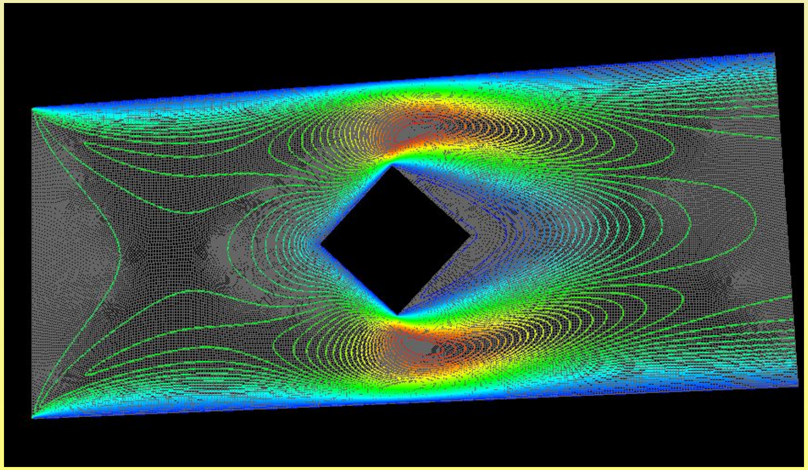
Benchmark Problems

1. 2D/3D Driven Cavity



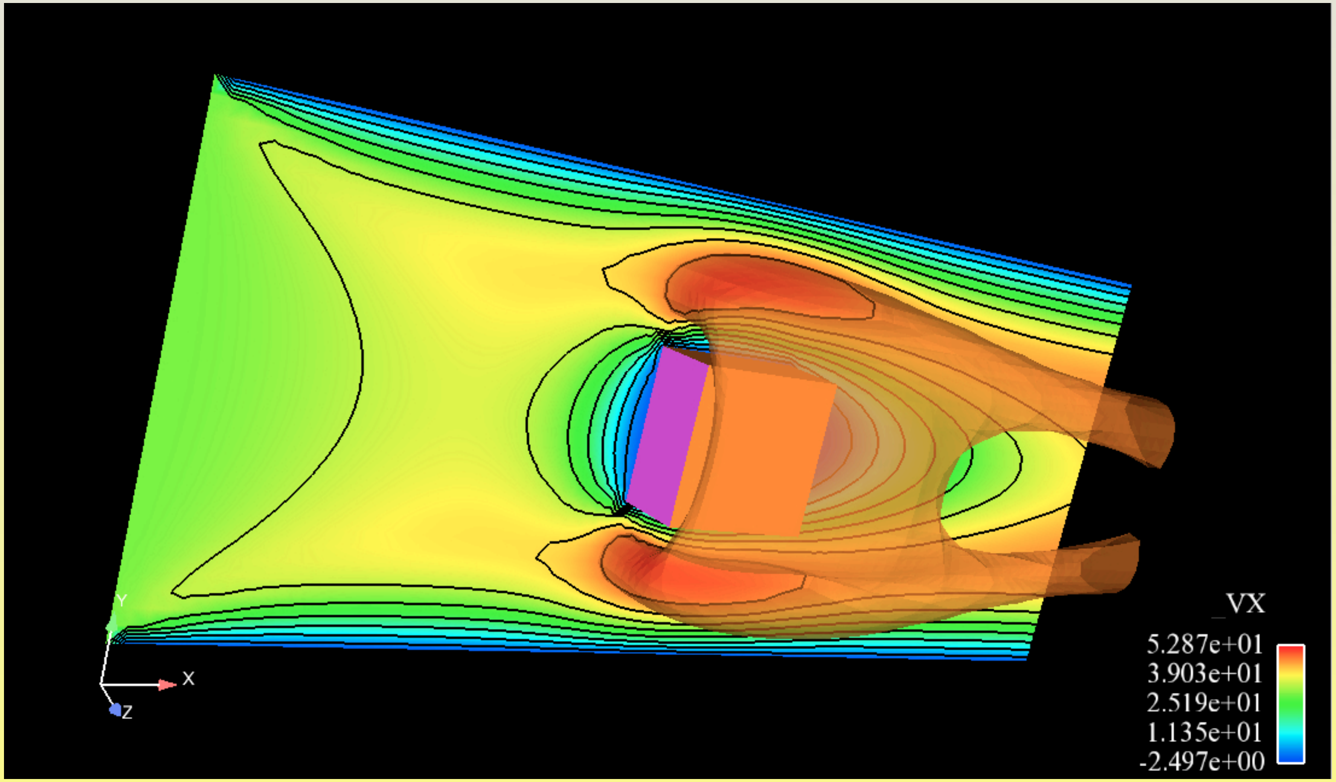
2. 2D flow over a diamond obstruction on an unstructured grid





Benchmark Problems

3. 3D flow over a cube obstruction

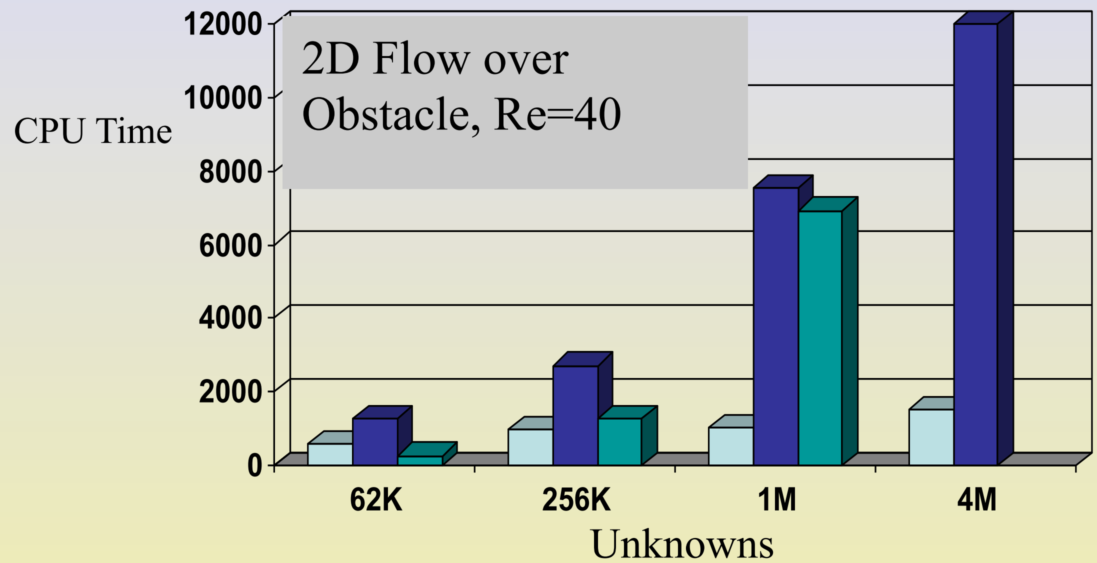


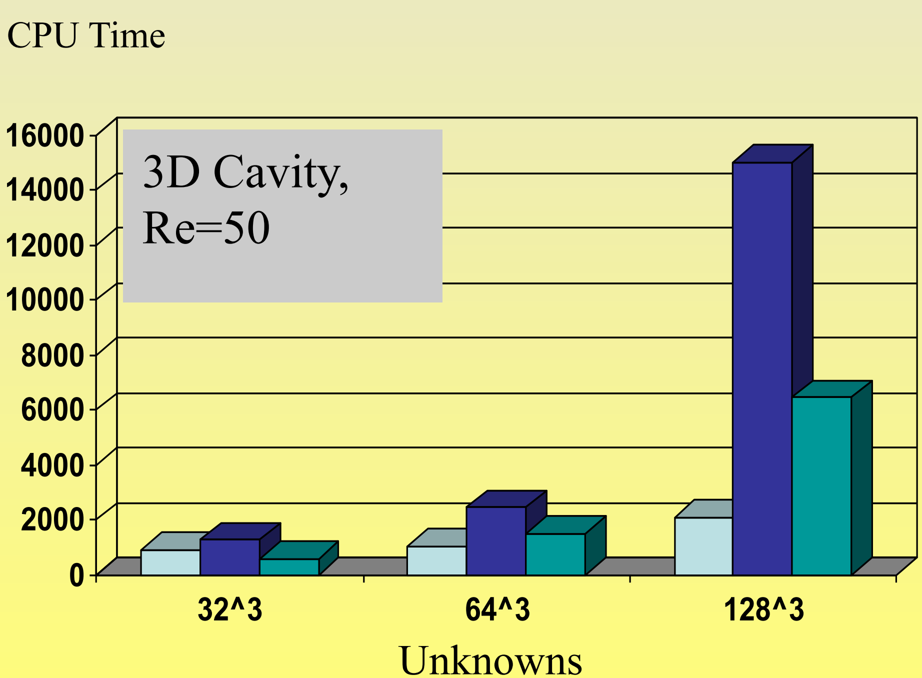
Representative Performance Results

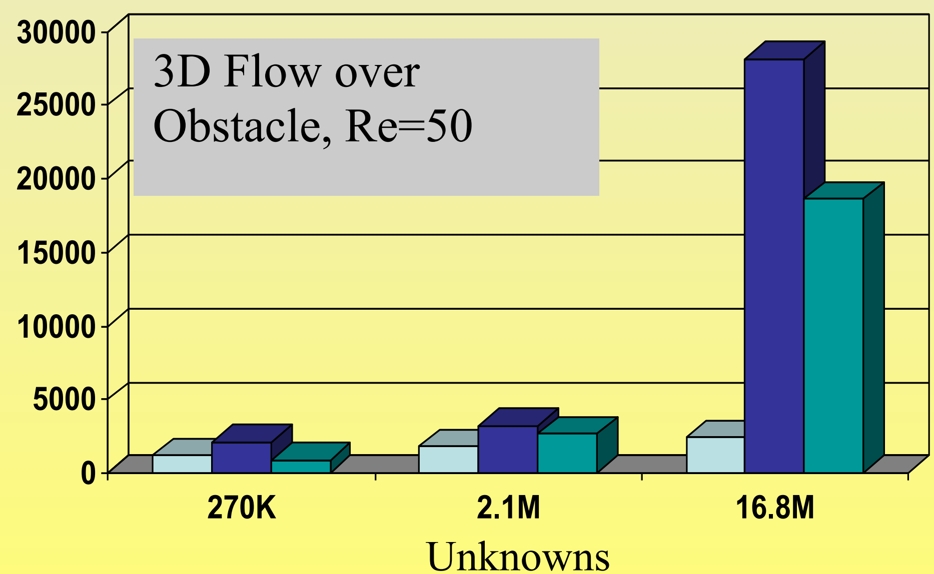
Pressure conv-diff

Simple

Domain decomposition







More recently developed interest: PDEs posed with uncertainty

Example: diffusion equation $-\nabla \cdot (a \nabla u) = f \;\text{ in } \mathcal{D}$

Assumptions:

1. Spatial correlation of random field: For $x, y \in \mathcal{D}$:

   Random field $a(x,\omega)$

   Mean $\mu(x) = E(a(x,\cdot))$

   Variance $\sigma(x) = E(a(x,\cdot)^2) - \mu^2$

   Covariance function

   $$c(x,y) = E(\,(a(x,\cdot) - \mu(x))\,(a(y,\cdot) - \mu(y))\,)$$

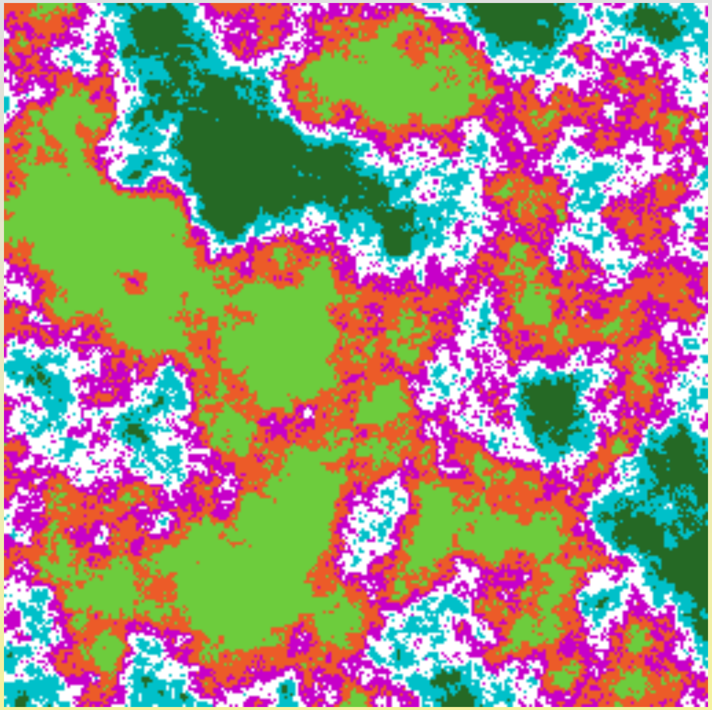
   is finite

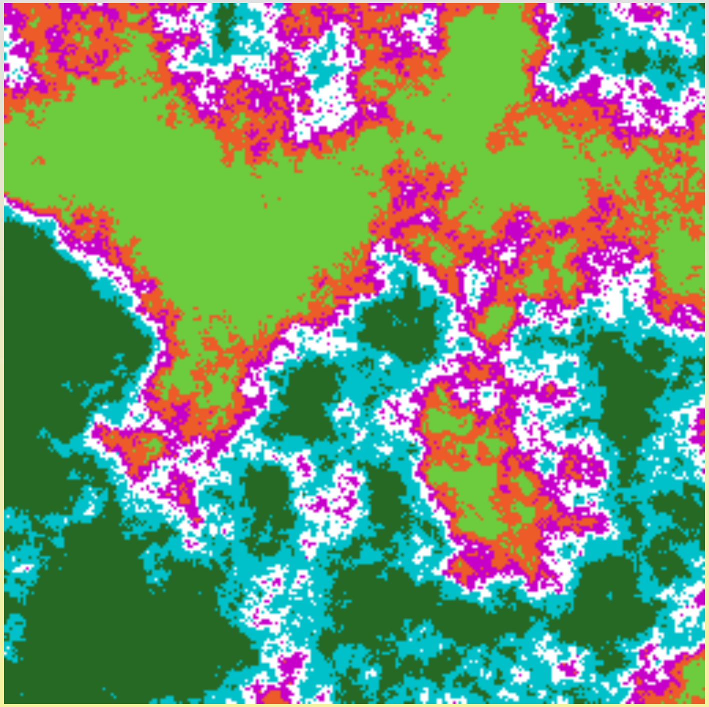
   vs. *white noise*, where c is a δ-function

2. Coercivity $0 < \alpha_1 \le a \le \alpha_2 < \infty$

   $\Longrightarrow$ Problem is well-posed

Depictions: Random Data on Unit Square





Monte-Carlo Simulation

Sample $a(x,\omega)$ at all $x \in \mathcal{D}$, solve in usual way

Standard weak formulation: find $u \in H^1_E(\mathcal{D})$ such that

$$a(u,v) = \ell(v)$$

for all $v \in H^1_{E_0}(\mathcal{D})$,

$$a(u,v) = \int_{\mathcal{D}} a \nabla u \cdot \nabla v \, dx, \qquad \ell(v) = \int_{\mathcal{D}} f \, v \, dx$$

Multiple realizations (samples) of $a(x,\cdot)$ $\longrightarrow$
Multiple realizations of u $\longrightarrow$
Statistical properties of u

Problem: convergence is slow, requires many solves
Seek: more efficient alternatives

For this class (and your careers):

- Importance of
  - Ideas
  - Communication
    - Writing
    - Presentation
    - Communication with your advisor
    - Communication with us
- I look forward to working with you