

AOSC 615. Advanced Methods in Data Assimilation

Suggested Model Description

1. Lorenz Model with 3 Variables
2. Lorenz Model with 40 Variables
3. Point Vortex Model
4. SPEEDY Model

Elements of Data Assimilation: Lorenz Model (1963)

- Model ($N=3$)

$$\mathbf{x}_k = \mathbf{m}_{k,k-1}(\mathbf{x}_{k-1})$$

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1)$$

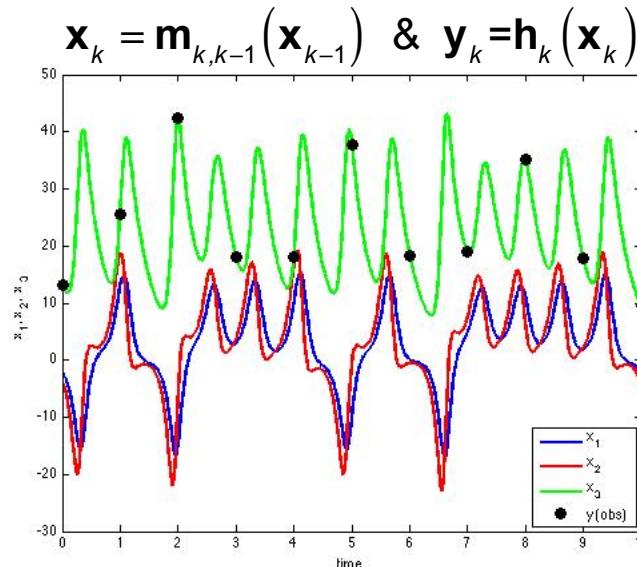
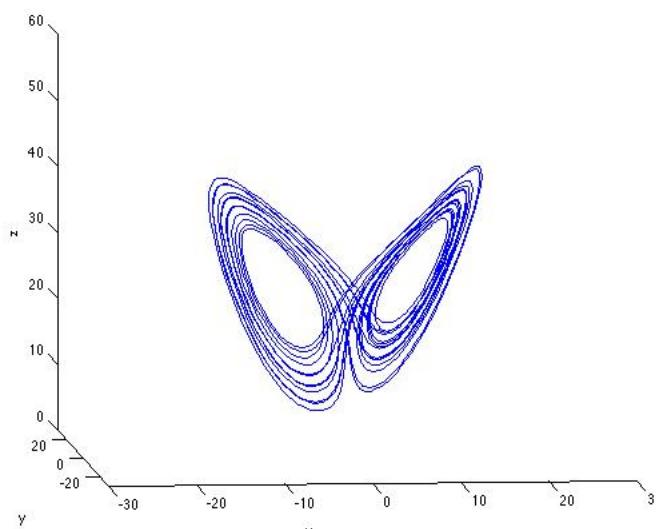
$$\frac{dx_2}{dt} = x_1(\rho - x_3) - x_2$$

$$\frac{dx_3}{dt} = x_1x_2 - \beta x_3$$

- Observation

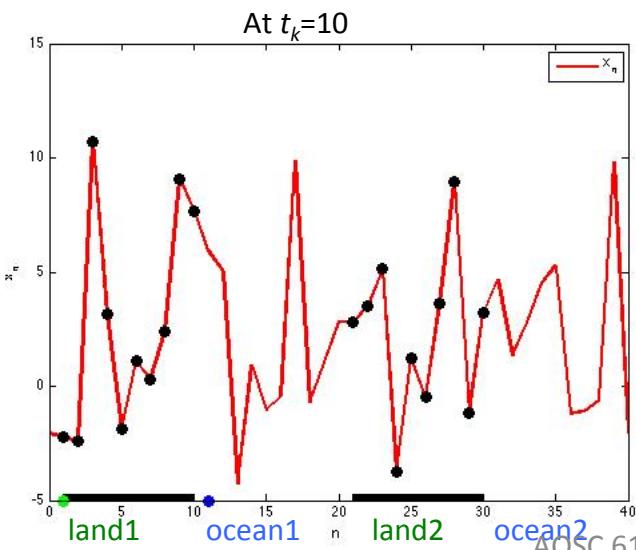
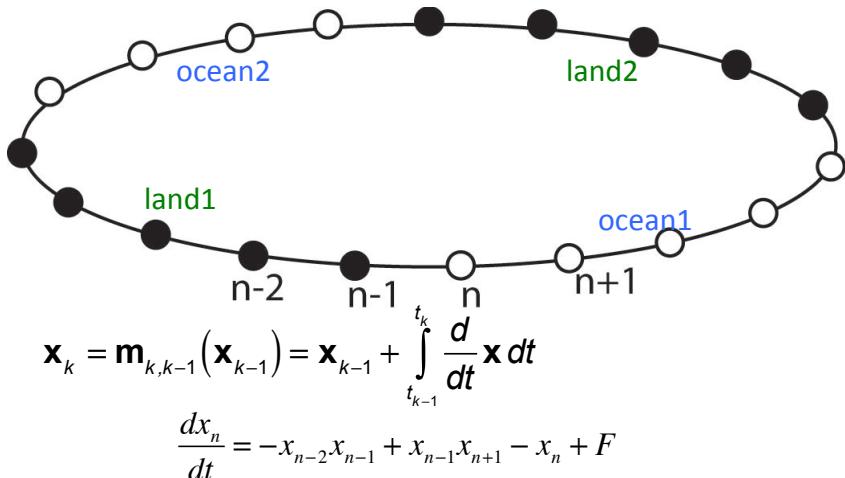
Ex) observation of the 3rd variable

$$y_k = \mathbf{x}_3(t_k) = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{\mathbf{H}_3} \underbrace{\begin{pmatrix} x_1(t_k) \\ x_2(t_k) \\ x_3(t_k) \end{pmatrix}}_{\mathbf{x}(t_k)} = \mathbf{H}_3 \mathbf{x}(t_k)$$



Lorenz '95 Model

- Model (typically $N=40$)



- Observation

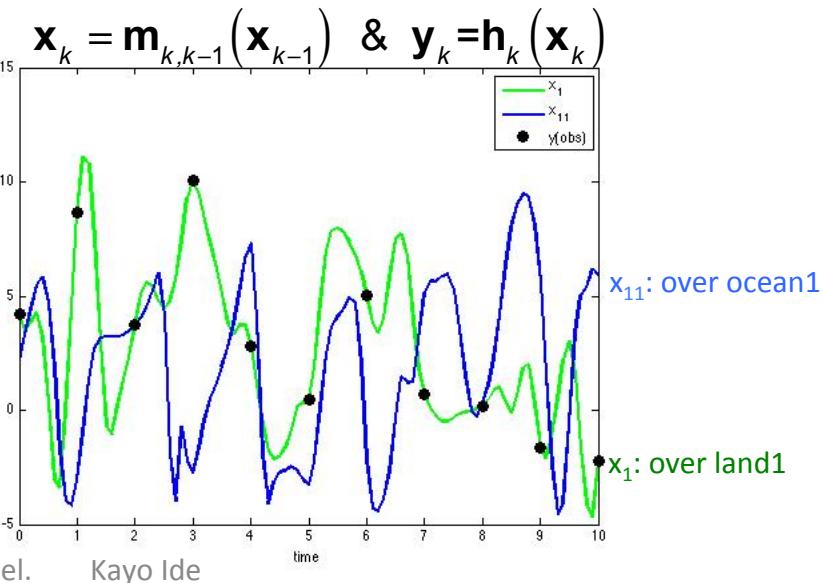
$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k)$$

Ex) observation over the lands

$$\mathbf{y}_k = \begin{pmatrix} \mathbf{y}_{land1,k} \\ \mathbf{y}_{land2,k} \end{pmatrix} = \mathbf{H}_{land}(\mathbf{x}_k)$$

$$\mathbf{H}_{land} = \begin{matrix} land1 & ocean1 & land2 & ocean2 \\ \text{land1} & I & 0 & 0 \\ land2 & 0 & 0 & I & 0 \end{matrix}$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_{land1} \\ \mathbf{x}_{ocean1} \\ \mathbf{x}_{land2} \\ \mathbf{x}_{ocean2} \end{pmatrix}$$



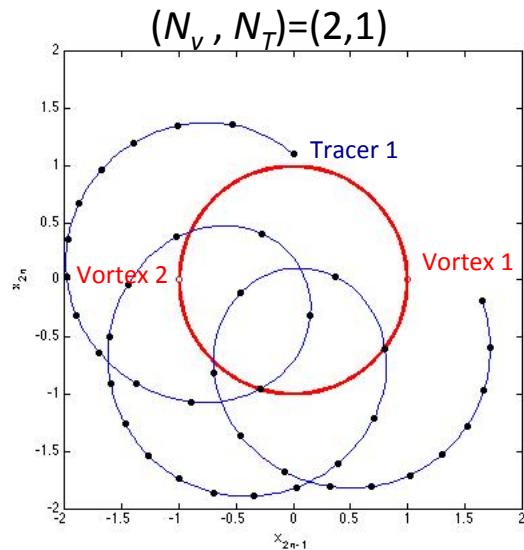
Point Vortices & Tracers

- Model: N_v vortices & N_T tracers

$$\mathbf{x}_k = \mathbf{m}_{k,k-1}(\mathbf{x}_{k-1}) = \mathbf{x}_{k-1} + \int_{t_{k-1}}^{t_k} \frac{d}{dt} \mathbf{x} dt$$

$$\begin{pmatrix} \frac{d}{dt} x_{v,n} \\ \frac{d}{dt} y_{v,n} \end{pmatrix} = \sum_{\substack{k=1 \\ k \neq n}}^{N_v} \frac{\Gamma_{v,k}}{2\pi} \frac{1}{|\mathbf{x}_{v,n} - \mathbf{x}_{v,k}|^2} \begin{pmatrix} -(y_{v,n} - y_{v,k}) \\ (x_{v,n} - x_{v,k}) \end{pmatrix}$$

$$\begin{pmatrix} \frac{d}{dt} x_{t,n} \\ \frac{d}{dt} y_{vt,n} \end{pmatrix} = \sum_{k=1}^{N_v} \frac{\Gamma_{v,k}}{2\pi} \frac{1}{|\mathbf{x}_{t,n} - \mathbf{x}_{v,k}|^2} \begin{pmatrix} -(y_{t,n} - y_{v,k}) \\ (x_{t,n} - x_{v,k}) \end{pmatrix}$$



AOSC 615. Project Model.

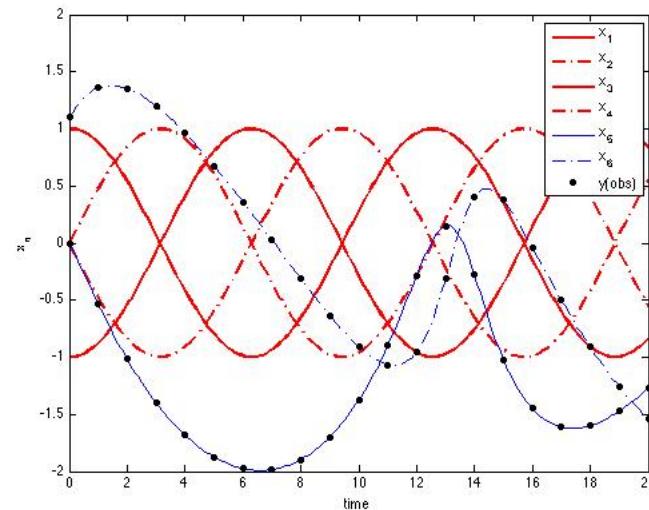
- Observation

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k)$$

Ex) observation of tracers

$$\mathbf{y}_k = \mathbf{y}_{tracer,k} = \mathbf{H}_{tracer}(\mathbf{x}_k)$$

$$\mathbf{H}_{tracer} = \begin{matrix} \text{tracer} & \text{tracer} \\ \text{vortex} & \text{tracer} \\ \mathbf{0} & \mathbf{I} \end{matrix} \quad \mathbf{x} = \begin{pmatrix} \mathbf{x}_v \\ \mathbf{x}_T \end{pmatrix}$$



Kayo Ide

SPEEDY: Simplified Parameterization, primitivE-Equation Dynamics

- Model developed for climate studies

- For computational efficiency, use of
 - Low resolution (Spectral model with T30L7)
 - Simplified physical parameterization
- Maintain the basic characteristics of an atmospheric General Circulation Model (GCM)

A data assimilation package has been developed by the UMD group and is available online

- Tutorial: <http://www.atmos.umd.edu/~ekalnay/SPEEDY-JunjieLiu/JunjieLiuTutorial.doc>
- State-of-art LETKF: <http://www.atmos.umd.edu/~miyoshi/letkfwiki/>

