Analysis of seasonal cycles in climatic trends with application to satellite observations of sea ice extent

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[1] We present a new technique to study the seasonal cycle of climatic trends in the expected value, variance, and other moments of the statistical distribution. The basis of the technique is multiple linear regression, but with periodic basis functions. The technique allows us to provide comprehensive information on statistical parameters of climate for every day of an observational period. Using daily data, the technique has no problems caused by different lengths of months or the leap-year cycle. Without needing to assume the stationarity of contemporary climate, the technique allows the study of statistical parameters of climatic records of arbitrary length. We illustrate the technique with applications to trends in the satellite observed variations of sea ice extent in the Northern and Southern Hemispheres. We show that a significant part of the variability in hemispheric sea ice extents for the period 1978–1999 is related to linear trends. *INDEX TERMS:* 1620 Global Change: Climate dynamics (3309); 1863 Hydrology: Snow and ice; 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 4215 Oceanography: General: Climate and interannual variability (3309)

1. Introduction

[2] Traditional climatology uses monthly averages and statistics estimated for 30-yr time intervals. This approach, however, has well-known limitations. Monthly averages incorporate different lengths for different months and different years (leap years). Using only 12 monthly averages is a very crude approximation of the seasonal variation for many meteorological variables and produces biases in the spectrum of meteorological records. Time series for specific months are too short to estimate the parameters of the statistical distribution of monthly averages properly.

[3] Here we introduce a new statistical approach to analyzing climate data that explicitly accepts the idea that long-term climatic records may contain trends in means, variances, covariances, and other moments of the statistical distribution of meteorological variables. We also explicitly permit seasonal cycles of these trends. This approach is designed to analyze observed past climate change and climate model output, but is not designed to be used to extrapolate the trends into the future.

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2. Theory

[4] We start by using daily observations instead of monthly averages. Let us denote

$$y(t) = Y(t) + y'(t); \tag{1}$$

where $y(t)$ is the observed value of y for day number t, $t = t_1, t_2, t_3, \ldots, t_n$; $Y(t)$ is the expected value of $y(t)$; and $y'(t)$ is the residual (anomaly). We can use as t the Julian day number, or choose any other specific date as the reference day ($t = 0$).

[5] A common approach in studies of the seasonal cycle is to use Fourier harmonics of the annual period to approximate the seasonal variation of the mean values of the observed data [*Anderson*, 1971; *Polyak*, 1975]. It is well known that climatic trends may be different for different seasons [*Mitchell*, 1961; *Vinnikov*, 1986; *Hansen and Lebedeff*, 1987; *Chapman and Walsh*, 1993; *Stammerjohn and Smith*, 1997; *Hansen et al.*, 1999; *Parkinson et al.*, 1999; *Jones et al.*, 1999; *Rigor et al.*, 2000; *Serreze et al.*, 2000]. Let us assume that the climatic trend in the expectation $Y(t)$ of the observed variable $y(t)$ is a periodic function of the annual period. Particularly, for a linear trend this means:

$$Y(t) = A(t) + B(t) \cdot t, \tag{2}$$

where $A(t) = A(t + T)$, $B(t) = B(t + T)$, and $T = 365.25$ days. Let us use a limited number of Fourier harmonics to approximate both periodic functions, $A(t)$ and $B(t)$:

$$Y(t) = a_0 + \sum_{k=1}^{K}\left[a_k \sin\left(\frac{2\pi kt}{T}\right) + b_k \cos\left(\frac{2\pi kt}{T}\right)\right] + \alpha_0 t + \sum_{m=1}^{M}\left[\alpha_m t \sin\left(\frac{2\pi mt}{T}\right) + \beta_m t \cos\left(\frac{2\pi mt}{T}\right)\right]. \tag{3}$$

We can estimate all the unknown multiple regression coefficients in (3) from the least squares condition:

$$\sum_{t=t_1}^{t_n}[y(t) - Y(t)]^2 = F(a_0, \ldots, a_K, b_1, \ldots, b_K, \alpha_0, \ldots, \alpha_M, \beta_1, \ldots, \beta_M) = \min. \tag{4}$$

The number of harmonics required for approximation of $A(t)$ and $B(t)$, K, and M, should be chosen from independent considerations or should be estimated from analyses of the same data.

[6] The expression (3) permits us to calculate expected values $Y(t)$ of $y(t)$ for each day of the time interval (t_1, t_n) and to calculate

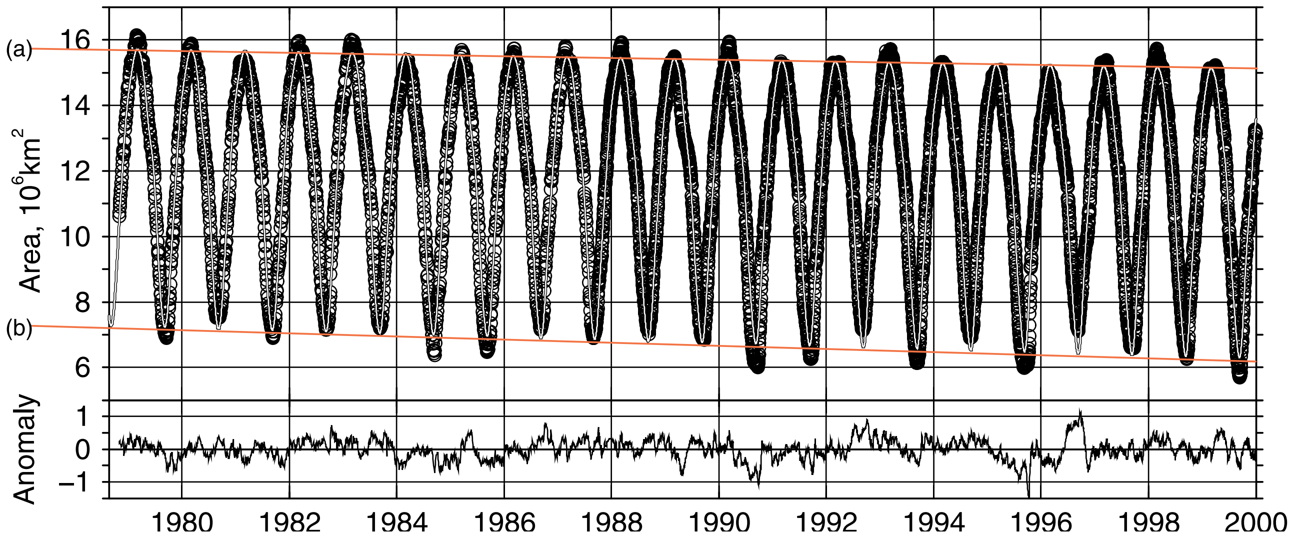

Figure 1. Northern Hemisphere sea ice extents derived from satellite passive-microwave data. Small circles are observed daily extents. The expected values are plotted in the white curve, and the daily anomalies are plotted in the thin line in the bottom panel. The envelope lines (a) and (b) are for annual maximum and minimum expected daily extents.

the residuals $y'(t)$. Then, using the same technique for the variables $y'(t)^2$, $y'(t)^3$, $y'(t)^4$, $y'(t)y'(t - lag)$, $x'(t)y'(t)$, etc., we can evaluate the seasonality of trends in variance and other moments of the statistical distribution of the variables $y(t)$ and any other variable $x(t)$. A different number of harmonics of the annual cycle may be needed to approximate the seasonality of different moments. The coordinate functions in (3) are not required to be orthogonal, but they are approximately orthogonal if there are not too many missing data in the record and the reference day (t = 0) is chosen in the middle of the time interval (t_1, t_n). Polynomial or other specific functions can be used instead of a linear model for the trends.

3. Application to Satellite Observations of Sea Ice Extent

[7] There have been significant changes in sea ice extent during the past two decades [*Johannessen et al.*, 1995; *Bjørgo et al.*, 1997; *Cavalieri et al.*, 1997; *Stammerjohn and Smith*, 1997; *Parkinson*

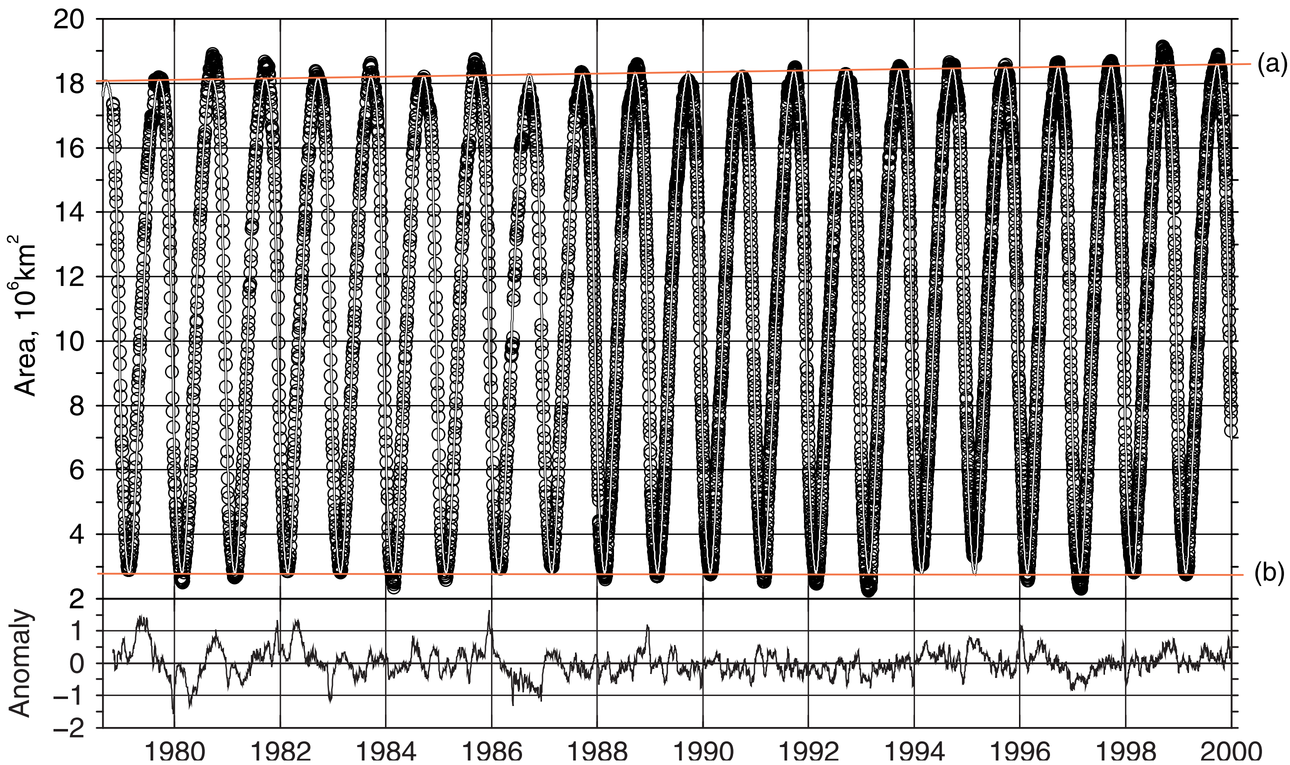

Figure 2. Same as Figure 1, but for Southern Hemisphere sea ice extents.

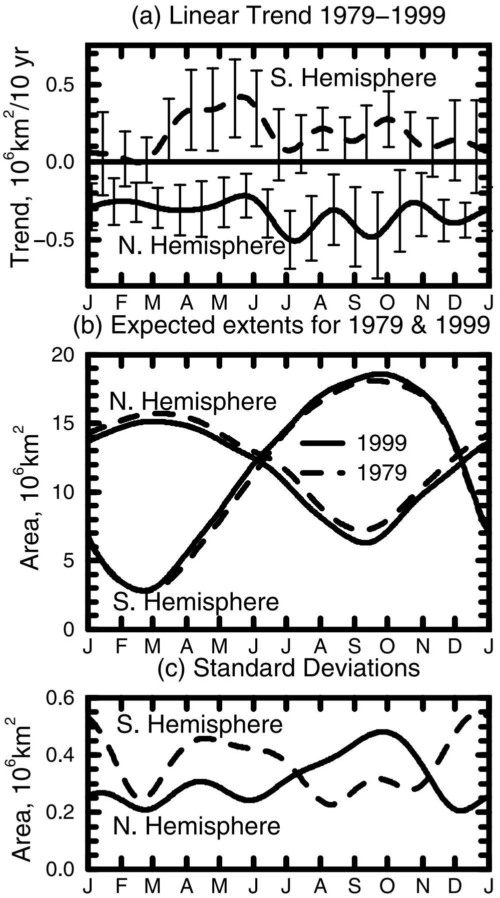

Figure 3. Sea ice extents in the Northern and Southern Hemispheres: (a) linear trend 1979–1999, (b) expected extent for years 1979 and 1999, (c) standard deviations of daily extents, 1979–1999.

et al., 1999], and these are likely due at least in part to impacts humans have had on the climate system [*Vinnikov et al.*, 1999]. The best information source for climatic monitoring of sea ice extent is passive microwave satellite observations. Daily observations have recently been reanalyzed for every other day for the years 1978–1987, using data from the Nimbus 7 Scanning Multichannel Microwave Radiometer (SMMR), and for every day for 1987–1999, using data from the Defense Meteorological Satellite Program (DMSP) Special Sensor Microwave/Imager (SSM/I). These data allow us to test our technique and to examine the seasonality of the trends of sea ice extent in the Northern and Southern Hemispheres (Figures 1–3).

[8] The observed daily sea ice extents in each hemisphere and the result of approximation of their expectations with (3) are given in the top panels of Figures 1 and 2, using $K = M = 6$. More than 6000 daily observations for about 21 years were used to evaluate the 26 coefficients in (3) for each of the hemispheres. We have more than 200 observations for each coefficient, but daily data are not independent and the real number of degrees of freedom for each coefficient is less than this value.

[9] Daily anomalies of sea ice extents are presented in the bottom panels of Figures 1 and 2. The top (a) and bottom (b) envelope lines of function $Y(t)$ from (3) in these figures show the trends in expected maximum and minimum sea ice extents. Seasonal variations of the linear trends in the Northern and Southern Hemisphere sea ice extents, the expected values of the extents at the beginning of the record (year 1979) and at the end of the record (year 1999), and standard deviations of the observed daily extents are given in Figure 3. Only four ($K = 4$) harmonics of the annual period were used to evaluate the seasonality in the variance. The record is currently too short (~21 years) to be used effectively for analysis of the trends of the variance and higher moments of the statistical distribution of sea ice extents. Maximum possible root mean squared errors of the trend estimates $B(t)$ are shown in Figure 3a with vertical bars. They have been estimated using a matrix of covariance of the unknown coefficients in (4) estimated from a least squares technique for independent observations, the estimates of seasonal standard deviations presented in Figure 3c, and estimates of the autocorrelation time scale of observed daily sea ice extent, ~50 days, which is close to the earlier estimate by *Walsh and Johnson* [1979]. This autocorrelation in the observed data makes confidence intervals for the trend estimates as much as 8 times larger than those for independent observations [*Polyak*, 1979; *Karol et al.*, 1976]. However, it would be more realistic to suppose that the errors are much smaller than those in Figure 3a. The trends in monthly averages from virtually the same data were found to be statistically significant [*Cavalieri et al.*, 1997; *Parkinson et al.*, 1999; *Parkinson and Cavalieri*, 2002].

4. Discussion and Conclusions

[10] Most previous analyses of trends in sea ice extents have produced statistics for monthly or yearly averages. Daily observations give us an opportunity to see additional interesting details in the same climatic records. Results of applying the new technique to sea ice extents yield the following conclusions:

4.1. Northern Hemisphere Sea Ice Extent

[11] The expected sea ice extent in the Northern Hemisphere has been decreasing during 1979–1999 for each day of the year with rates from -0.2×10^6 km^2/10 yr to -0.5×10^6 km^2/10 yr (Figures 3a and 3b). Consistently, although in less detail, *Parkinson and Cavalieri* [2002] find that monthly trends in the Northern Hemisphere ice extents have negative values for all 12 months, all with magnitudes of at least -0.23×10^6 km^2/10 yr.

[12] The expected maximum seasonal sea ice extent in 1999 was about 0.5×10^6 km^2 less than it was in 1979 (Figure 1, line (a)). This corresponds to a ~0.3° latitude poleward shift of the average position of the sea ice boundary at the time of its maximum seasonal expansion.

[13] The expected minimum seasonal sea ice extent in 1999 was about 1.0×10^6 km^2 less than it was in 1979 (Figure 1, line (b)). This corresponds to a ~1° latitude poleward shift of the average position of the sea ice boundary at the time of its maximum seasonal retreat.

[14] The average amplitude of the seasonal variation in sea ice extent has increased about 0.5×10^6 km^2 during the 21-year period 1979–1999, from $\sim 8.5 \times 10^6$ km^2 to $\sim 9.0 \times 10^6$ km^2 (Figure 1).

[15] Daily anomalies of sea ice extent (Figure 1) are an order of magnitude smaller than the seasonal variation of sea ice extent (about $8.5–9.0 \times 10^6$ km^2, Figure 1).

[16] Trend-related changes during the last two decades of the 20th Century have the same order of magnitude ($0–1 \times 10^6$ km^2) as daily anomalies of sea ice extent (Figure 1).

4.2. Southern Hemisphere Sea Ice Extent

[17] The expected sea ice extent in the Southern Hemisphere has been increasing during 1979–1999 for almost every day of the year, with rates up to $\sim 0.4 \times 10^6$ km^2/10 yr (Figures 3a and 3b). This asymmetry between Northern and Southern Hemisphere sea ice variation was predicted using a climate model forced by increasing greenhouse gases [*Manabe et al.*, 1992] and then observed through satellite microwave measurements [*Cavalieri et al.*, 1997].

[18] The expected seasonally maximum sea ice extent in 1999 is about 0.5×10^6 km^2 larger than it was in 1979 (Figure 2, line (a)). There is no noticeable trend in sea ice extent at the time of its maximum seasonal retreat (Figure 2, line (b)). The average amplitude of seasonal variation in sea ice extent has increased about

0.5×10^6 km^2 during the 21-year period, from $\sim 15.3 \times 10^6$ km^2 to $\sim 15.8 \times 10^6$ km^2. Thus the seasonal amplitude of sea ice extent has increased in both hemispheres, and in both cases by about 0.5×10^6 km^2 (Figures 1 and 2).

[19] Daily anomalies of sea ice extent in the Southern Hemisphere (Figure 2) are an order of magnitude smaller than the seasonal variation of sea ice extent (Figure 3b), the same as in the Northern Hemisphere.

[20] As in the Northern Hemisphere, daily anomalies in the Southern Hemisphere are generally between -1×10^6 km^2 and 1×10^6 km^2 (Figures 1 and 2), although the 1979–1999 trends are generally smaller in magnitude, as well as being opposite in sign (Figure 3a).

4.3. Further Development of the Technique

[21] Harmonic functions may not be optimal for approximation of seasonal variations of many climatic variables. Other classes of periodic functions may be used, and empirical statistically orthogonal functions should be considered as possible alternatives to harmonic functions.

[22] In some cases, significant seasonal variations can exist in the variance of climatic indices. In such cases, it might be useful to repeat the calculations using a "weighted least squares" with weights that are inversely proportional to the estimated variance.

[23] Trend estimates for each day of a year provide information about the seasonality of trends in moments of the statistical distribution of climatic variables. The same technique can be applied to analyze time series of observed climatic indices at every specific time of a day. In the case when we have hourly meteorological observations, we can put together the estimates for every hour and receive a full picture of seasonal and diurnal cycles in climatic trends. Joint analysis of the seasonal and diurnal cycles in climatic trends will be discussed in another paper.

[24] The existing traditional practice of climatic services is based on monthly averages and a 30-year period for normals. Even for a variable with an autocorrelation time scale of 50 days, such as sea ice, this technique provides high temporal resolution depictions of the seasonal cycles of the means, anomalies, and trends (Figures 1–3). Without needing to assume the stationarity of contemporary climate, our approach allows the study of statistical parameters of climatic records of arbitrary length. This opens a new opportunity to modernize existing climate services.

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