AOSC 652: Analysis Methods in AOSC

Assignment #6: Numerical Integration
Due: Monday, 10 October 2016 (at start of class)

Name: ___________________________

90 points total

Late penalty waived until Wed at 5 pm: after that, 20 pts per day

The first part of this assignment involves completing the changes to a FORTRAN code that computes the integral of data using Gaussian Quadrature. This part should be straightforward to complete for students who followed Monday and Wednesday's lectures.

1 (40 points). Copy file ~rjs/aosc652/week_06/gauss_integrate.f to your work area and add appropriate lines of code to subroutines fn1, fn2, fn3, and fn4 so that this code will compute the integral of functions fn1=3$\boldsymbol{x}^2$, fn2=5$\boldsymbol{x}^4$, fn3=9$\boldsymbol{x}^8$, and fn4=13$\boldsymbol{x}^{12}$, from $\boldsymbol{x}$ = **0** to $\boldsymbol{x}$ =**1.** using Gaussian integration.

Your new code must reflect the fact that for Gaussian integration, the integrating variable used by this numerical technique, denoted $\boldsymbol{y}$, always goes from −1. to 1. (you can blame Gauss for this inconvenience)

To get program gauss_integrate.f to work, you must "uncomment" and complete the FORTRAN statements that contain ???. These statements relate $\boldsymbol{x}$ and $\boldsymbol{y}$ and describe the ratio d$\boldsymbol{x}$/d$\boldsymbol{y}$. Once these statements are completed, Gaussian integration will properly evaluate the various integrals from **0** to **1.**, which is the interval over which we would like to evaluate these functions for this exercise.

a) Make the required changes to the four subroutines (fn1, fn2, fn3, and fn4) and fill in the following table:

Function	2 nodes	10 nodes	Error, 10 nodes
3 x^2			
5 x^4			
9 x^8			
13 x^{12}			

b) If we are to compute the integral of a non-linear function using an approach with 10 intervals, and if ***accuracy*** of the resulting integration was ***essential***, which approach would we choose: Simpson's rule or Gaussian Integration?

c) In class on Wednesday, a series of figures were shown to detail the reason for the respective accuracy (or lack thereof) of the integral of fn4, found using the Trapezoidal rule and Simpson's rule.

Prepare a new figure showing the value of fn4, with the **y-axis** ranging from **–1** to **13** and the x-axis ranging from **0** to **1**. The value of fn4 should be plotted as a **solid line**. Add to this figure points that denote:

> i) the place along the x-axis where fn4 is evaluated for the 10 node Gaussian integration (use a particular symbol)
>
> ii) the place along the x-axis where fn4 is evaluated for the 2 node Gaussian integration (use a different symbol)
>
> To properly place these points, you should use the same variable transformation that was relied upon to complete part a) above.

Based on this figure, answer **why the evaluation of the integral of fn4 using 10 node Gaussian quadrature works so well whereas evaluation using the 2 node approach fails?**

For this part of the assignment, please fill in the table above, please answer the question just above, please provide a hard copy of the requested plot, and please turn in all code used to include the assignment, including your edited version of file gauss_integrate.f, printed using enscript and showing the full path name.

2. (50 points) In class on Monday, we noted numerical evaluation of integrals is central to modern Atmospheric and Oceanic Science.

Often pressure and temperature can be measured more accurately than geometric altitude. This situation has changed somewhat with the advent of GPS (Global Positioning System) technology, but many research aircraft and balloon platforms still operate under the premise that pressure and temperature can be determined more accurately than altitude.

Here, we will explore the calculation of altitude, given pressure and temperature, by integrating the following equation that relates these three quantities:

$$\Delta z = \int_{p_2}^{p_1} \frac{R_{air}}{g} T \, d\ln(p) \qquad \text{Eqn (1)}$$

Eqn (1), known as the ***hypsometric equation***, is given in most calculus-based meteorology text books. It appears as equation 1.18 in my edition of *An Introduction to Dynamic Meteorology* by James R. Holton, except the quantity R_{air}/g has been moved outside of the integral sign: i.e.:

$$\Delta z = \frac{R_{air}}{g} \int_{p_2}^{p_1} T \, d\ln(p) \qquad \text{Eqn (2)}$$

Below we will ask you to evaluate the mathematical maneuver that relates Eqn (2) to Eqn (1).

We shall explore the integration of the hypsometric equation using "known" values of altitude, pressure, and temperature: i.e., the U.S. Standard Atmosphere of 1976.

Please copy files:

~rjs/aosc652/week_06/altitude_find.f and

~rjs/aosc652/week_06/atm.us45std.dat

to your work area. The code contains several lines marked with ???

These lines of code *must be replaced* with code written in proper syntax so that upon compilation, the program will compute altitude, for indices ialt varying from 1 to 41, based on the integration of Eqn (1) using the trapezoidal rule. The program outputs results of the calculation to unit 99. You are welcome to rename the output of the program, either by opening a file with a specific name and unit # 99 within FORTRAN, or at the Linux command line.

Also *please add comments to the code* where requested.

As for many of our exercises, you should be able to quickly determine, by inspection of a particular file, the expected value of altitude. You are welcome to consult this website:

http://www.engineeringtoolbox.com/standard-atmosphere-d_604.html#

should you so desire for the completion of this exercise, or any other public website for that matter.

Once your coding is complete, please answer the following questions:

a) How close is your calculated value of altitude to the expected value?

b) Speculate as to why these values of altitude might differ?

c) Why did we use the trapezoidal rule to carry out the integration? Could we have used Simpson's rule?

d) How different is altitude if found using Eqn (2) rather than Eqn (1) for the “standard value” of g at mean sea level?

Note: to answer this question you will need to modify the code, making use of variable “grav_standard”.

e) What are the consequences of numerical representation of the hypsometric equation using Eqn (2) rather than Eqn (1)?

For this part of the assignment, please turn in a hard copy of your code printed using enscript and showing the full path name, as well as any output of the code (printed same way) used to answer these questions.

Extra credit (max 10 points … we are looking for specific answers to receive full credit).

Code ~rjs/aosc652/week_06/data_integrate.f contains a subroutine that allows for integration of a function using Simpson's rule, whether the set of points specified for the evaluation of the integral is even or odd. Traditionally, Simpson's rule can only be applied if the number of evaluation points is odd.

How did we get around this restriction?

Why do we compare diff1 and diff2, and choose a different interval for the evaluation of zint_extra in the code?

We purposefully left off comments from this segment of the code, so that you can try to decipher what is happening. All too often, you will need to decipher uncommented (or poorly commented code) in your career, without the benefit of being able to readily speak with the author of the code. It will always be good for *you to add comments to your codes*, particularly as a benefit should you return to a piece of software long after it was originally written ☺.