Assignment #9: Fourier Series and Spectral Analysis AOSC 652

Due: Monday, 31 October 2016 (at start of class)

Name: ___________________________
130 points total
Late penalty: 10 pts per day

> This assignment involves computation of the Power Spectrum of the Vostok Climate record, one of the most important and measurements in all of the Atmospheric and Oceanic Sciences. You may complete the computation of the Power Spectrum using either Python, IDL, or Matlab and you can form the requested graphical elements using either hppltd, Python, IDL, or Matlab. You are welcome to check your result using the FORTRAN code fourier_analysis.f, should you so desire, or an analog of subroutine power_simple in this code that you implement in Python, IDL, or Matlab.

File **~rjs/aosc652/week_09/vostok_climate_record.dat** contains a time series of temperature reconstructed from the Vostok Ice core that was drilled to a depth of 3600 meters above the surface of Vostok Lake in East Antarctica (78°S, 106°E). Temperature for the past 422,766 years has been estimated from measurements of the isotopic composition of ice, as described by Petit *et al., Nature*, 1999. The actual data in this file were obtained from the World Data Center for Paleoclimatology located in Boulder, Co. The data can be found at:

> http://www.ncdc.noaa.gov/paleo/icecore/antarctica/vostok/vostok_isotope.html

Please copy file **vostok_climate_record.dat** to your work area and explore the contents of this file.

You may complete the computation of the power spectrum for this assignment using either Python, IDL or Matlab. You have already used programs such as **~rjs/aosc652/week_09/fourier_analysis.f** and **~rjs/aosc652/week_09/sunspot.py and ~rjs/aosc652/week_09/sunspot.pro** to compute the *normalized power spectrum* of the sunspot number time series. In all that follows, we shall again look at the *normalized power spectrum*, this time for the Vostok ice core.

a) (30 points) Compute ***and plot*** the *normalized power spectrum* of the Vostok Temperature Record, in coordinates of both frequency and time, using the raw data set unmodified, for:

> i) the coordinate of frequency, from a starting point of 5×10^{-6} yr^{-1}, to an end frequency of 6×10^{-5} yr^{-1}, in steps of every 10^{-6} yr^{-1}
>
> ii) the coordinate of time, from a start time of 1000 years, to an end time of 200,000 years, in steps of every 1000 years. For the time plot, have the left hand side of the x-axis correspond to 200,000 years and the right hand side correspond to 1000 years, so that the plot is roughly comparable to the Petite et al. figure shown below.
>
> Based on these two plots, for what frequency and time period does the maximum power of the Vostok temperature record occur? How do your power versus frequency and power versus time plots compare to the results of Petit *et al.*, 1999 (their Figure 4a, given below for your convenience)?

Notes:

Petite *et al.* showed their results in a single plot. You may proceed with two plots, one for frequency and the other for time.

If you complete this assignment in IDL, the FFT (Fast Fourier Transform) function requires date to be on an evenly spaced grid. To interpolate the Vostok temperature record to an evenly spaced grid, consider adding to your IDL program lines such as:

```
num_pts=10000.
year_min=149.
year_max=422766.
year_grid=year_min+lindgen(num_pts+1)*(year_max-year_min)/num_pts
data_gridded=interpol(year,data,year_grid)
```

where data contains the raw Vostok temperature record &
year is the time grid of the raw Vostok temperature record

If you complete this assignment in Python, please note the data also needs to be placed on a uniform grid, as described by Jeff in class, using code such as:

```
year_grid = np.linspace(year_min, year_max, num_pts_grid)
temp_grid = np.interp(year_grid, yearOrg, tempOrg)
```

If you complete this assignment in Matlab, please read all of Chapter 8 of Moler carefully and realize the data also needs to be placed on a uniformly spaced grid.

Understanding *nuances* such as whether data need to be evenly spaced, and devising solutions to meet such a requirement, are essential skills of a Numerical Analyst. If the IDL or Python FFT program is expecting input data to be evenly spaced but is given unevenly spaced data, the results will not make sense. If you are curious, you can pass the raw data to the IDL or Python FFT routine and examine the results. If you happen to do this, feel free to include a relevant figure and a brief commentary in the hardcopy of your assignment (no extra credit this week, however).

For your convenience, here is the key figure from Petite *et al.* :

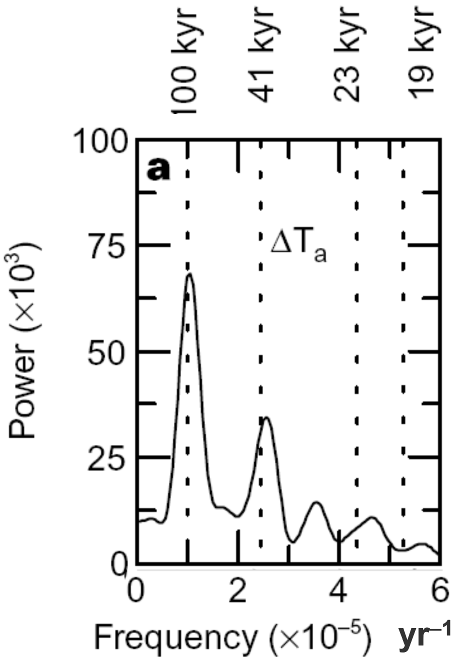

Figure 4a. Spectral properties of the Vostok time series of temperature. The normalized power spectrum (arbitrary units) is shown as a function of frequency (cycles/yr) and as a function of time (kyr, or thousands of years). The spectral analysis was done using the Blackman-Tukey method. Vertical lines correspond to the orbital periods of 100 kyr, 41 kyr, 23 kyr and 19 kyr years. From Petit *et al.*, *Nature*, 1999.

b) (50 points) As we discussed in class, the time series of temperature is too short to be properly analyzed for its power spectrum using the method we have applied to the much longer dial tone and sunspot time series. To obtain a scientifically meaningful result, we must analyze the data using either a correlation method (e.g., the Blackman-Tukey method used by Petit *et al.*) or we must "condition" the data by multiplying the time series by a Taper Function.

Repeat your analysis of the power spectrum of the Vostok Temperature Record, for the same start and end values of both frequency and time given above, using the ***Hanning Cosine Taper Function*** to ***condition the data***. As noted in class, the Hanning Cosine Taper Function is given by:

$$\text{Taper}(t) = 0.5\,(1 - \cos(2\pi\ t / T))$$

where T is the total length of the time interval under consideration.

Please note that here we are asking you to implement your own code within either Python, IDL, or Matlab that will first multiply the raw data by the cosine function described below, then to compute the power spectrum on the resulting product of the raw data × Taper (t)

Prepare two plots as for part a), showing the power spectrum found using the conditioned data.

Based on the four plots you have prepared so far:

i) How does the Power Spectrum found using the Hanning Cosine Taper function differ from the Power Spectrum found using the unconditioned data?

ii) How do results found using the Hanning Cosine Taper function compare to the results of Petit *et al*. (1999)?

c) (30 points) Programs like Python, IDL, and Matlab contain several functions that will condition data for you. Implement the Hanning function in your code, then repeat the analysis using this internal Hanning function. Finally, either state or show using additional plots how your results compare to those found using the Hanning Cosine Taper Function that you coded for part b).

d) (20 points) Prepare two more plots: the raw data that constitute the Vostok Temperature Record and the value of the product of the Vostok Temperature Record × the Hanning Cosine Taper function. Using these plots, comment on:

i) How does the Taper method work and why are the results different than those found using the unconditioned data?

ii) To what portion of the ***actual*** Vostok temperature record is the Power Spectrum found using the Taper method *most* sensitive? Please provide the answer and brief support for your answer.

iii) To what aspect of the ***actual*** Vostok temperature record is the Power Spectrum found using the Taper method *least* sensitive? Please provide the answer and brief support for your answer.

Please turn in at least six plots [two each for parts a), b), and d)], written (or typed) answers to the questions, plus all code (with full pathnames) used to complete the assignment. You are welcome to consult the Petit et al. paper, or any other material (other than notes from a prior year of this class ☺) for completion of this exercise.