Analysis Methods in Atmospheric and Oceanic Science

AOSC 652

Numerical Integration

Week 6, Day 3

- **Review HW #5**
- **Brief discussion of factors that control global climate and future warming**
- **General help with HW #6**

7 Oct 2016

AOSC 652: Analysis Methods in AOSC

Least Squares Fitting:

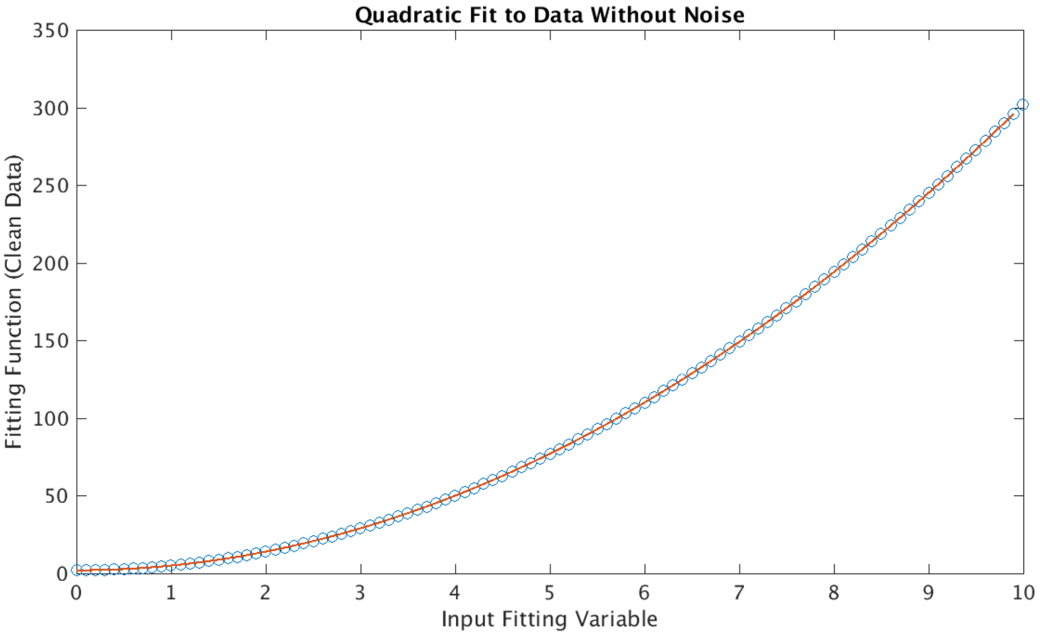
Fit to $y = 2.0 + 3.0\ x^2$

Coefficients are:

$a = 2.00001$

$b = -2.0 \times 10^{-5}$

$c = 3.0$



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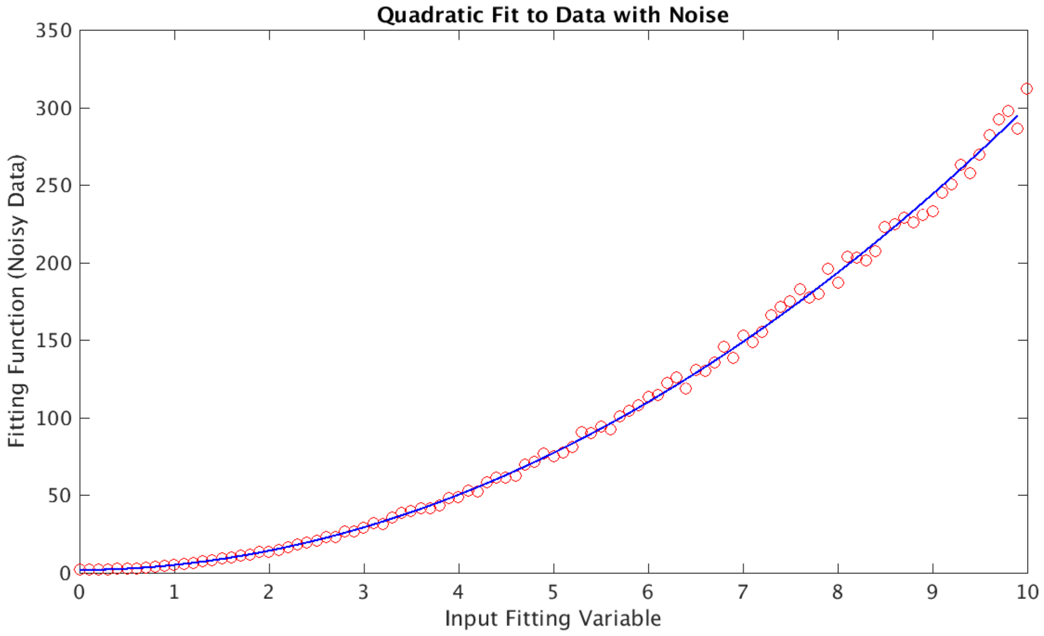

Fit to y = 2.0 + 3.0 x^2

Coefficients are:

a = 1.772
b = 0.289
c = 2.958

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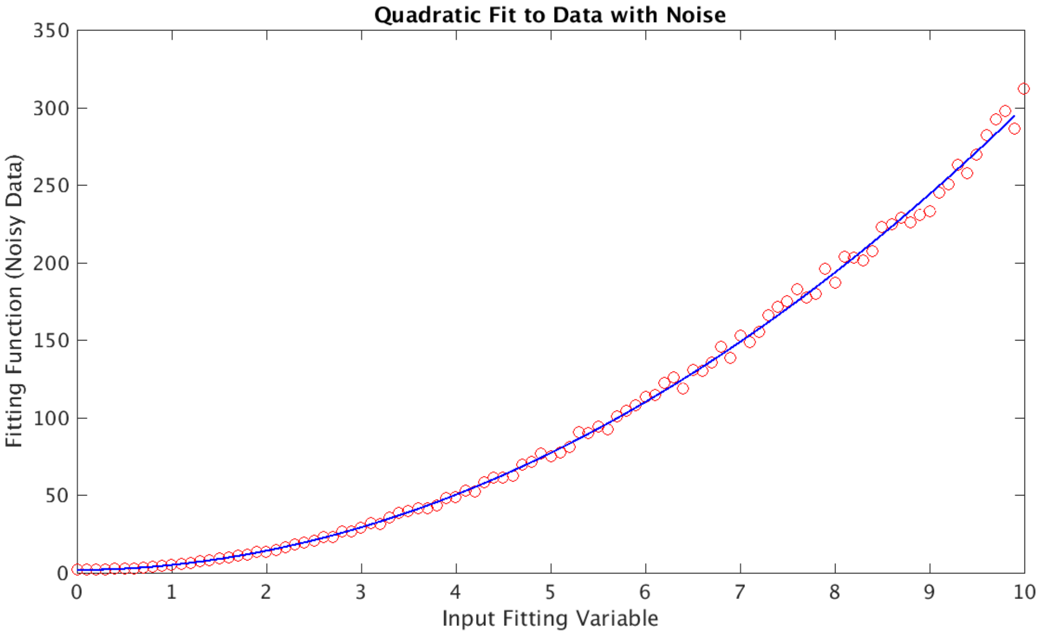
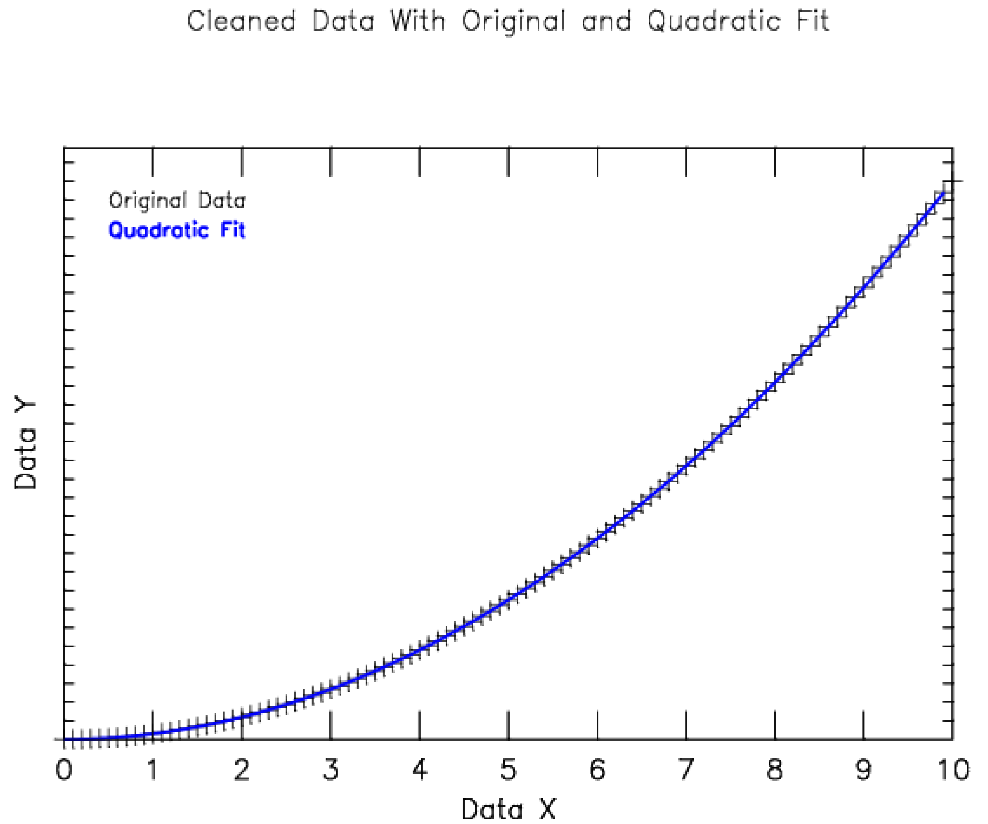


Do the plots "make sense"?

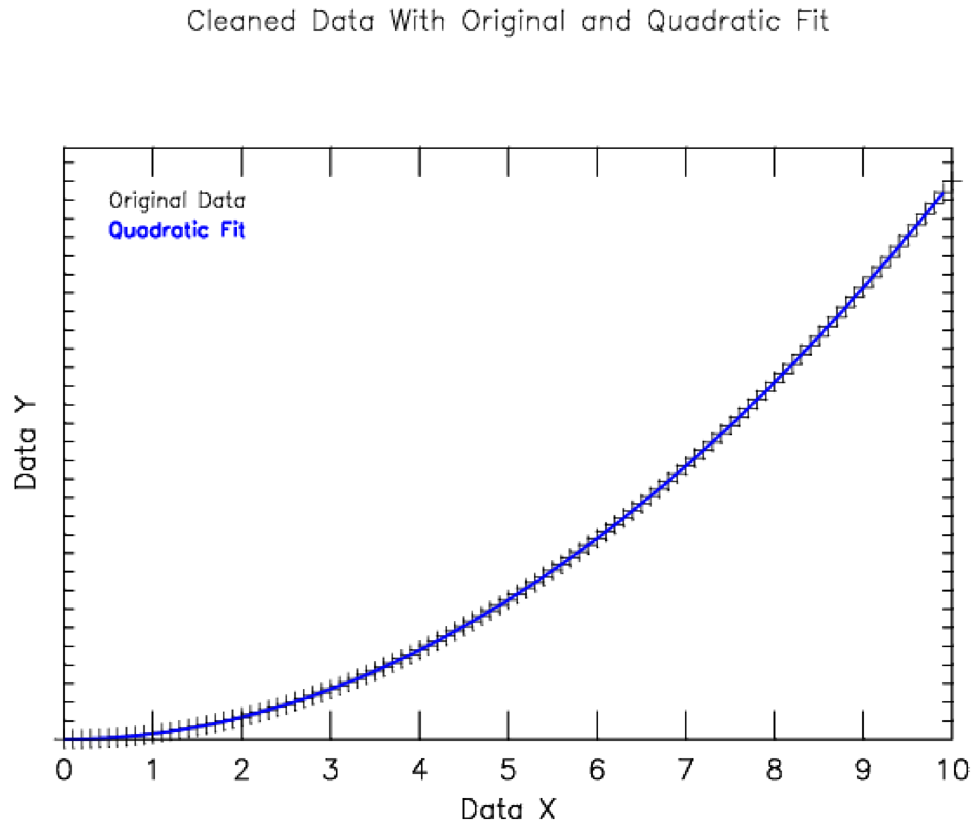
Yes, the fits look similar, despite significant differences in coefficients a & b, because the coefficient c controls the quadratic nature of the curve



Contents of stncl.* file:

```
quad_function.dat
2,1
   -1.00000
   0
   0
   2,  320,   20,   10
   0
Data Y
   0
   0
   0,   10,    1,    1
   0
Data X
   0
  120.00000
  180.00000
```

7 Oct 2016



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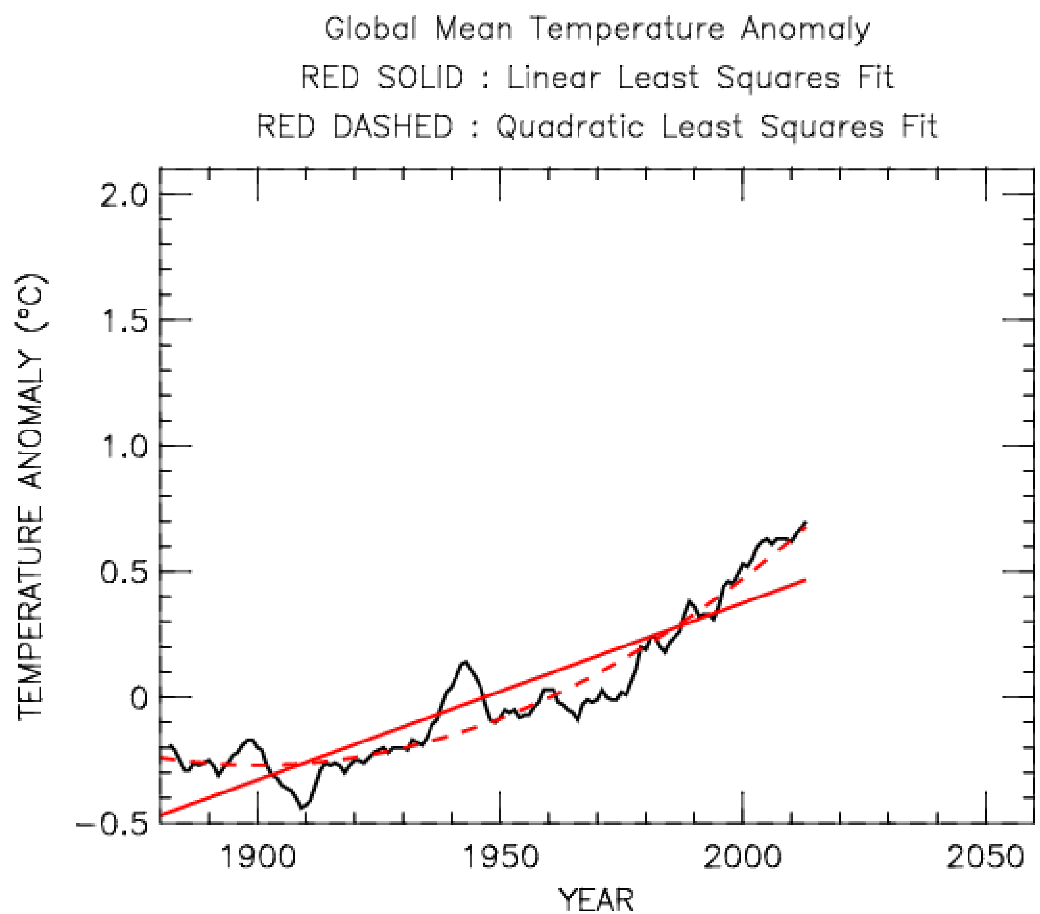
Least Squares Fitting:



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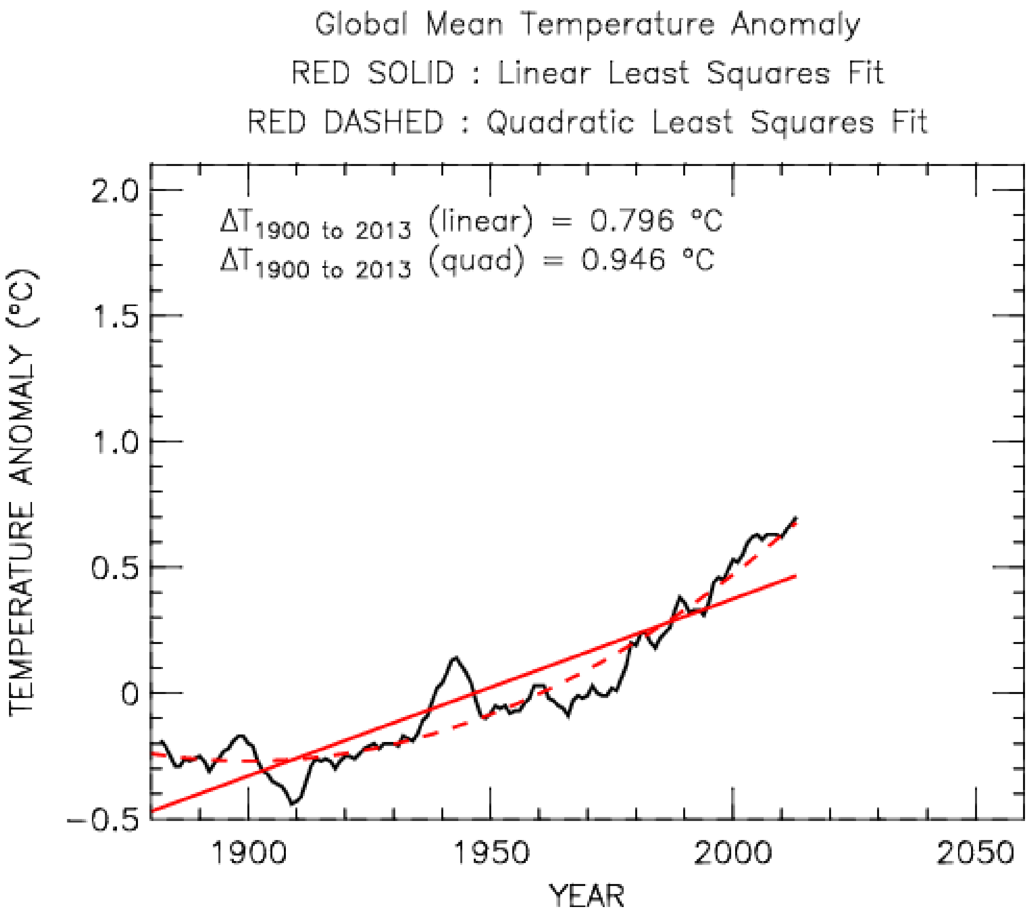

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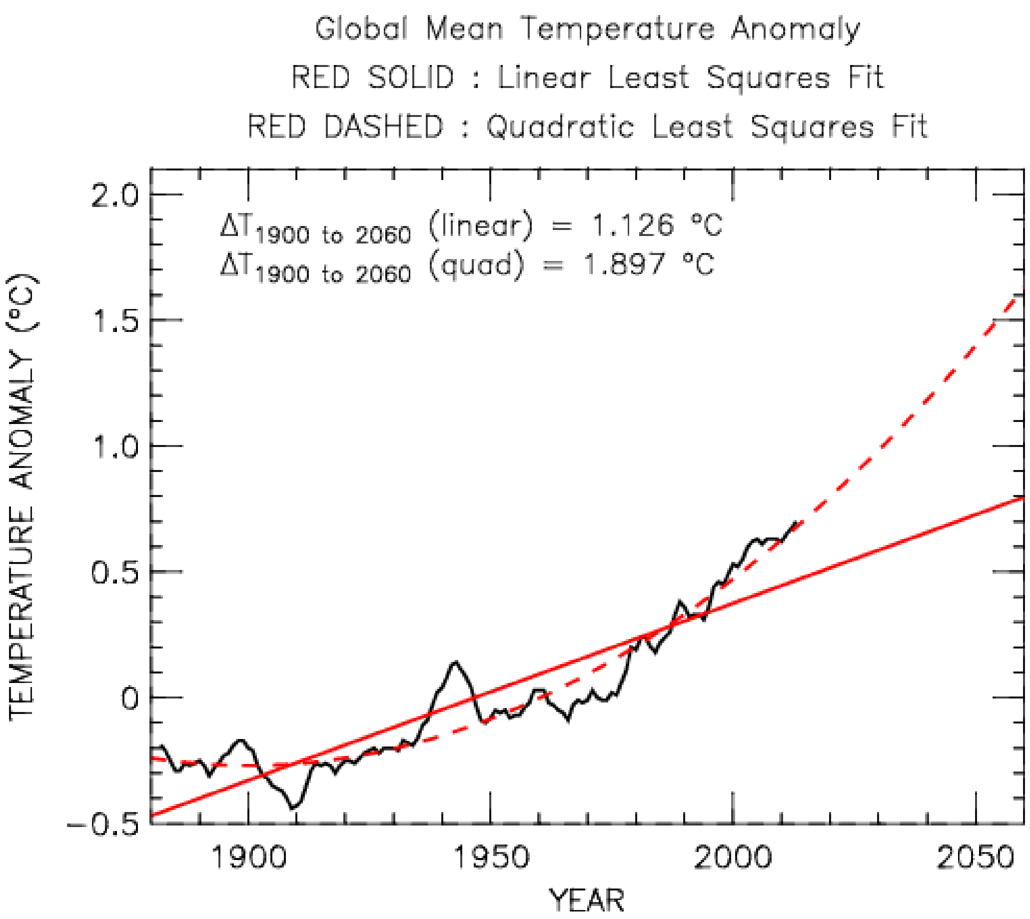

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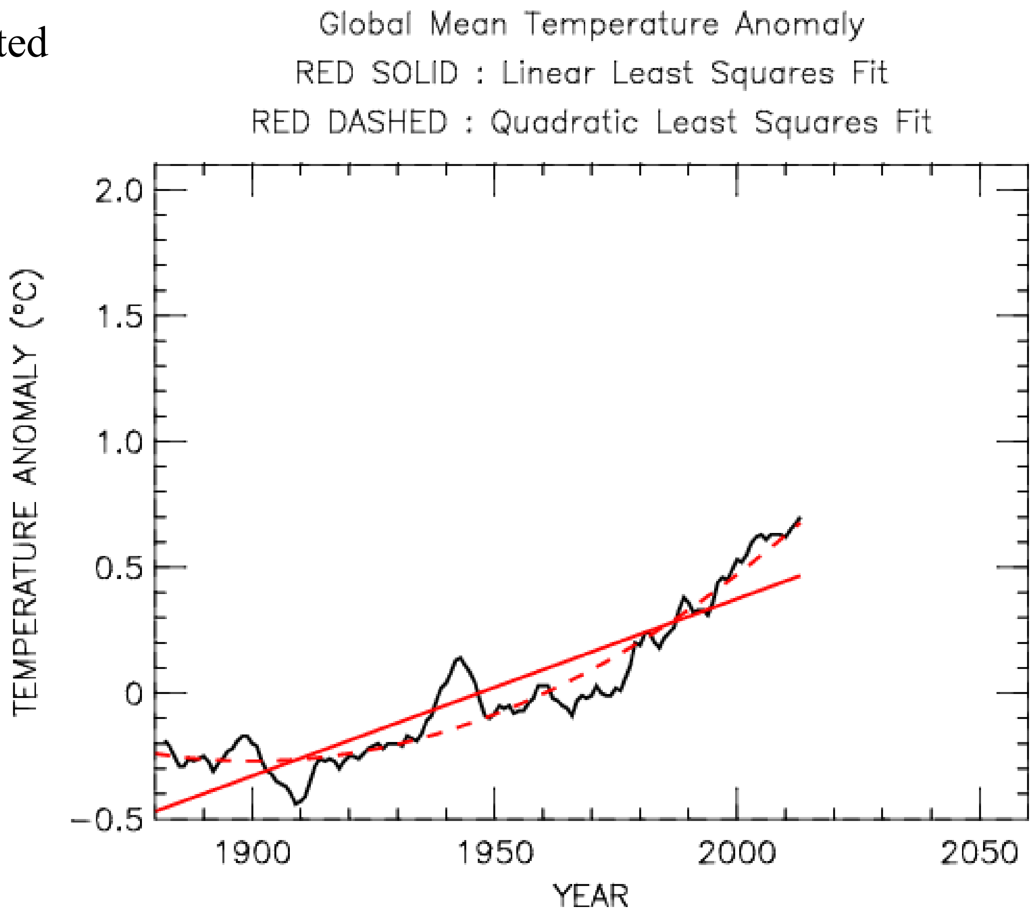
Least Squares Fitting:

$$\chi^2 = \frac{1}{N - 1 - \ \# \text{ of Fitting Parameters}} \sum_{i=1}^{N} \left(\frac{y_i - Fit(x_i)}{\sigma_i} \right)^2$$

where σ_i is the uncertainty associated with each measurement.



What do you expect for the respective values of χ^2 ?

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Least Squares Fitting:

$$\chi^2 = \frac{1}{N - 1 - \# \text{ of Fitting Parameters}} \sum_{i=1}^{N} \left(\frac{y_i - Fit(x_i)}{\sigma_i} \right)^2$$

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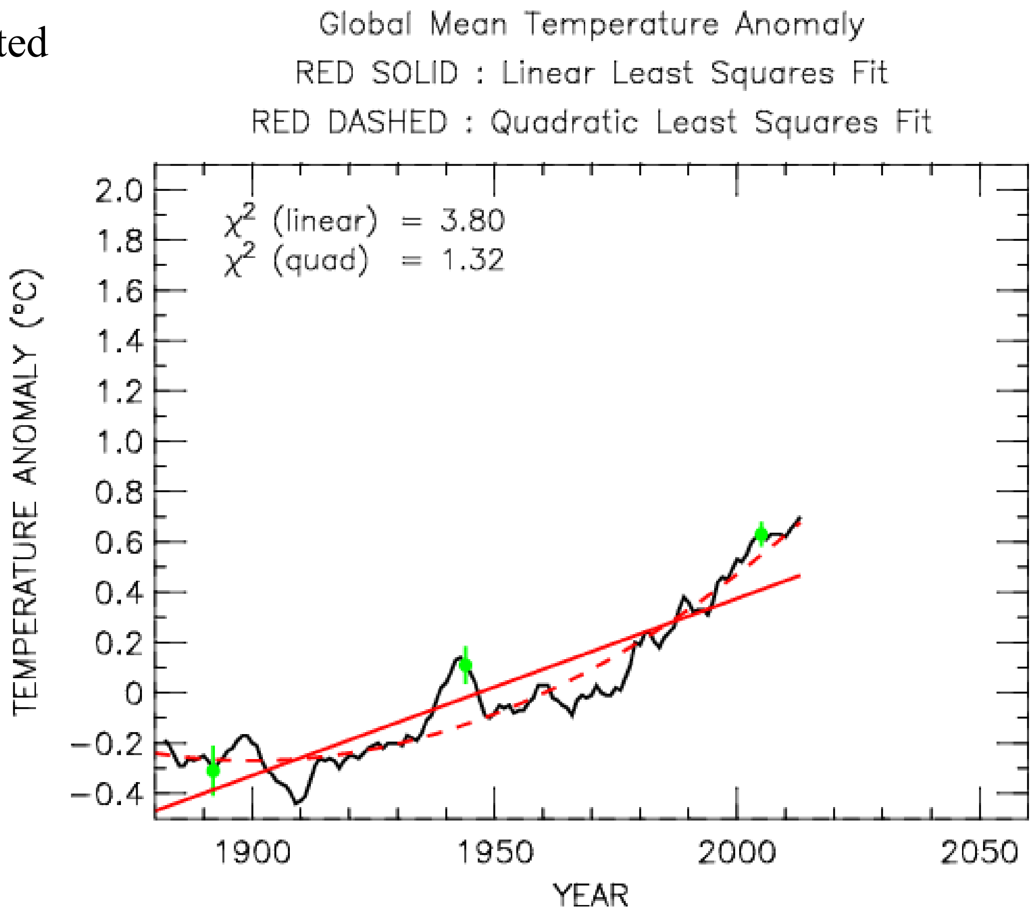


Quadratic fit follows data more closely than lienar fit; therefore, $\chi^2_{\text{QUADRATIC}} < \chi^2_{\text{LINEAR}}$

AOSC 652: Analysis Methods in AOSC

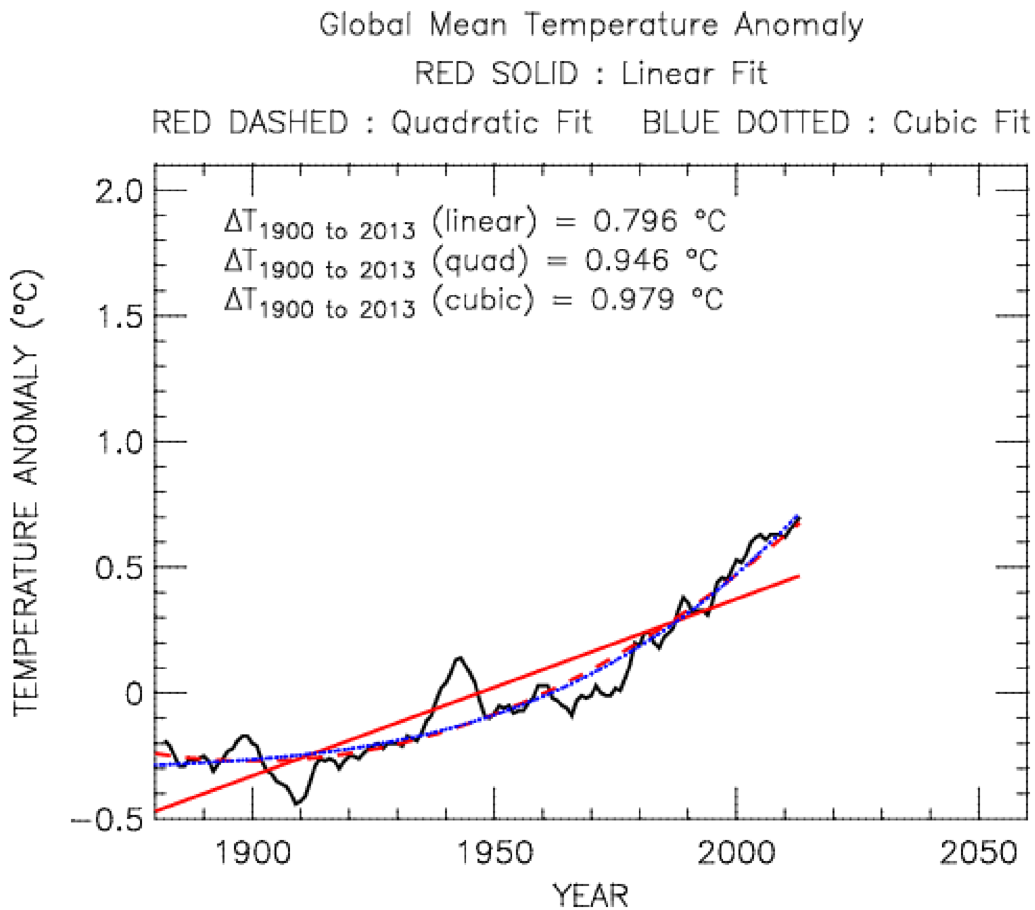
Cubic Least Squares:

$$y = a + b\,x + c\,x^2 + \,d\,x^3$$

Minimize:

$$Cost \text{ Funtion } \equiv \sum_{i=1}^{N} (a + b\,x_i + c\,x_i^2 + d\,x_i^3 - y_i)^2$$

See pages 666–670 of Press et al. for cubic fit



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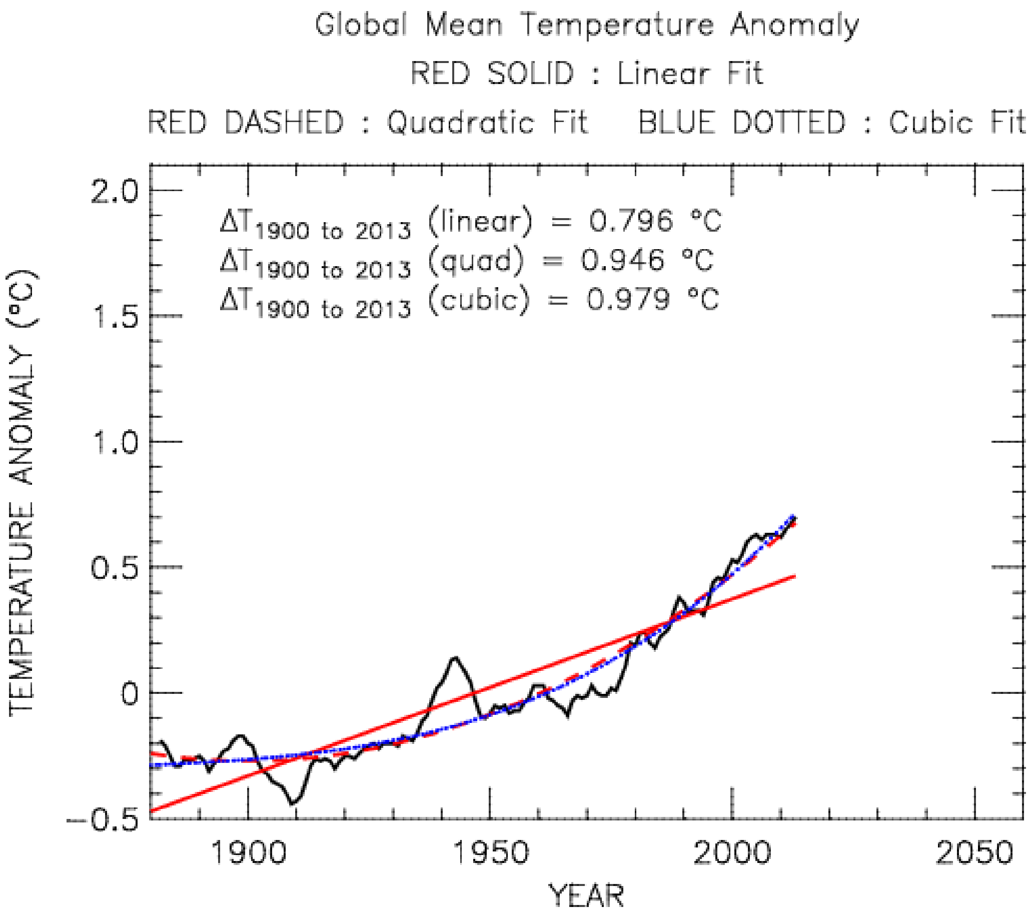

What do you expect for the respective values of χ^2 ?

AOSC 652: Analysis Methods in AOSC

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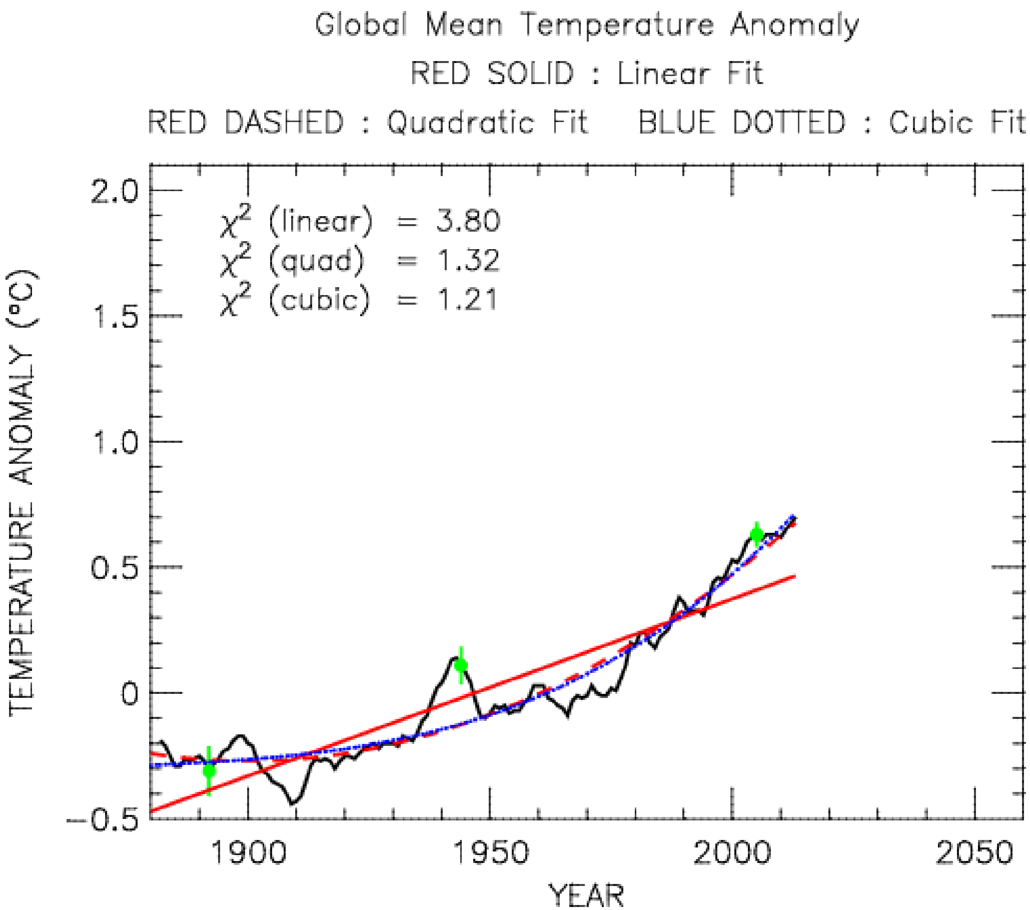
See pages 666–670 of Press et al. and computer code for cubic fit:

~rjs/aosc652/week_05/
data_fit.instructor.f



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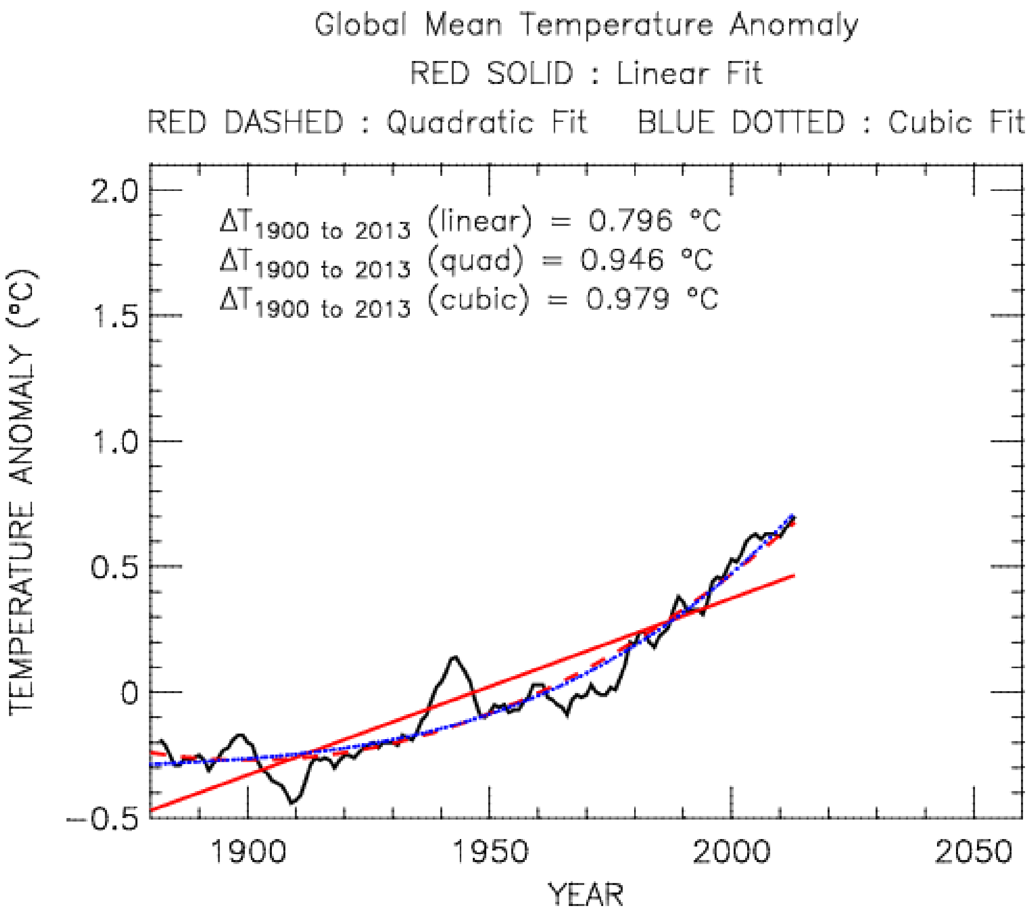

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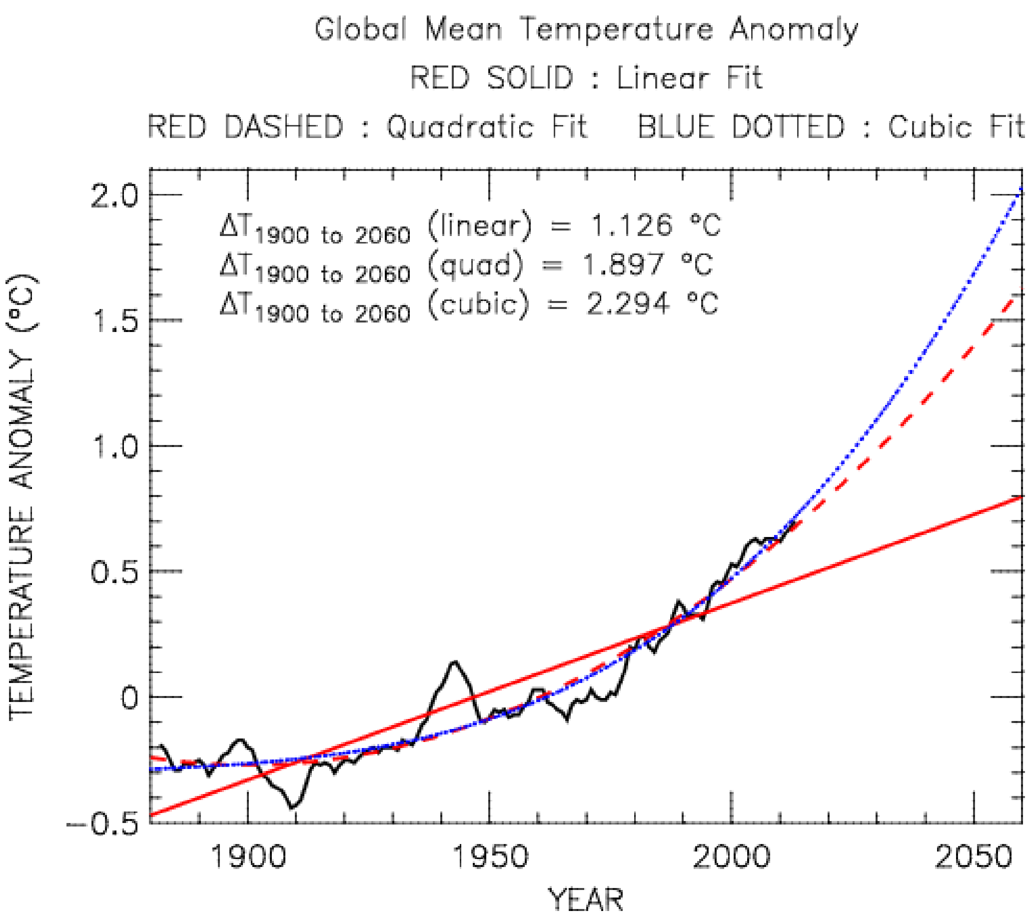
Clearly the projection of ΔT depends on the functional form of the fit !



AOSC 652: HW 05 Points Subtracted

Use of integer variable for floating point operation (i.e., narray), rather than float(narray) **In future will be 5 point deduction**	1
No units, ΔT = part of answer	2
Use of integer for floating point operation (i.e., 2060) rather than floating point (i.e., 2060**.**) **In future will be 5 point deduction**	3
Part 1: No discussion of > 10% difference, two values of ***a***	3
Part 2: No comparison of linear and least squares fits to actual data (after all, you made a figure)	3
Full range of data not shown on plot	At least 3
No comments in subroutine quad_lst_sqr **Comments are your friend**	5
Use of single precision inside of quad_lst_sqr, rather than double precision	5
No units, ΔT figure	5
Statement that functional fits don't matter for ΔT extrapolation	At least 5
Provision of incorrect numbers for ΔT extrapolation that bear no relation to figure	At least 8

AOSC 652: Analysis Methods in AOSC

Coding notes:

```
      subroutine quad_lst_sqr(xval,yval,narray,a_coeff,b_coeff,c_coeff)
      implicit double precision(a-h,o-z)
      dimension xval(narray),yval(narray)
      Real deter,x11,x12,x13,x21,x22,x23,x31,x32,x33
C       This program calculates quadratic least squares fit
C       to a given set of data passed in variables xval and yval.
C       The program returns the coefficients of the equation
C       y=a_coeff+b_coeff*x+c_coeff*x*x, for the best fit to the data.
C
C       Inputs: xval, yval, narray
C
C       Outputs: a_coeff,b_coeff,c_coeff
C
C       Initial definition of terms in matrix to calculate quadratic
C       least squares fit.

...

C       Calculate the determinant for the matrix
      deter=narray*(x2sum*x4sum-x3sum*x3sum)
     +  -xsum*(xsum*x4sum-x3sum*x2sum)+x2sum*(xsum*x3sum-x2sum*x2sum)
      write(*,*)'determinant =',deter

C       Find the different values of the inverse matrix

      x11=(1/deter)*(x2sum*x4sum-x3sum*x3sum)
      x12=(-1/deter)*(xsum*x4sum-x3sum*x2sum)
      x13=(1/deter)*(xsum*x3sum-x2sum*x2sum)
      x21=(-1/deter)*(xsum*x4sum-x3sum*x2sum)
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AOSC 652: Analysis Methods in AOSC

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      implicit double precision(a-h,o-z)
      dimension xval(narray),yval(narray)
      real*8 deter,x11,x12,x13,x21,x22,x23,x31,x32,x33
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AOSC 652: Analysis Methods in AOSC

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C     Inputs: xval, yval, narray
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C     Outputs: a_coeff,b_coeff,c_coeff
C
C     Initial definition of terms in matrix to calculate quadratic
C     least squares fit.

...

C     Calculate the determinant for the matrix
      deter=float(narray)*(x2sum*x4sum-x3sum*x3sum)
     + -xsum*(xsum*x4sum-x3sum*x2sum)+x2sum*(xsum*x3sum-x2sum*x2sum)
      write(*,*)'determinant =',deter

C     Find the different values of the inverse matrix

      x11=(1./deter)*(x2sum*x4sum-x3sum*x3sum)
      x12=(-1./deter)*(xsum*x4sum-x3sum*x2sum)
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AOSC 652: Analysis Methods in AOSC

Coding notes:

```
      subroutine quad_lst_sqr(xval,yval,narray,a_coeff,b_coeff,c_coeff)
      implicit double precision(a-h,o-z)
      integer xi,yi
      parameter (ndim=3)
      dimension xval(narray),yval(narray)
      dimension m(ndim,ndim),z(ndim),ei(ndim,ndim)
     ! real*8 D
C      This program calculates quadratic least squares fit
C      to a given set of data passed in variables xval and yval.
C      The program returns the coefficients of the equation
C      y=a_coeff+b_coeff*x+c_coeff*x*x, for the best fit to the data.
C
C      Inputs: xval, yval, narray
C
C      Outputs: a_coeff,b_coeff,c_coeff
C
      ! Define M matrix
      do xi=1,ndim
         z(xi)=0.0
        do yi=1,ndim
          m(xi,yi)=0.0
         enddo
      enddo
      do xi=1,narray
        m(1,1)=m(1,1)+1.
        m(2,1)=m(2,1)+xval(xi)
        m(3,1)=m(3,1)+xval(xi)**2.
        m(1,2)=m(1,2)+xval(xi)
        m(2,2)=m(2,2)+xval(xi)**2.
        m(3,2)=m(3,2)+xval(xi)**3.
        m(1,3)=m(1,3)+xval(xi)**2.
        m(2,3)=m(2,3)+xval(xi)**3.
        m(3,3)=m(3,3)+xval(xi)**4.
        z(1)=z(1)+yval(xi)
        z(2)=z(2)+yval(xi)*xval(xi)
        z(3)=z(3)+yval(xi)*xval(xi)**2.
      enddo
```

FORTRAN coding notes:

- **At top of a subroutine:**

  real a_matrix(nmax, mmax)

  will create a real*4 array called a_matrix

  Must use real*8

  or dimension (coupled w/ implicit double precision)

- **Please remember "." after integers, if integers are used in floating point calcs**

- **Please use float(narray) if integer narray is to be used in floating pt calcs**

Comments are particularly helpful as your coding becomes more complex

AOSC 652: Analysis Methods in AOSC

Coding notes:

```
% Creating Axes (not usually needed for most plots)                    % New
% Type and enter 'help axes' in the command window for details
ax1 = axes('Position',[0 0 1 1],'Visible','off');
ax2 = axes('Position',[.1 .3 .85 .65],'box','on');

axes(ax2); % Activate the smaller plot area axes                        % New
hold on;   % This setting prevents new plots from clearing previous plots   % New

if plot_mode == 1
   h1=plot(year,glo_5yr,'k.','LineWidth',2);
   h2=plot(year2,y_lin_fit,'r');
   h3=plot(year2,y_quad_fit,'b');
end

% Calculating Differences in Temperature Anomalies

linear_dif_2013_1900 = y_lin_fit(134) - y_lin_fit(21)
linear_dif_2060_1900 = y_lin_fit(181) - y_lin_fit(21)
quad_dif_2013_1900 = y_quad_fit(134) - y_quad_fit(21)
quad_dif_2060_1900 = y_quad_fit(181) - y_quad_fit(21)

% set the axis ranges
axis([1880 2060 -.6 1.8]);

% Add axis labels
xlabel('Year');
ylabel('Temperature Anomaly (C)');

% Add a title
title('5 Year Running Mean of the Global Average Temperature Anomaly');
```

Coding notes:

```
% Creating Axes (not usually needed for most plots)                % New
% Type and enter 'help axes' in the command window for details
ax1 = axes('Position',[0 0 1 1],'Visible','off');
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% Add a title
title('5 Year Running Mean of the Global Average Temperature Anomaly');
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   h2=plot(year2,y_lin_fit,'r');
   h3=plot(year2,y_quad_fit,'b');
end

% Calculating Differences in Temperature Anomalies

linear_dif_2013_1900 = y_lin_fit(find(year2==2013)) - y_lin_fit(find(year2==1900))
linear_dif_2060_1900 = y_lin_fit(find(year2==2060)) - y_lin_fit(find(year2==1900))
quad_dif_2013_1900 = y_quad_fit(find(year2==2013)) - y_quad_fit(find(year2==1900))
quad_dif_2060_1900 = y_quad_fit(find(year2==2060)) - y_quad_fit(find(year2==1900))

% set the axis ranges
axis([1880 2060 -.6 1.8]);

% Add axis labels
xlabel('Year');
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title('5 Year Running Mean of the Global Average Temperature Anomaly');
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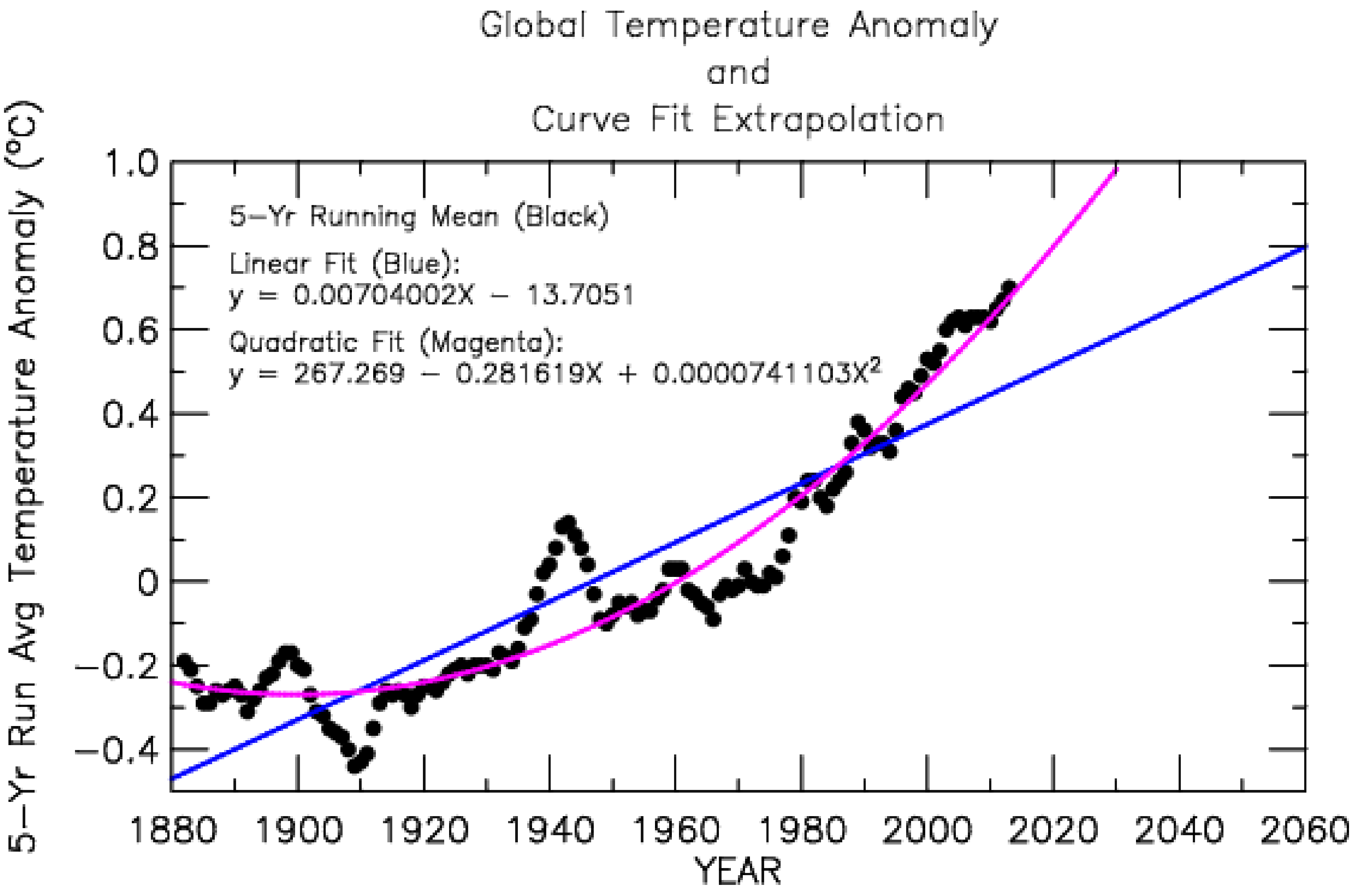
AOSC 652: Analysis Methods in AOSC

Let's look at a few more plots:



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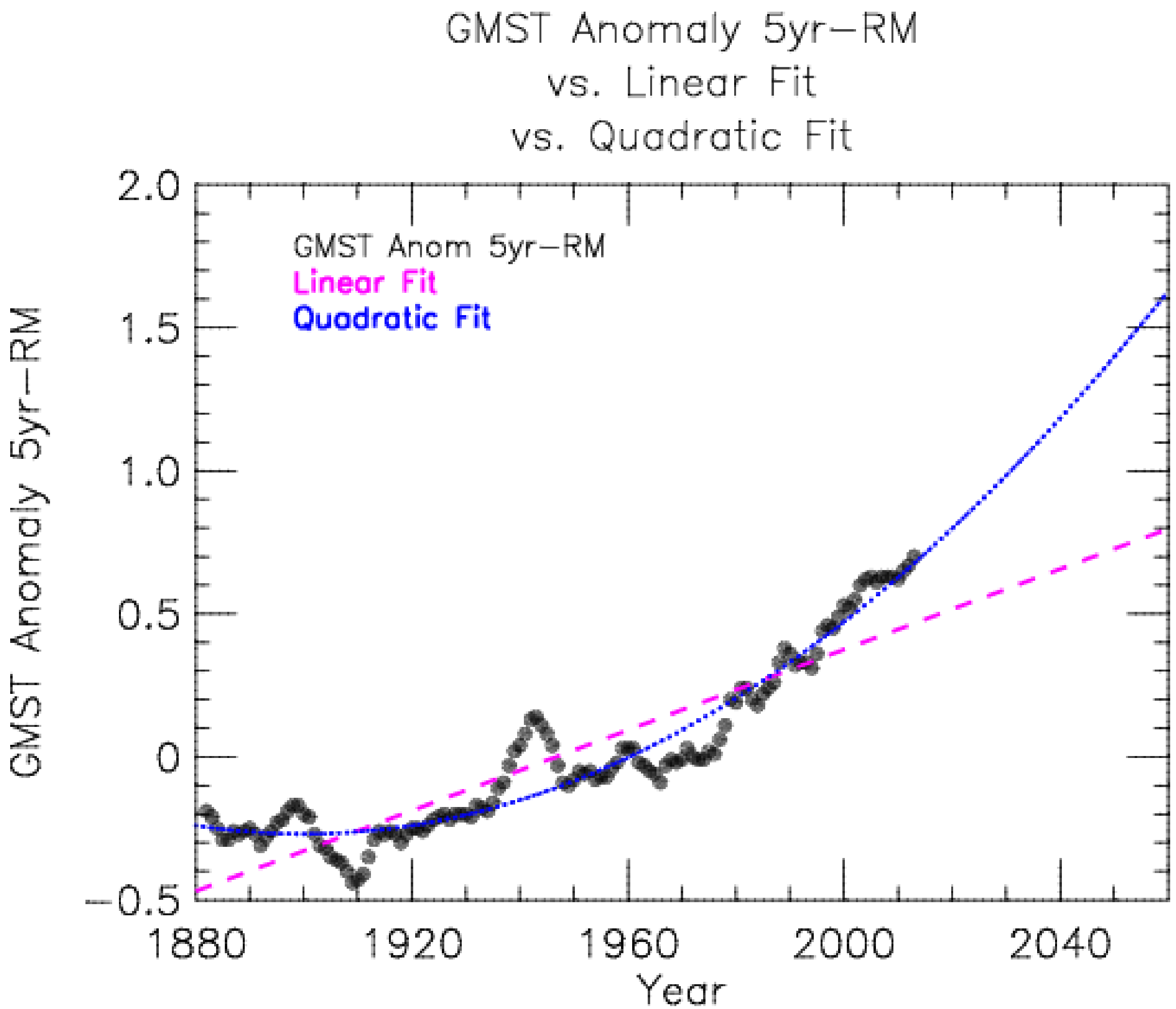

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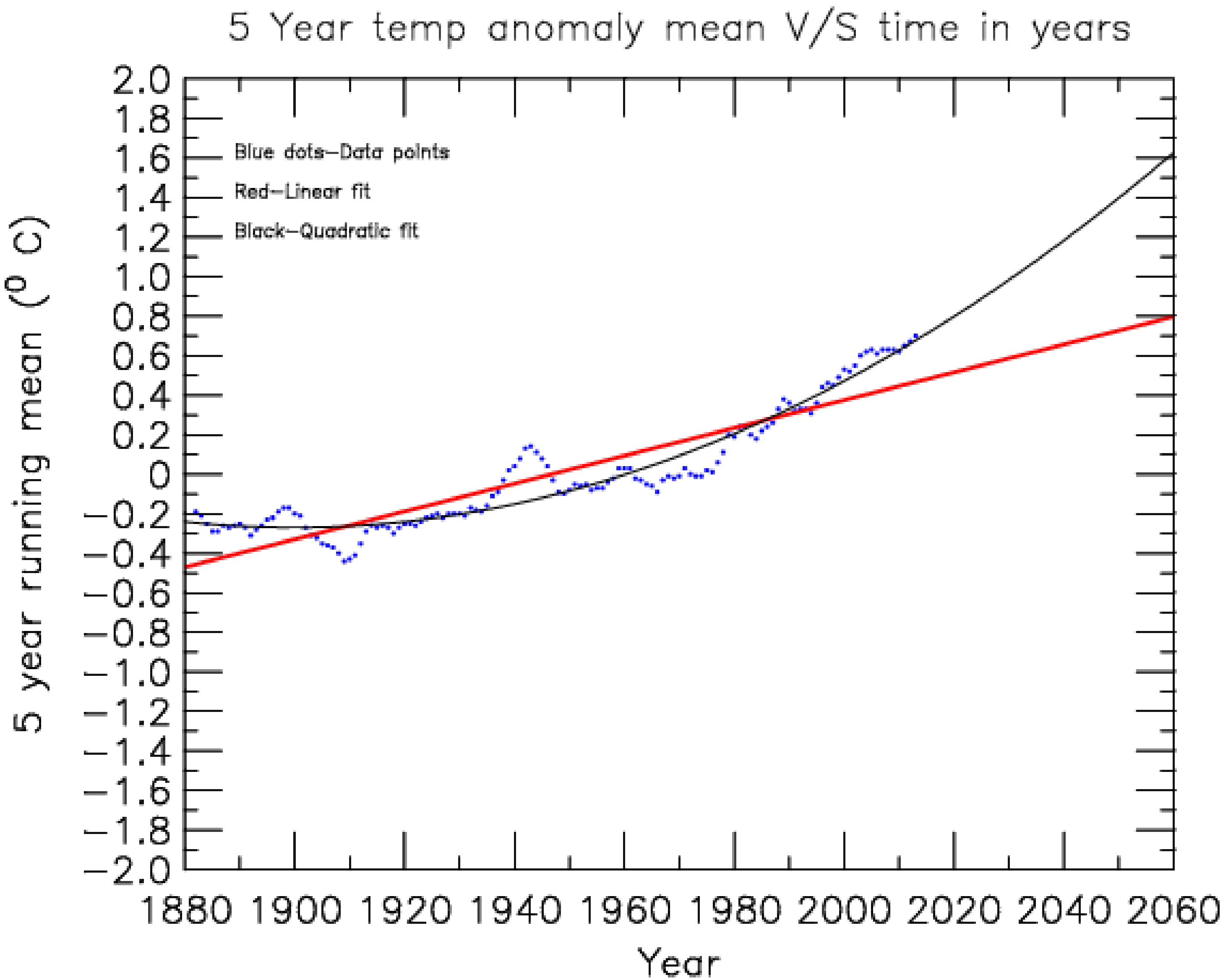

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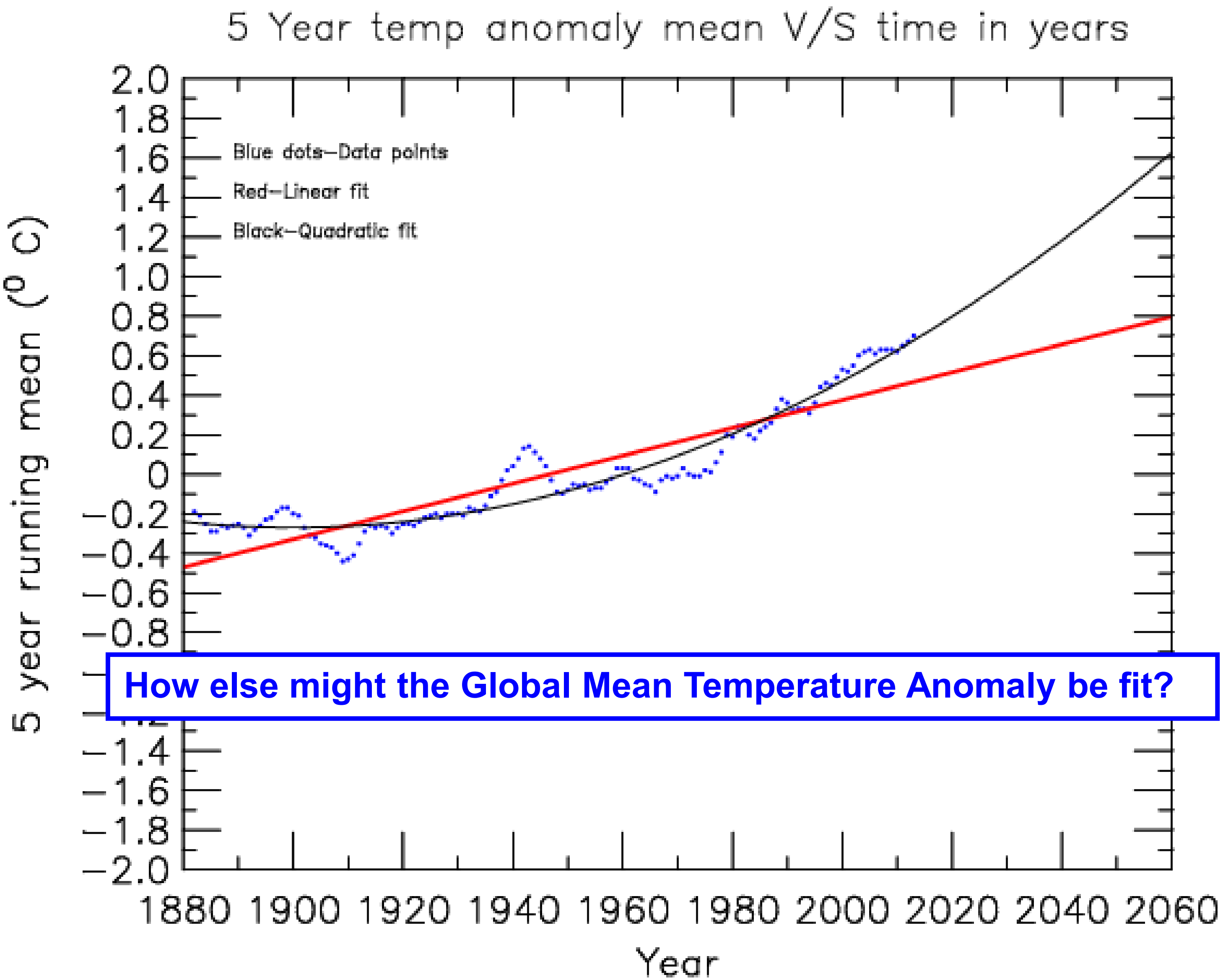
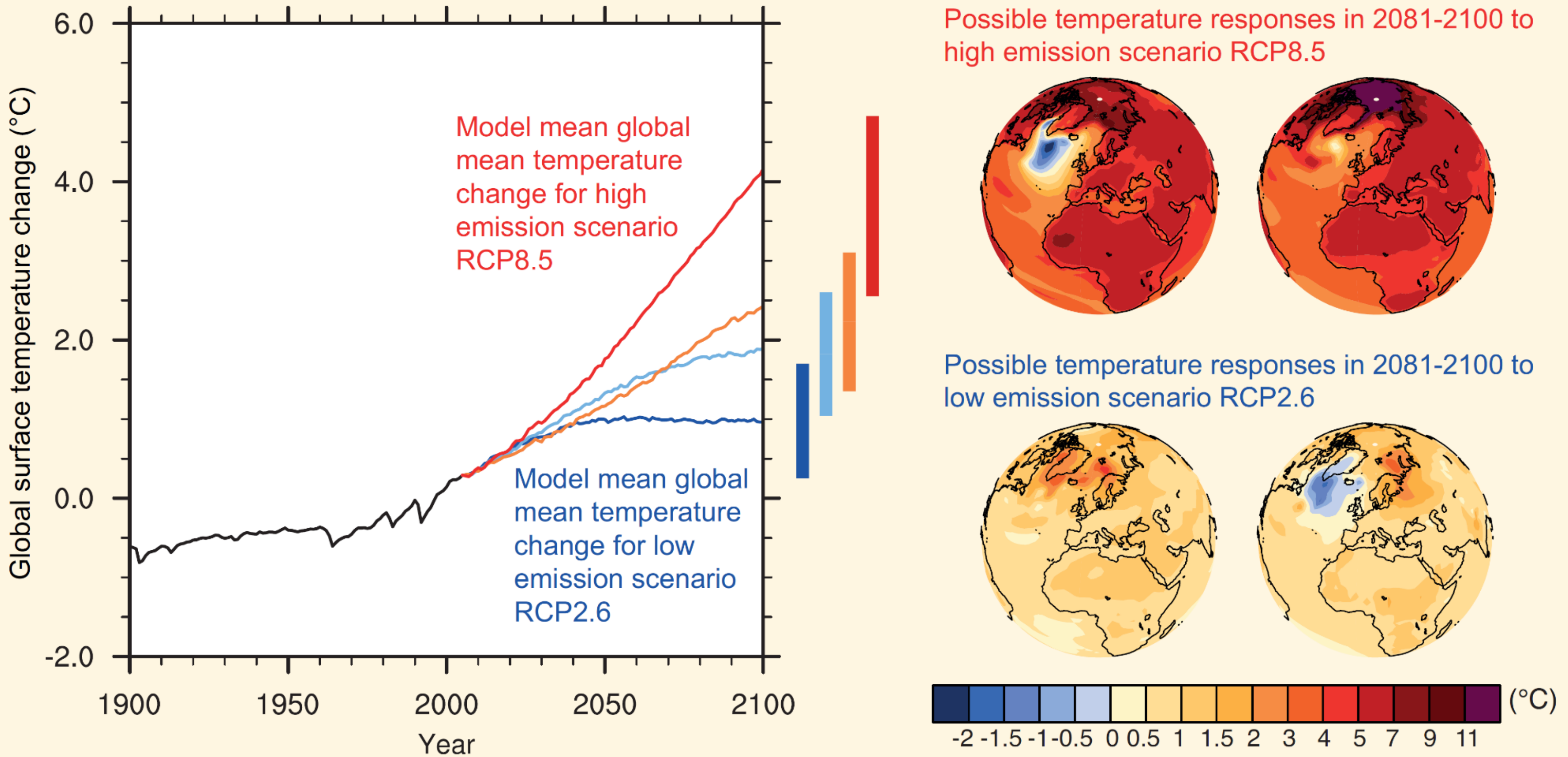


How else might the Global Mean Temperature Anomaly be fit?

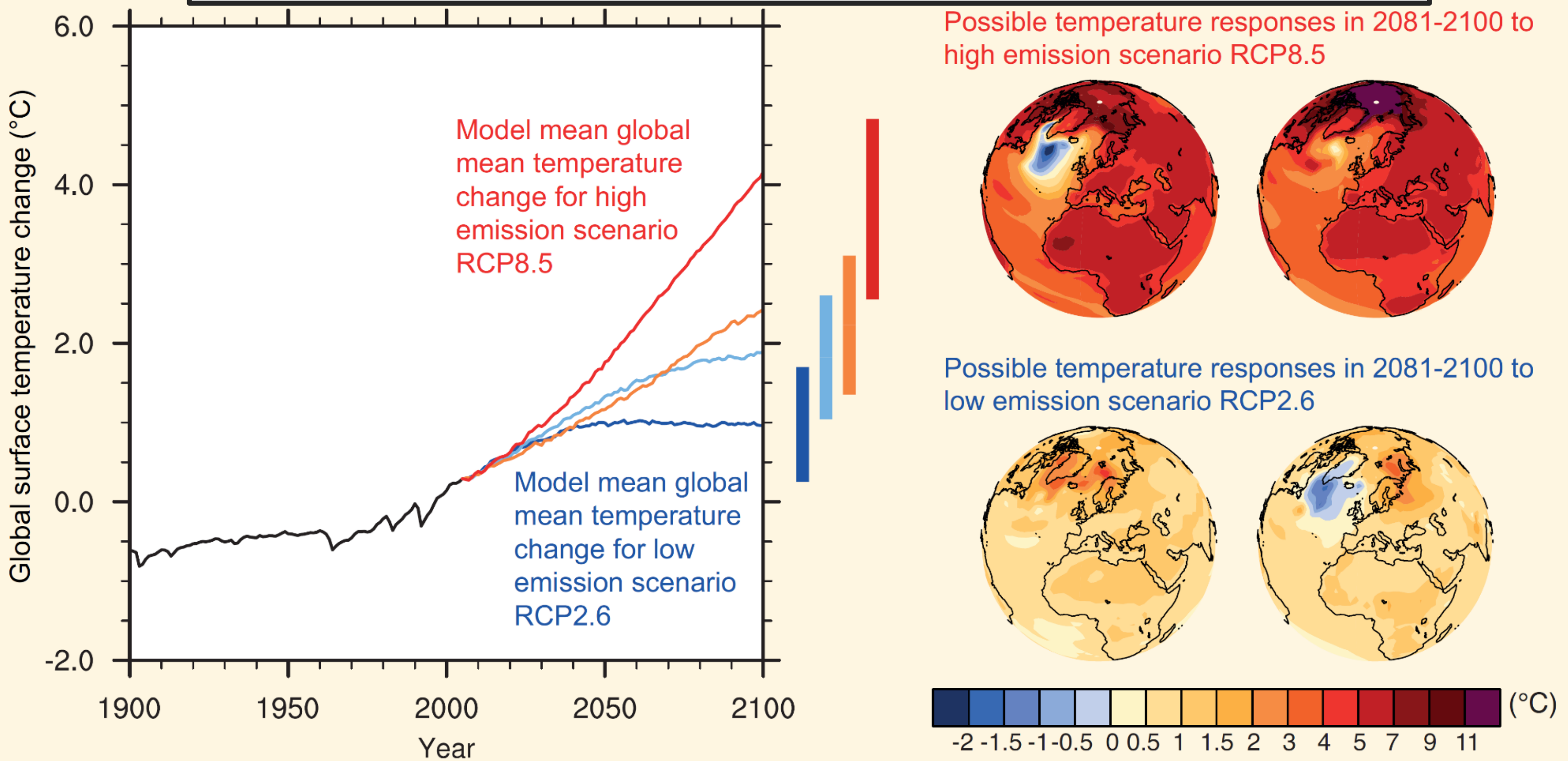
Global Warming Projections, IPCC (2013)



FAQ 12.1, Figure 1 | Global mean temperature change averaged across all Coupled Model Intercomparison Project Phase 5 (CMIP5) models (relative to 1986–2005) for the four Representative Concentration Pathway (RCP) scenarios: RCP2.6 (dark blue), RCP4.5 (light blue), RCP6.0 (orange) and RCP8.5 (red); 32, 42, 25 and 39 models were used respectively for these 4 scenarios. *Likely* ranges for global temperature change by the end of the 21st century are indicated by vertical bars. Note that these ranges apply to the difference between two 20-year means, 2081–2100 relative to 1986–2005, which accounts for the bars being centred at a smaller value than the end point of the annual trajectories. For the highest (RCP8.5) and lowest (RCP2.6) scenario, illustrative maps of surface temperature change at the end of the 21st century (2081–2100 relative to 1986–2005) are shown for two CMIP5 models. These models are chosen to show a rather broad range of response, but this particular set is not representative of any measure of model response uncertainty.

Question 12.1, IPCC, 2013

Are these projections accurate ?



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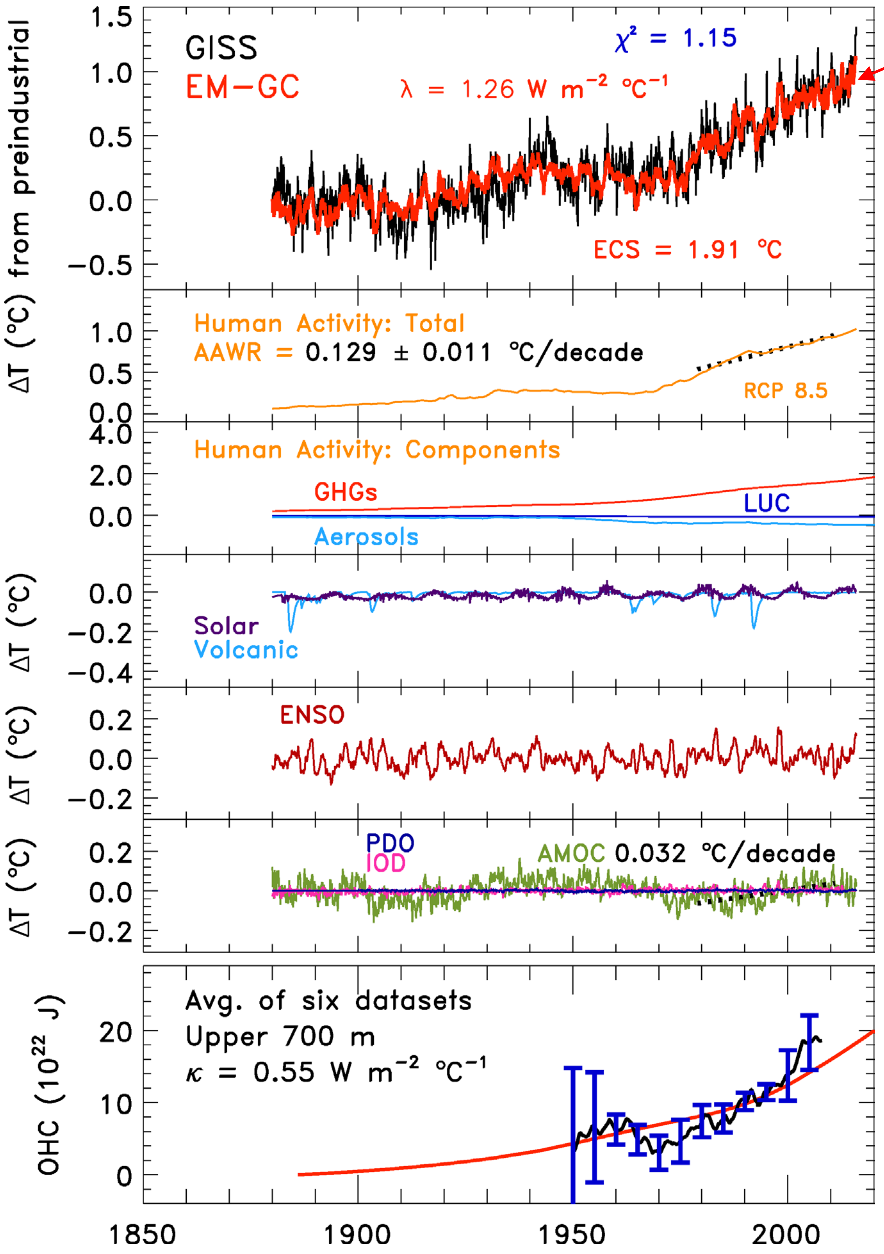
Question 12.1, IPCC, 2013

Aerosol RF = –0.9 W m^{-2}



$$\Delta T_{MDL\,i} = (1+\gamma_{TOT})\,(GHG\ RF_i + NAA\ RF_i) / \lambda_P + C_0 + C_1\times SOD_{i-6} + C_2\times TSI_{i-1} + C_3\times ENSO_{i-2} + C_4\times AMOC_i + C_5\times PDO_i + C_6\times IOD_i - Q_{OCEAN\,i} / \lambda_P$$

where

λ_P = 3.2 W m^{-2} / °C

$1+\gamma_{TOT} = \{1 - \Sigma(\text{Feedback Parameters}) / \lambda_P\}^{-1}$

NAA RF = net RF due to anthropogenic aerosols

SOD = Stratospheric optical depth

TSI = Total solar irradiance

ENSO = Multivariate El Niño South. Osc Index

AMOC= Atlantic Meridional Overturning Circulation

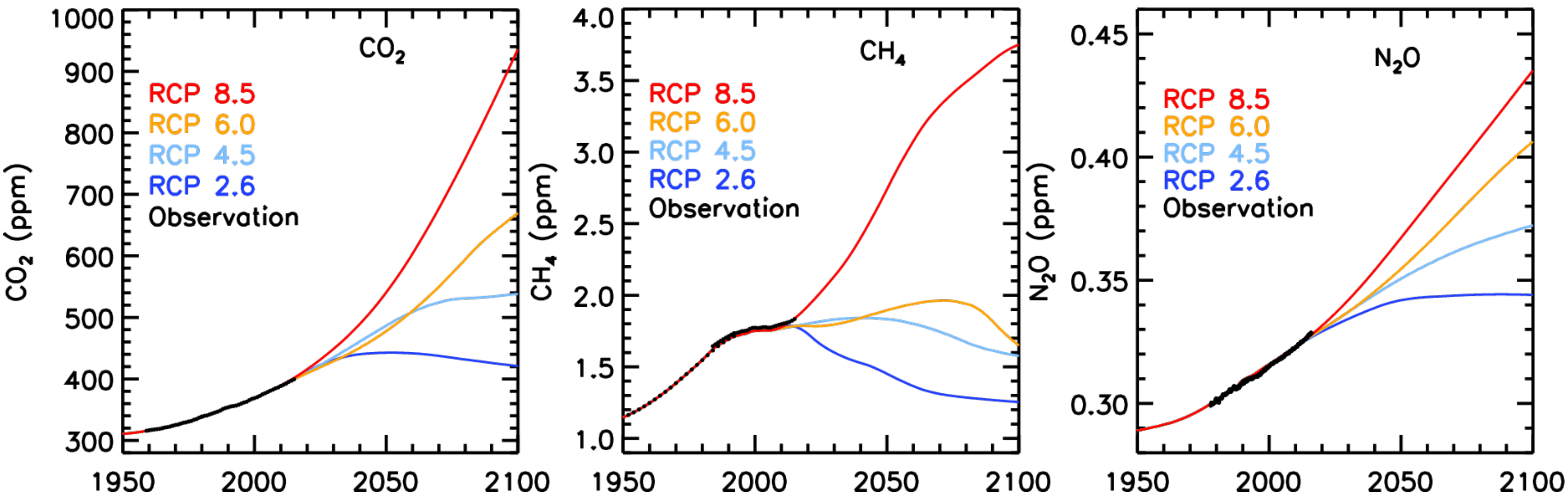
PDO = Pacific Decadal Oscillation

IOD = Indian Ocean Dipole

Q_{OCEAN} = Ocean heat export

ECS: Equilibrium Climate Sensitivity
rise in global mean surface T, for CO_2 doubling

Climate Projections Driven by Prescribed GHG Levels



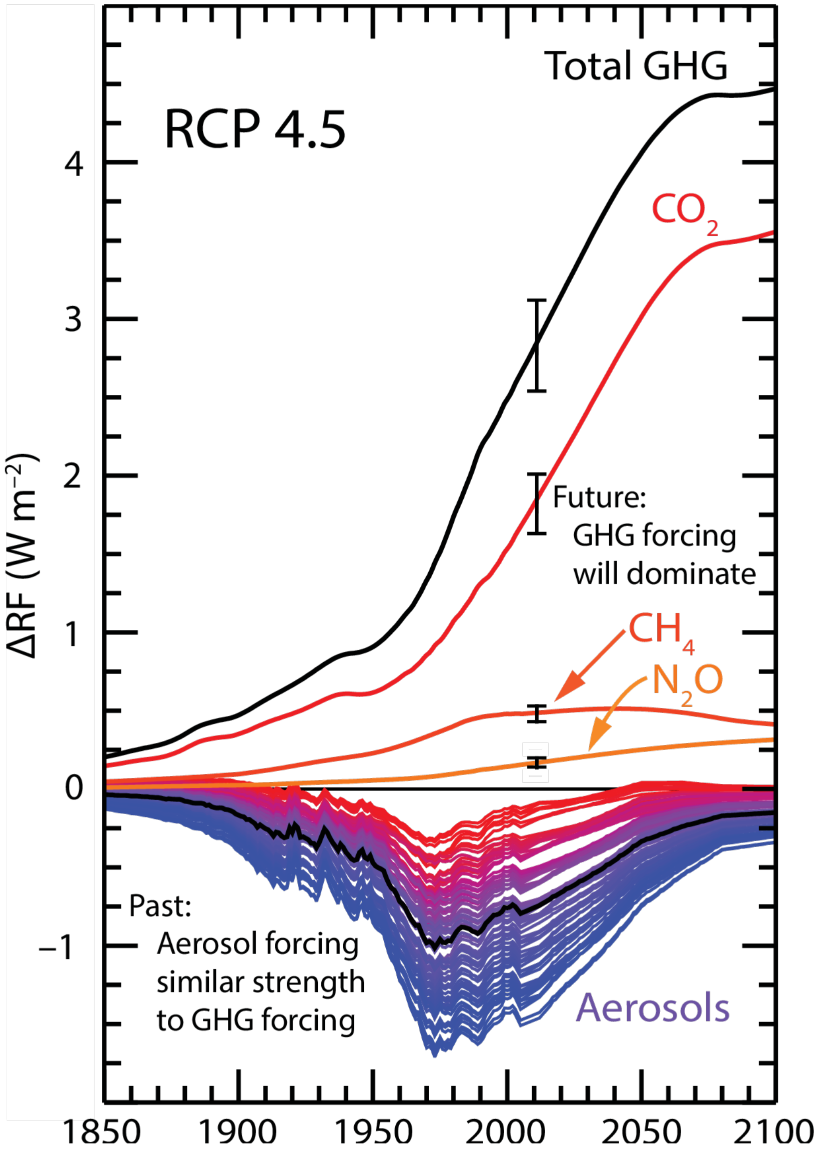
- **RCP**: Representative Concentration Pathway
  Number represents W m^{-2} RF of climate that occurs at end of this century, for each scenario

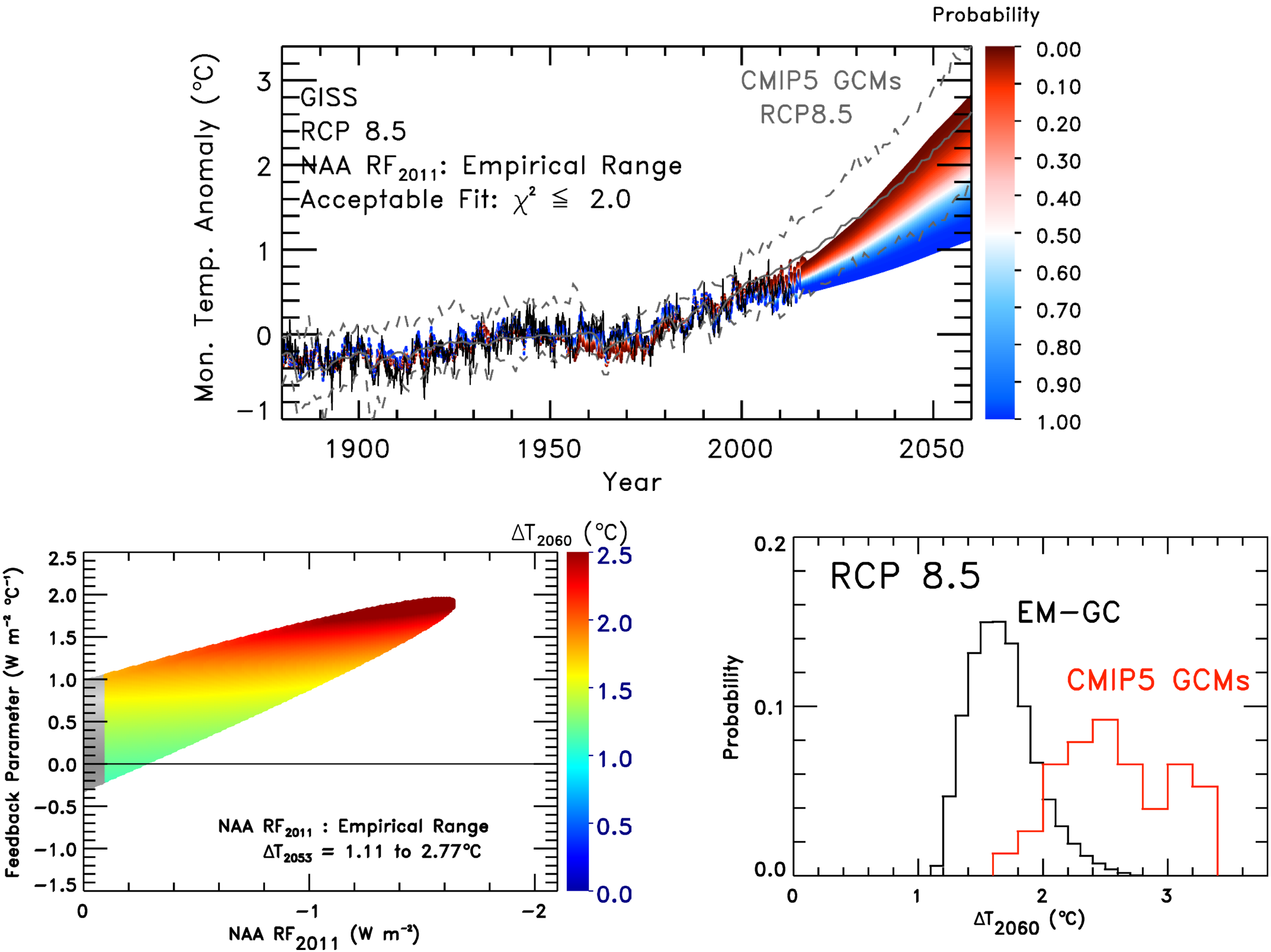
- Time series of CO_2, CH_4, N_2O as well as CFCs, O_3 etc provided to climate model groups

35

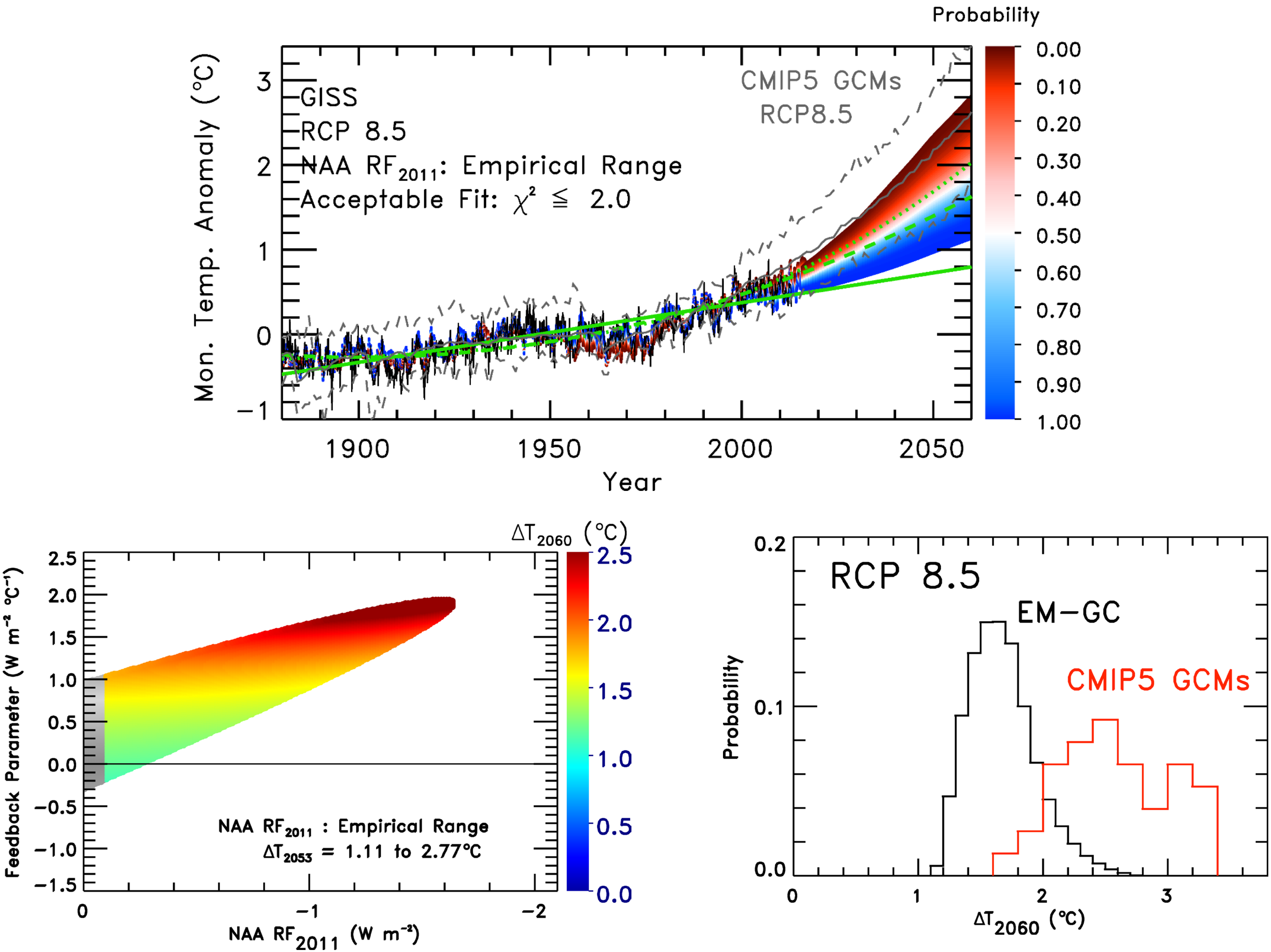
In the past, tropospheric aerosols have offset GHG induced warming, but precise offset is not well known.

In the future, this “mask” is going away due to air quality concerns.





Probability
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
GISS
RCP 8.5
NAA RF_{2011}: Empirical Range
Acceptable Fit: $\chi^2 \leqq 2.0$
CMIP5 GCMs
RCP8.5
Mon. Temp. Anomaly (°C)
3
2
1
0
−1
1900
1950
2000
2050
Year
ΔT_{2060} (°C)
2.5
2.0
1.5
1.0
0.5
0.0
Feedback Parameter ($W\ m^{-2}\ °C^{-1}$)
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
NAA RF_{2011} : Empirical Range
ΔT_{2053} = 1.11 to 2.77°C
0
−1
−2
NAA RF_{2011} ($W\ m^{-2}$)
RCP 8.5
EM−GC
CMIP5 GCMs
Probability
0.2
0.1
0.0
0
1
2
3
ΔT_{2060} (°C)


Probability
GISS
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CMIP5 GCMs
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Mon. Temp. Anomaly (°C)
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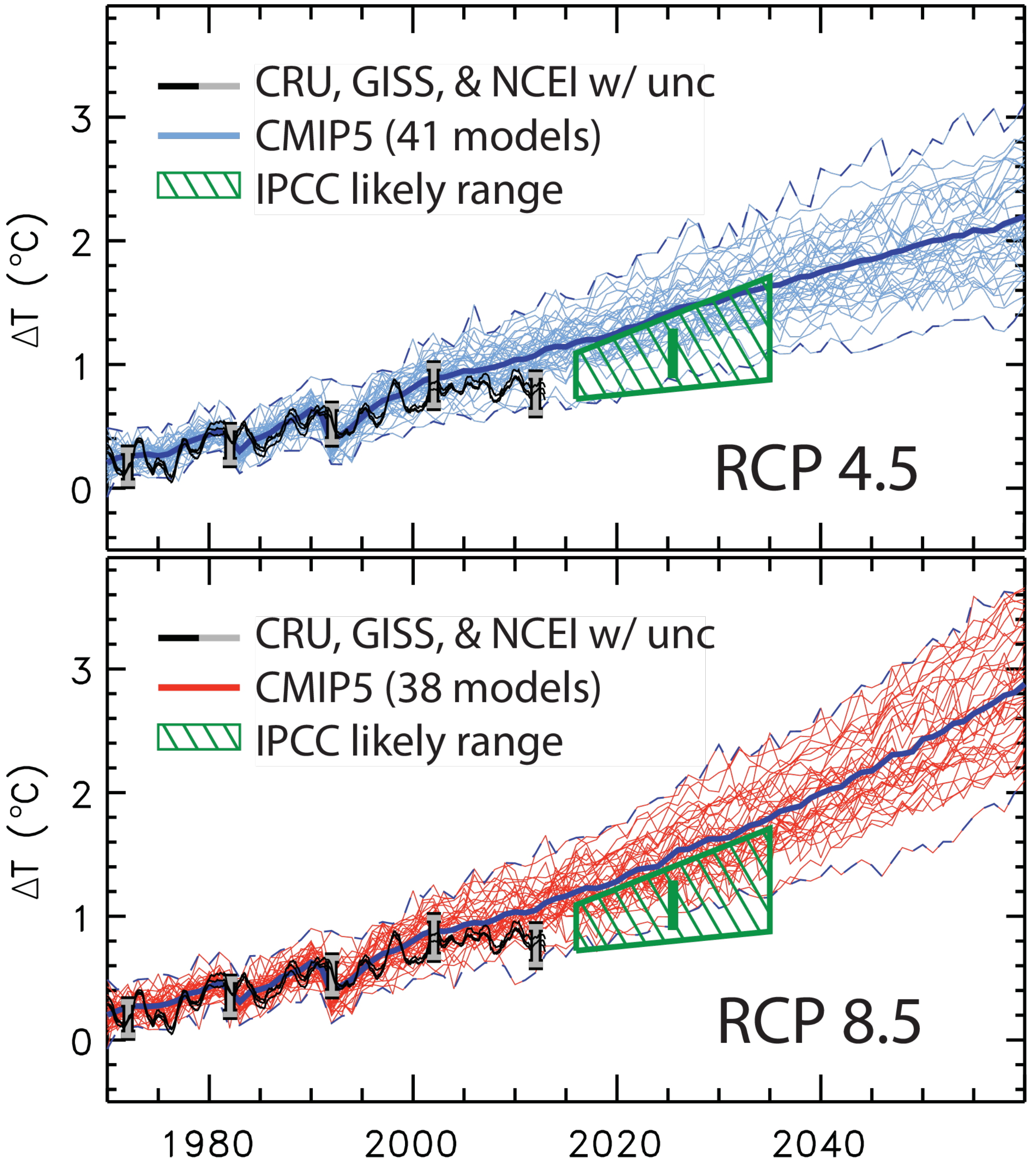
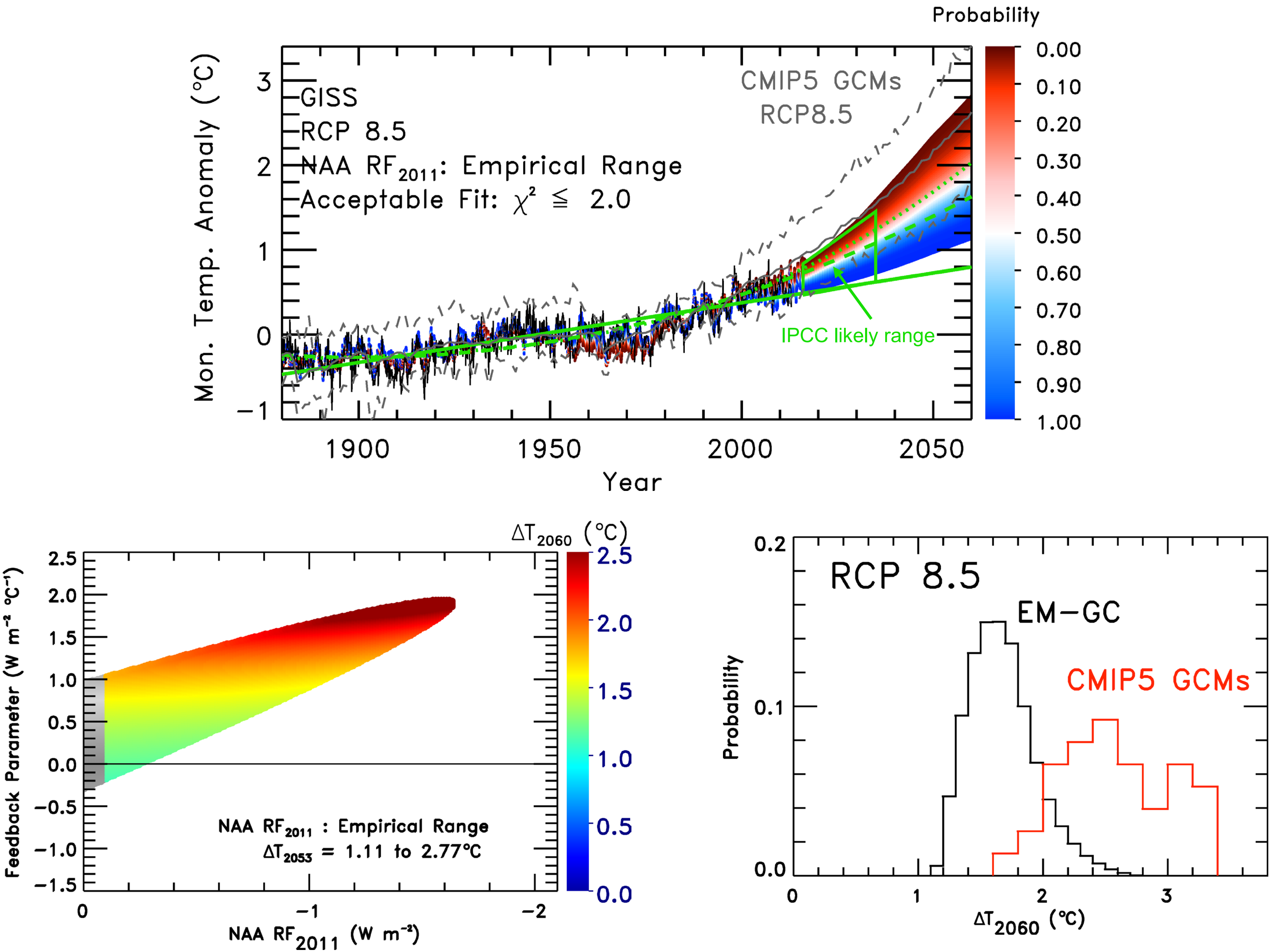


CRU, GISS, & NCEI w/ unc
CMIP5 (41 models)
IPCC likely range
RCP 4.5
CRU, GISS, & NCEI w/ unc
CMIP5 (38 models)
IPCC likely range
RCP 8.5
ΔT (°C)
0
1
2
3
1980
2000
2020
2040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