

# Analysis Methods in Atmospheric and Oceanic Science

AOSC 652

Introduction to Python and IDL

Week 8, Day 3

- **Review prior assignment**
- **General help with HW #8**

21 Oct 2016

# AOSC 652: Analysis Methods in AOSC

## Root Finding and Function Minimization

– Newton-Raphson:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

– Secant:

$$x_{i+1} = x_i - \frac{f(x_i) [x_i - x_{i-1}]}{f(x_i) - f(x_{i-1})}$$

- Some functions difficult or impossible to differentiate
- Initial guess for  $x_1$  and  $x_0$  can be crucial for whether algorithm converges

# AOSC 652: Analysis Methods in AOSC

## Three reasons program failed to find root:

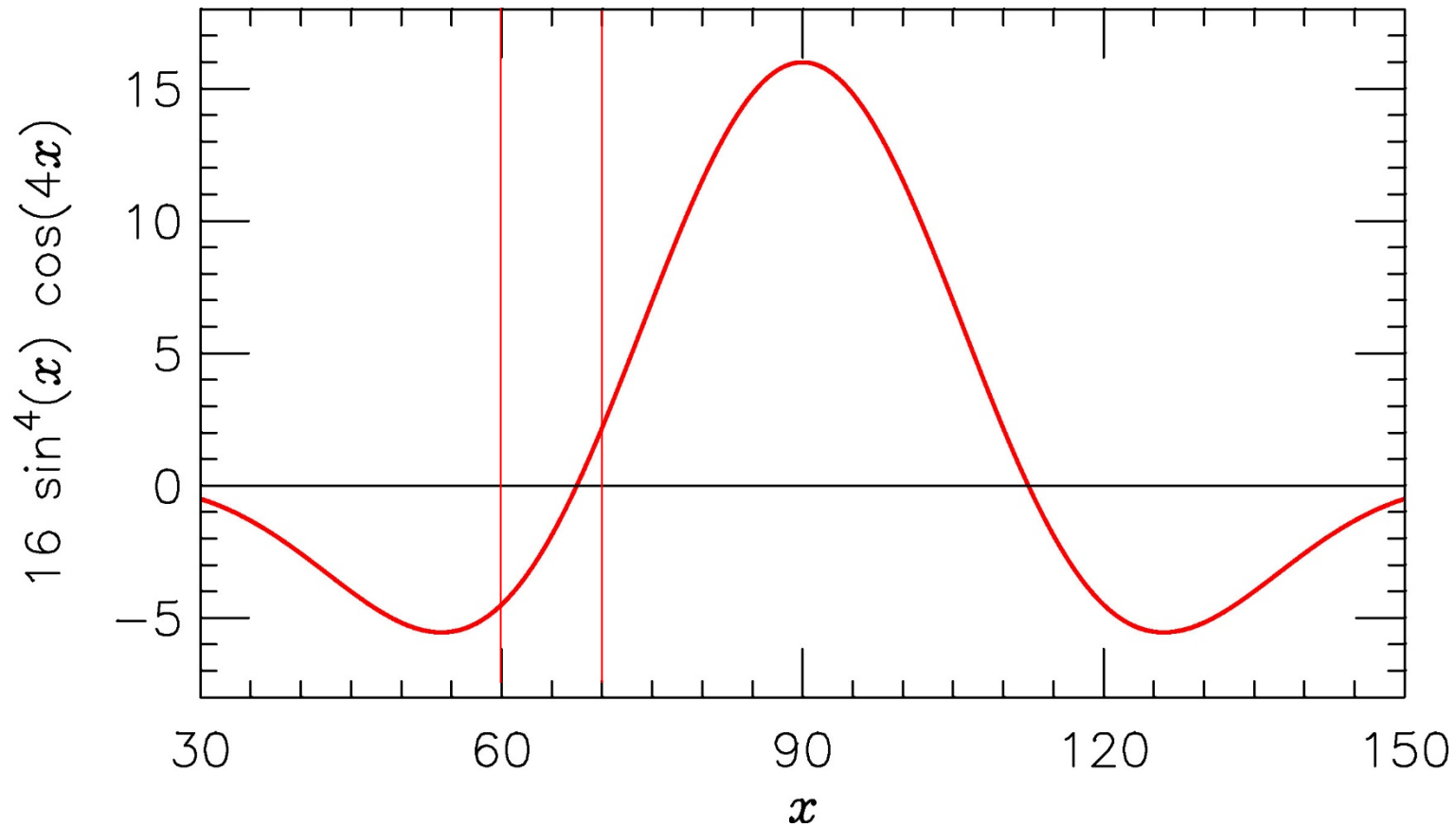
nconv = 1 : Attempt to find root caused guess to leave the interval

nconv = 2 : Maximum number of iterations exceeded

nconv = 3 : Update to root (i.e.,  $\Delta x$ ) equals zero

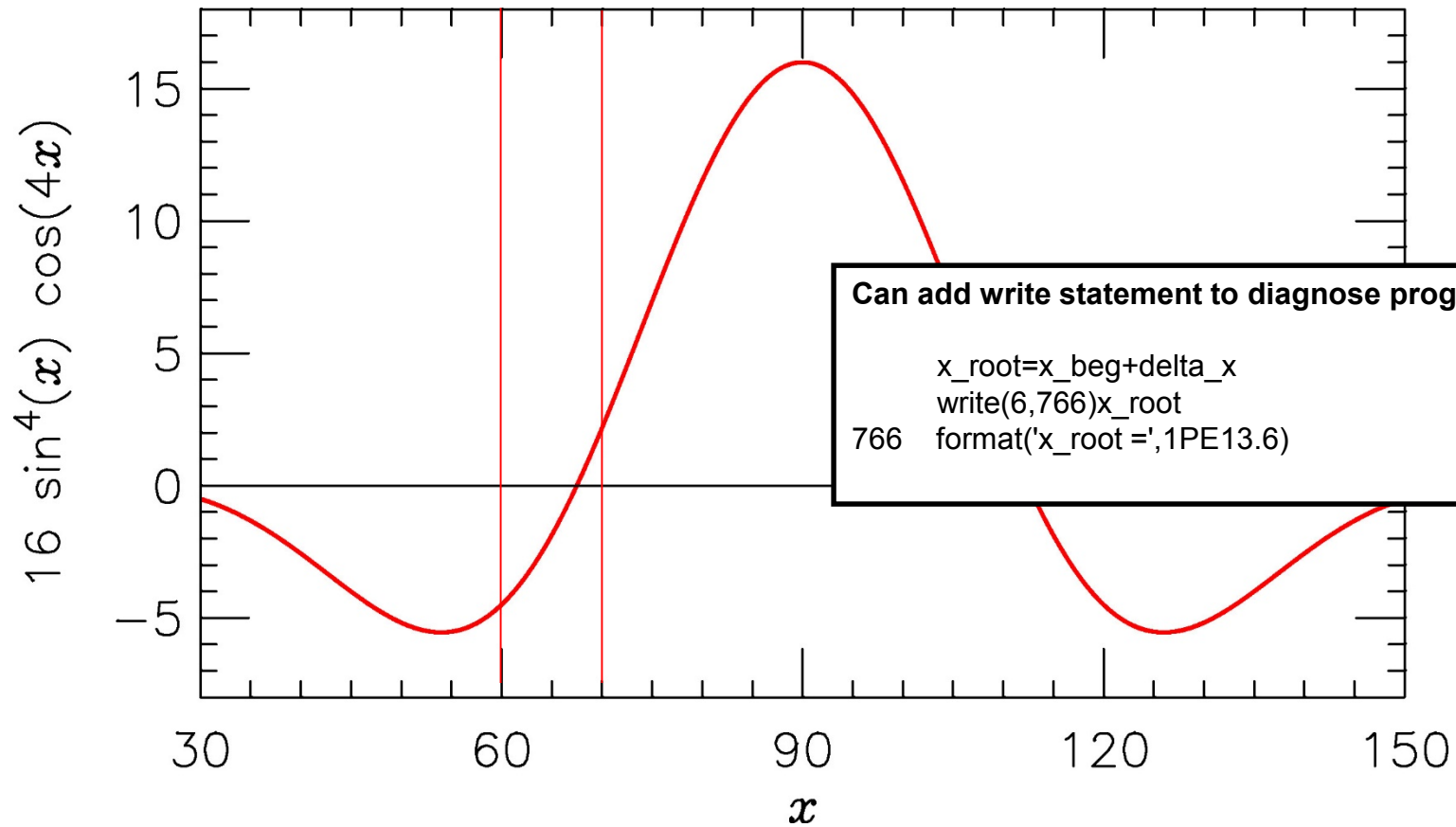
# AOSC 652: Analysis Methods in AOSC

## Assignment #7



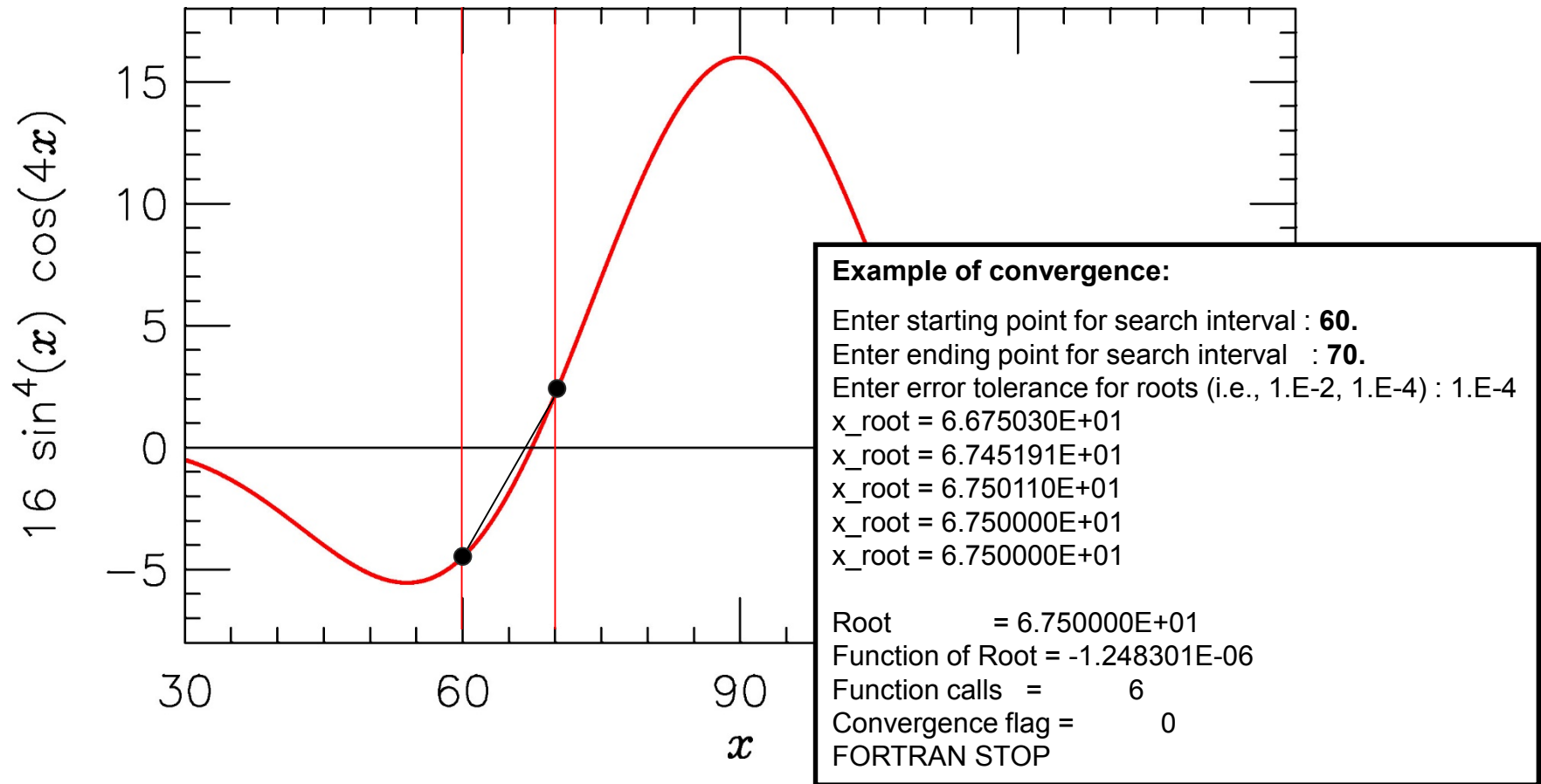
# AOSC 652: Analysis Methods in AOSC

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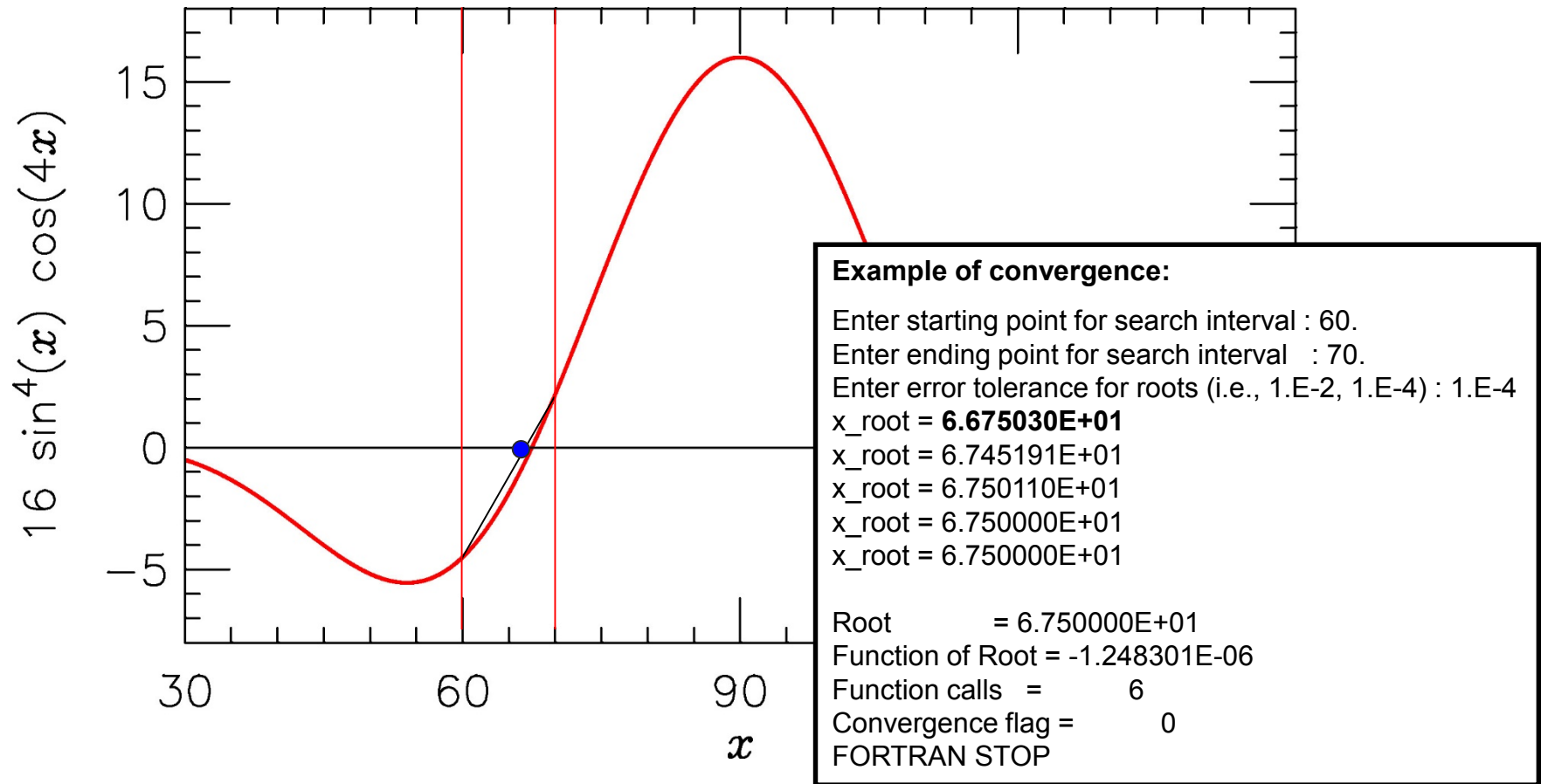
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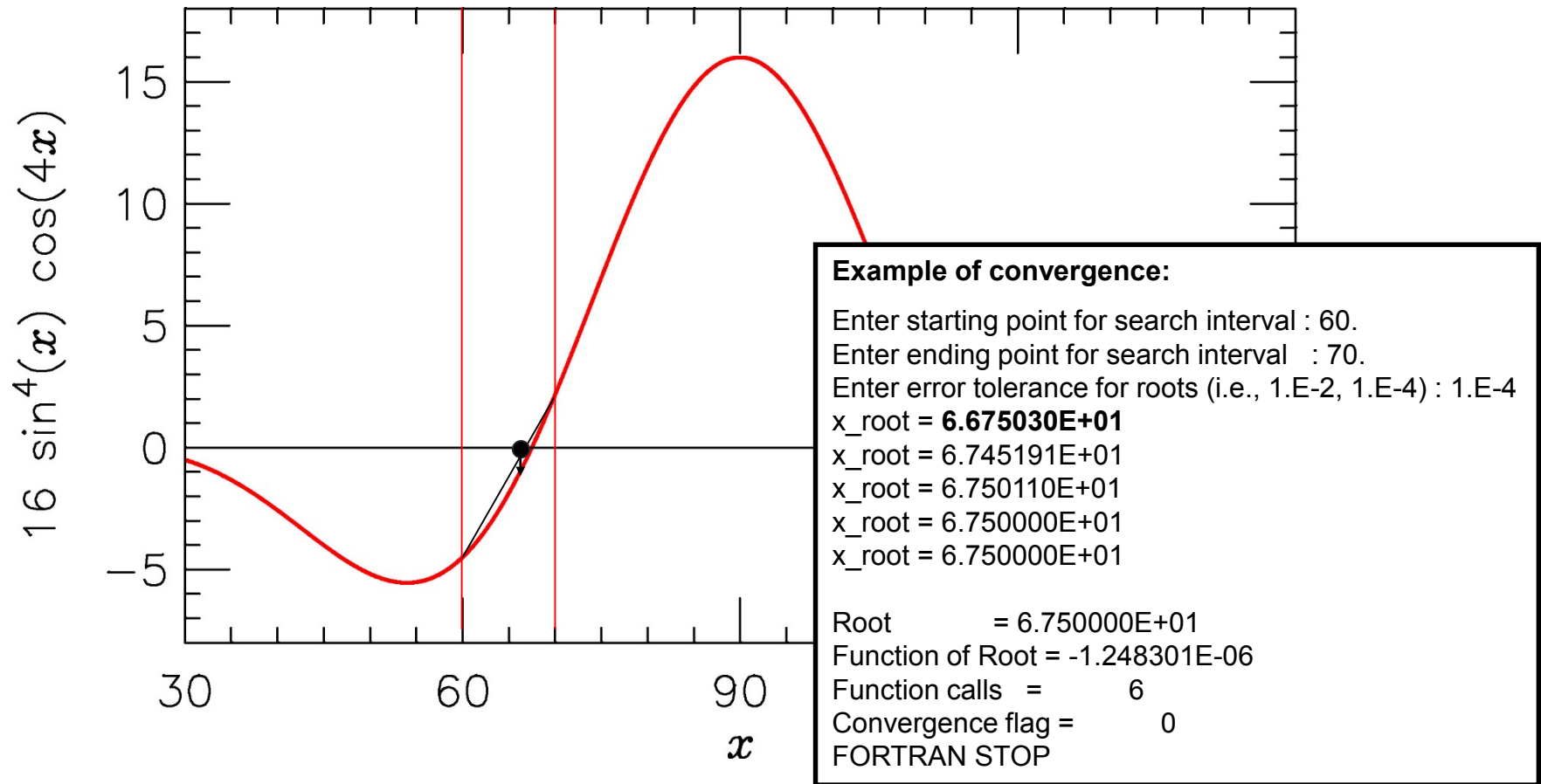
# AOSC 652: Analysis Methods in AOSC

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# AOSC 652: Analysis Methods in AOSC

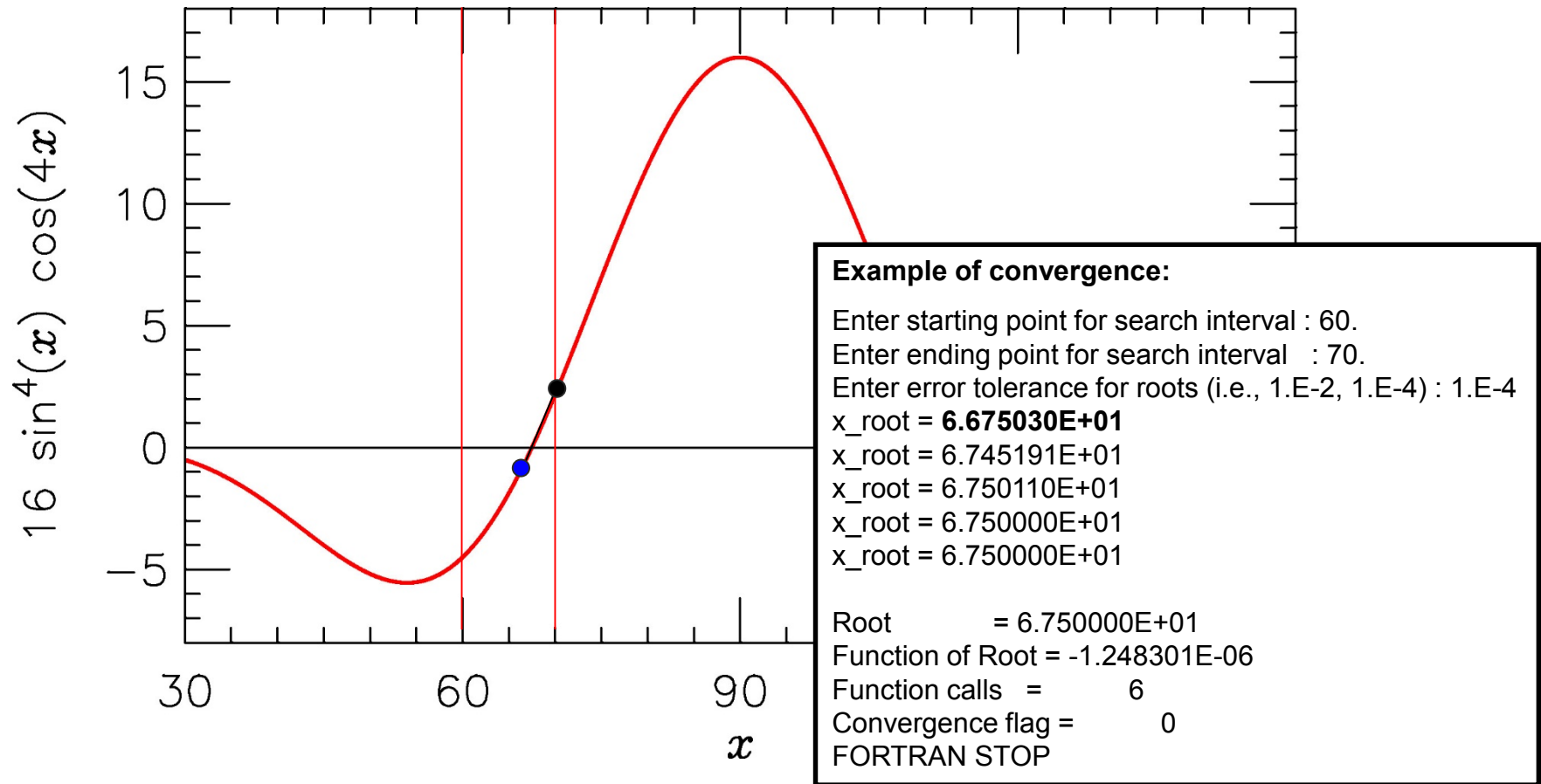
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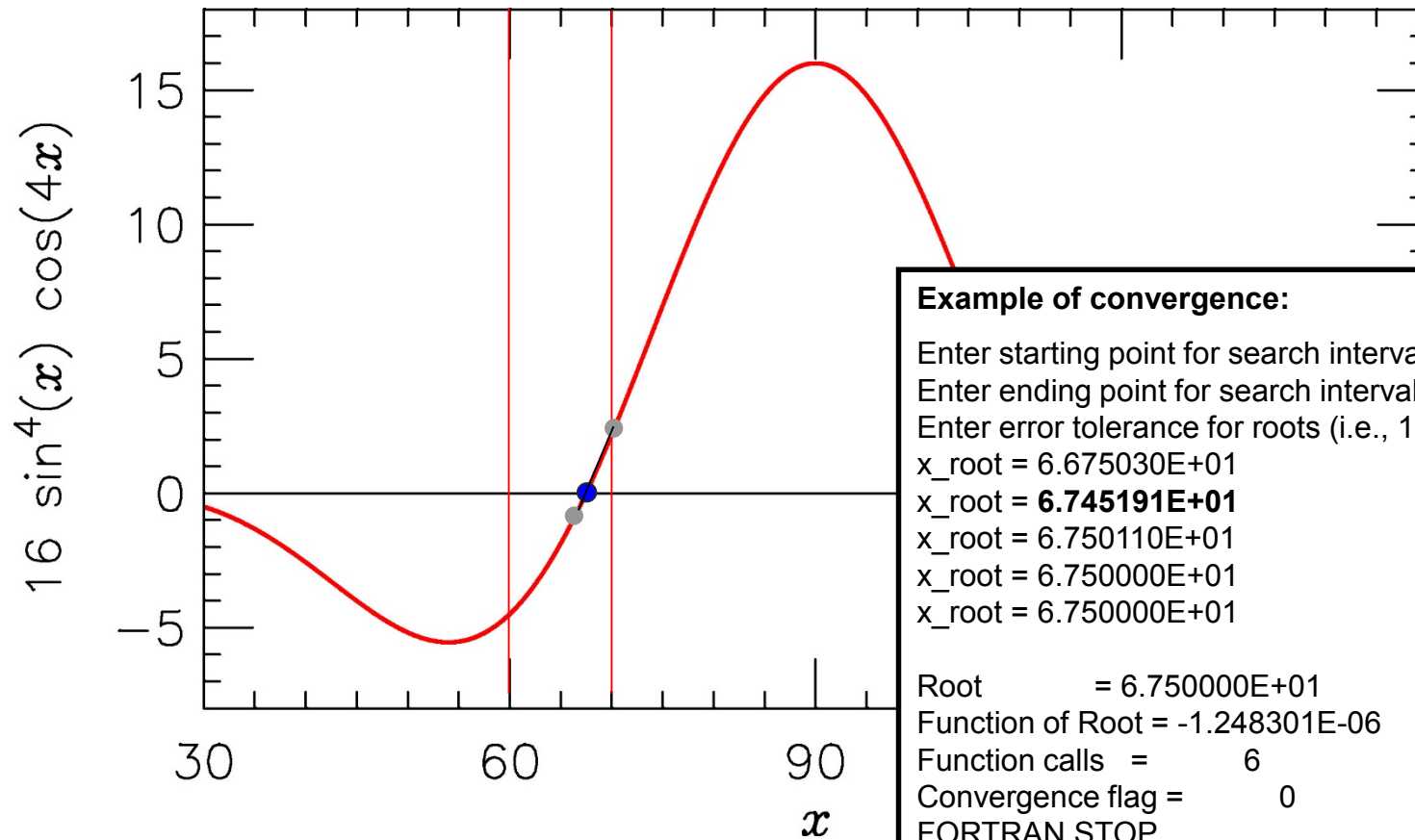
# AOSC 652: Analysis Methods in AOSC

## Assignment #7



# AOSC 652: Analysis Methods in AOSC

## Assignment #7



**Example of convergence:**

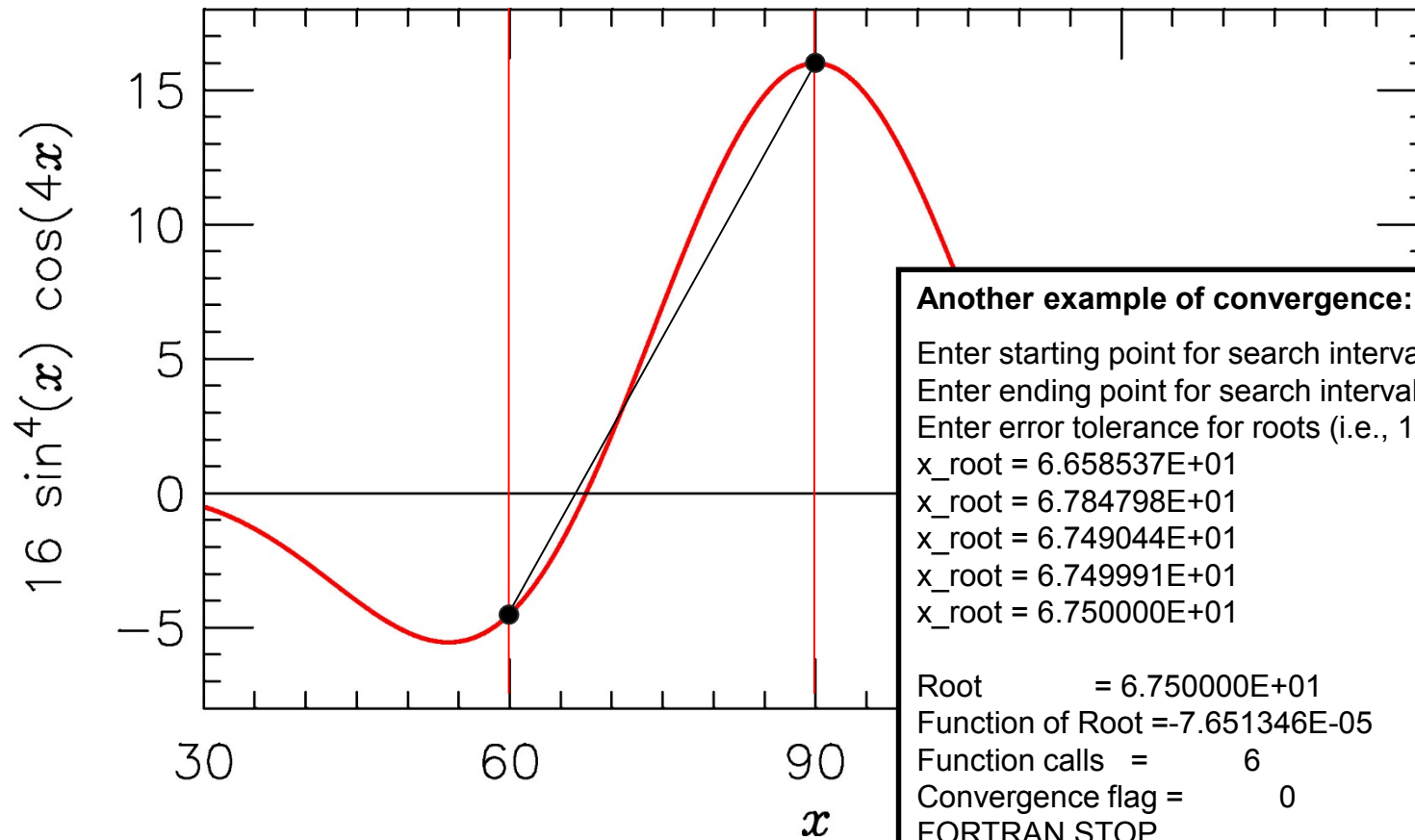
Enter starting point for search interval : 60.  
Enter ending point for search interval : 70.  
Enter error tolerance for roots (i.e., 1.E-2, 1.E-4) : 1.E-4

x\_root = 6.675030E+01  
x\_root = **6.745191E+01**  
x\_root = 6.750110E+01  
x\_root = 6.750000E+01  
x\_root = 6.750000E+01

Root = 6.750000E+01  
Function of Root = -1.248301E-06  
Function calls = 6  
Convergence flag = 0  
FORTRAN STOP

# AOSC 652: Analysis Methods in AOSC

## Assignment #7



**Another example of convergence:**

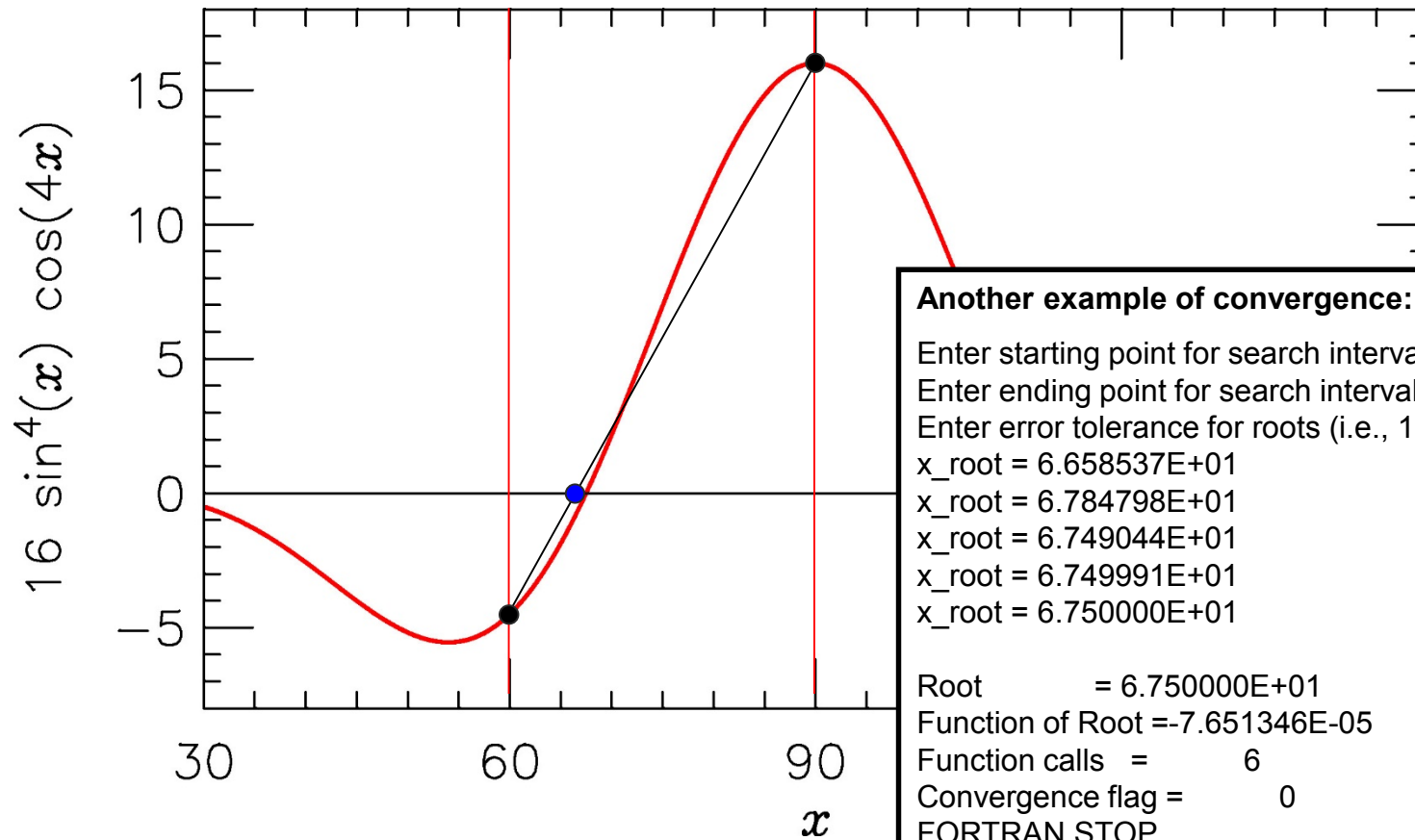
Enter starting point for search interval : **60.**  
Enter ending point for search interval : **90.**  
Enter error tolerance for roots (i.e., 1.E-2, 1.E-4) : 1.E-4

x\_root = 6.658537E+01  
x\_root = 6.784798E+01  
x\_root = 6.749044E+01  
x\_root = 6.749991E+01  
x\_root = 6.750000E+01

Root = 6.750000E+01  
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FORTRAN STOP

# AOSC 652: Analysis Methods in AOSC

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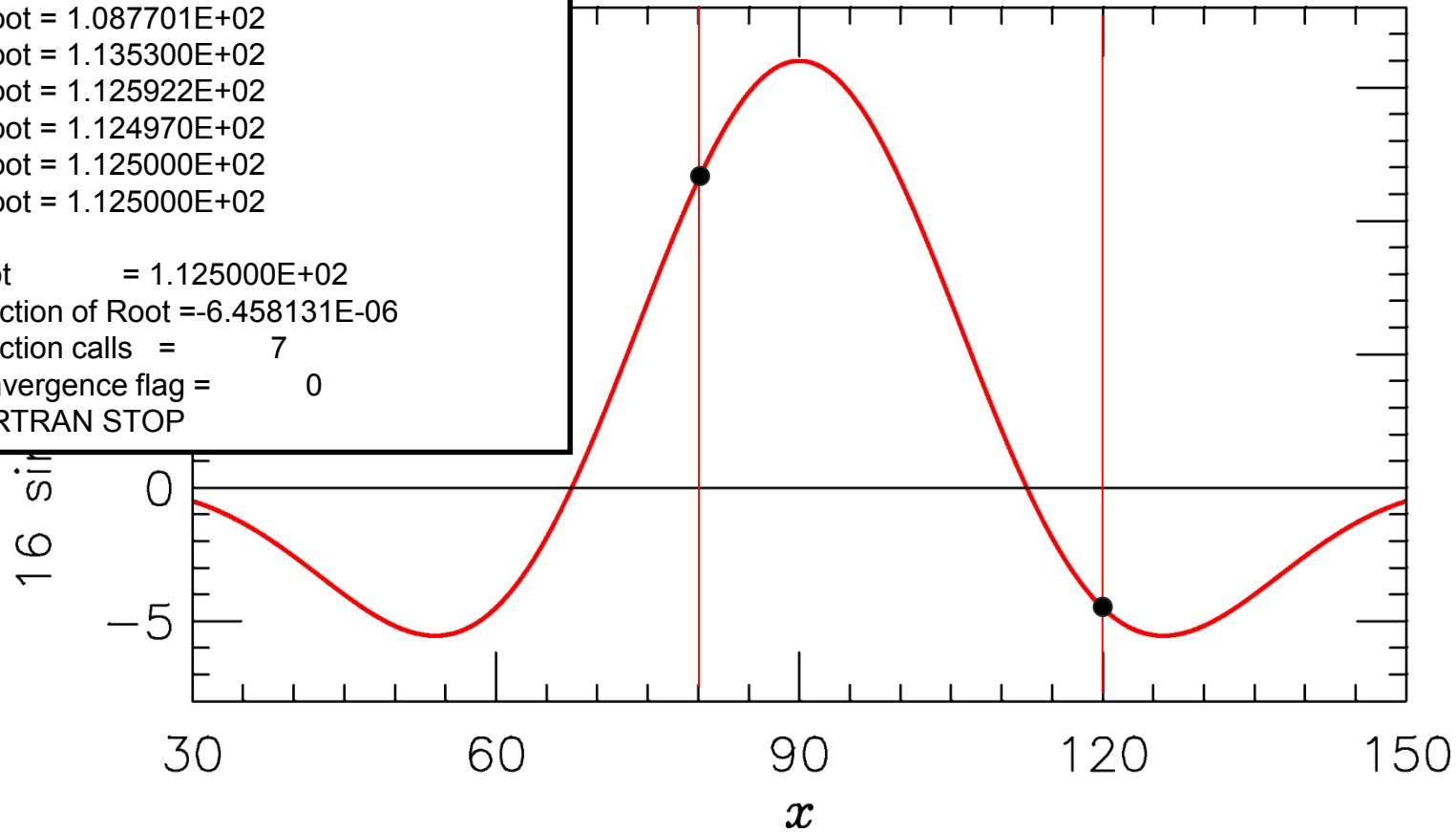
Root = 6.750000E+01  
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FORTRAN STOP

# AOSC 652: Analysis Methods in AOSC

**Yet another example of convergence:**

```
Enter starting point for search interval : 80.  
Enter ending point for search interval  : 120.  
Enter error tolerance for roots (i.e., 1.E-2,  
1.E-4) : 1.E-4  
x_root = 1.087701E+02  
x_root = 1.135300E+02  
x_root = 1.125922E+02  
x_root = 1.124970E+02  
x_root = 1.125000E+02  
x_root = 1.125000E+02
```

```
Root      = 1.125000E+02  
Function of Root = -6.458131E-06  
Function calls =      7  
Convergence flag =    0  
FORTRAN STOP
```

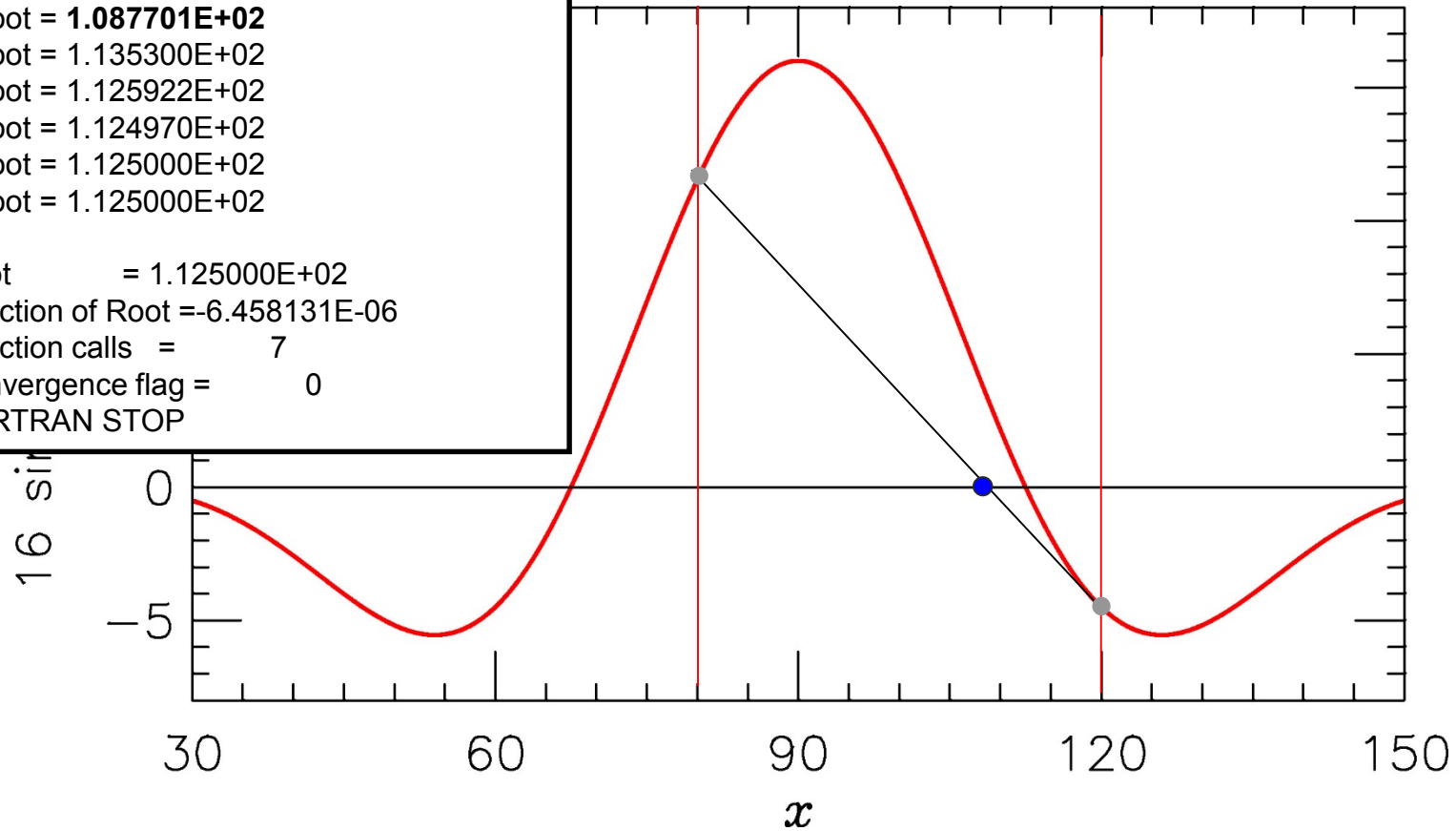


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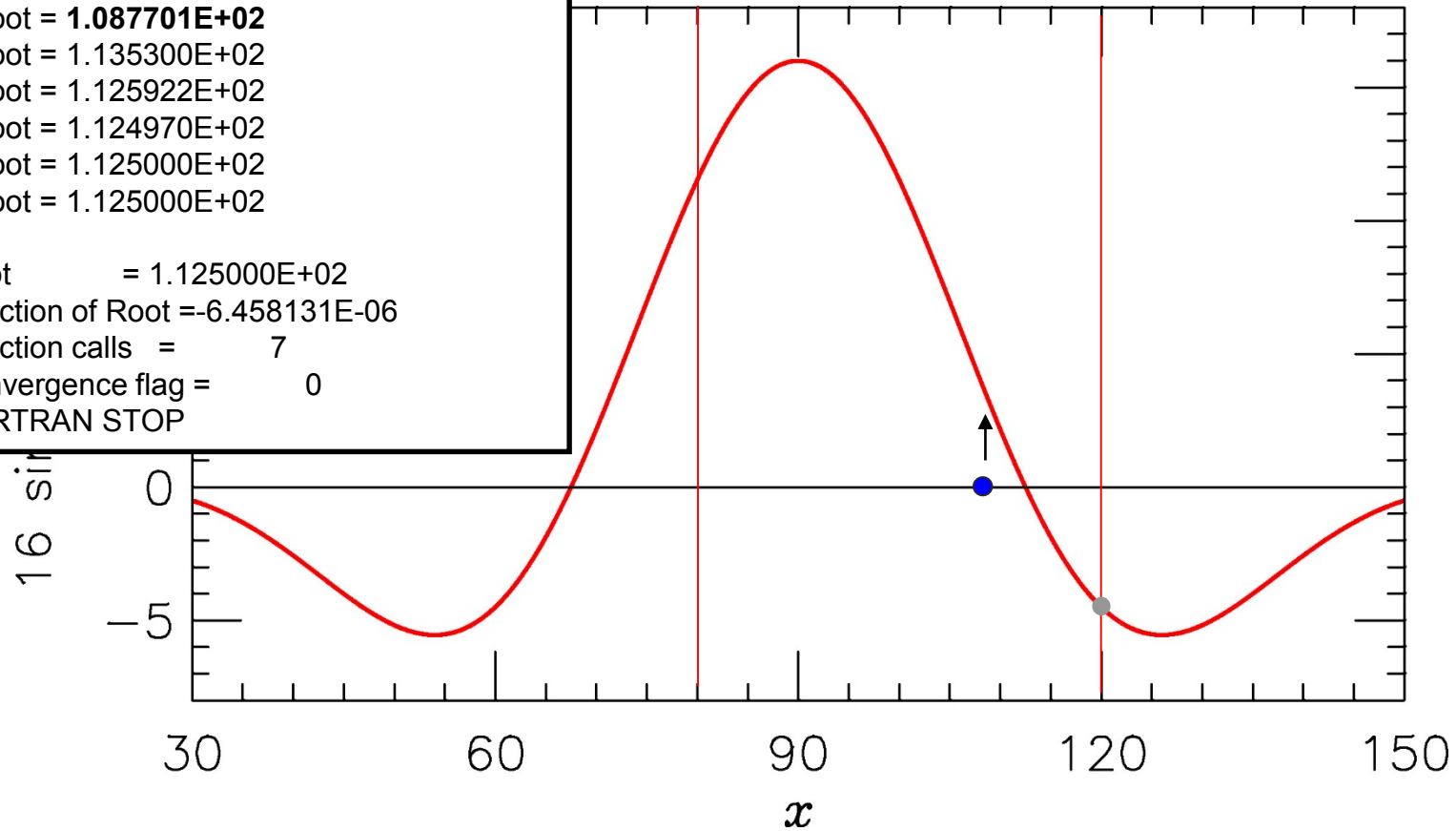
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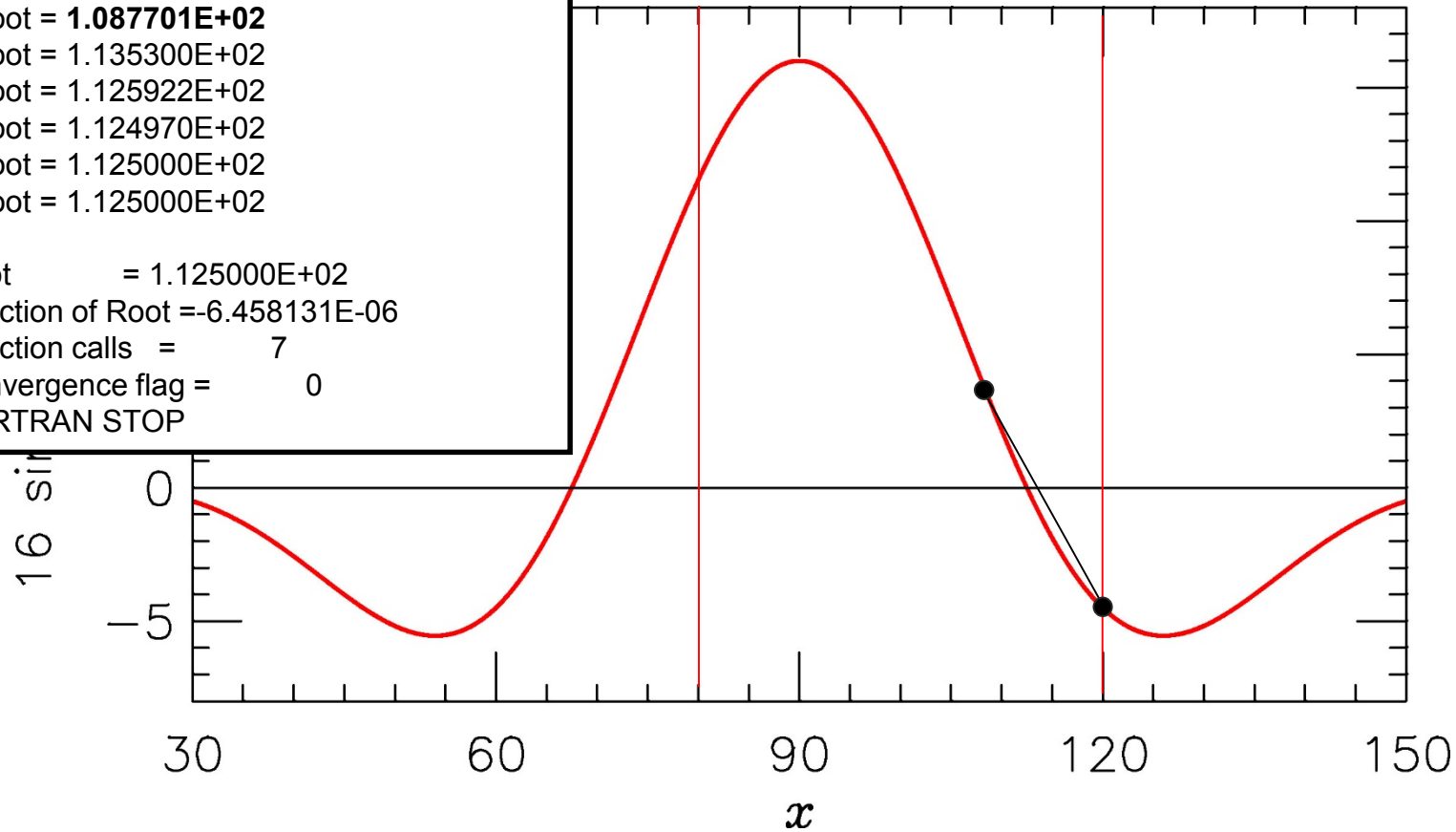
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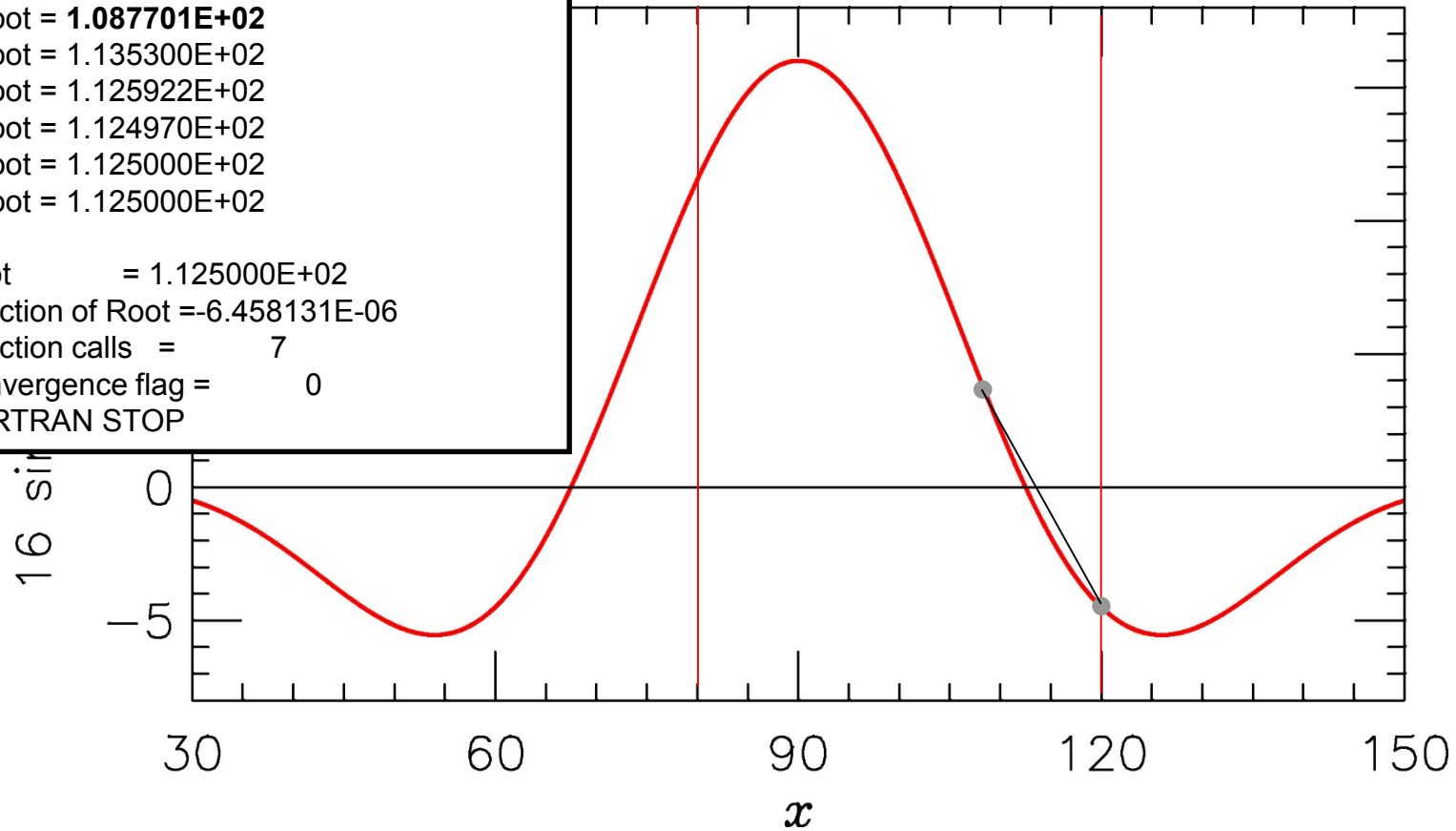
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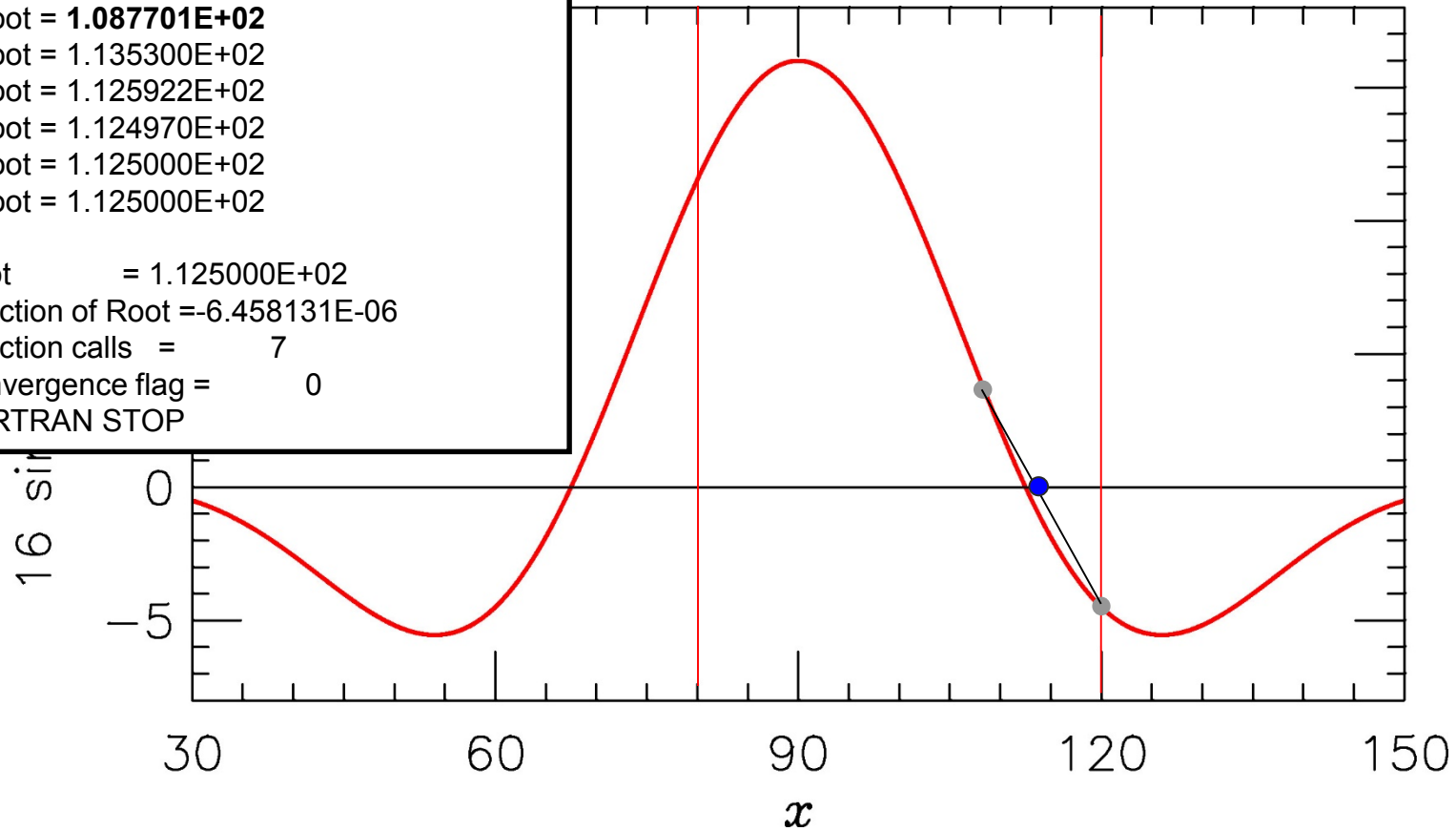
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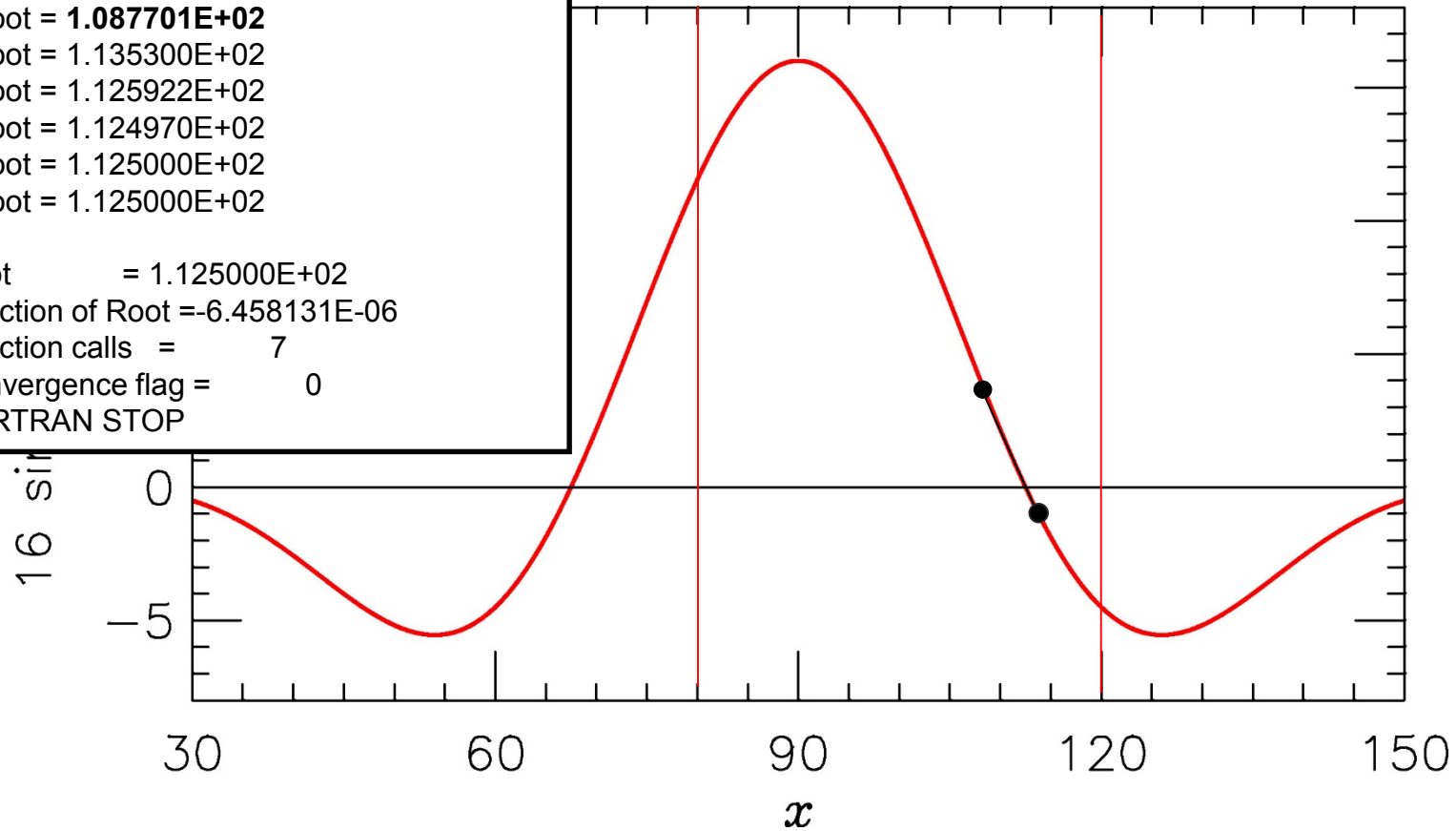
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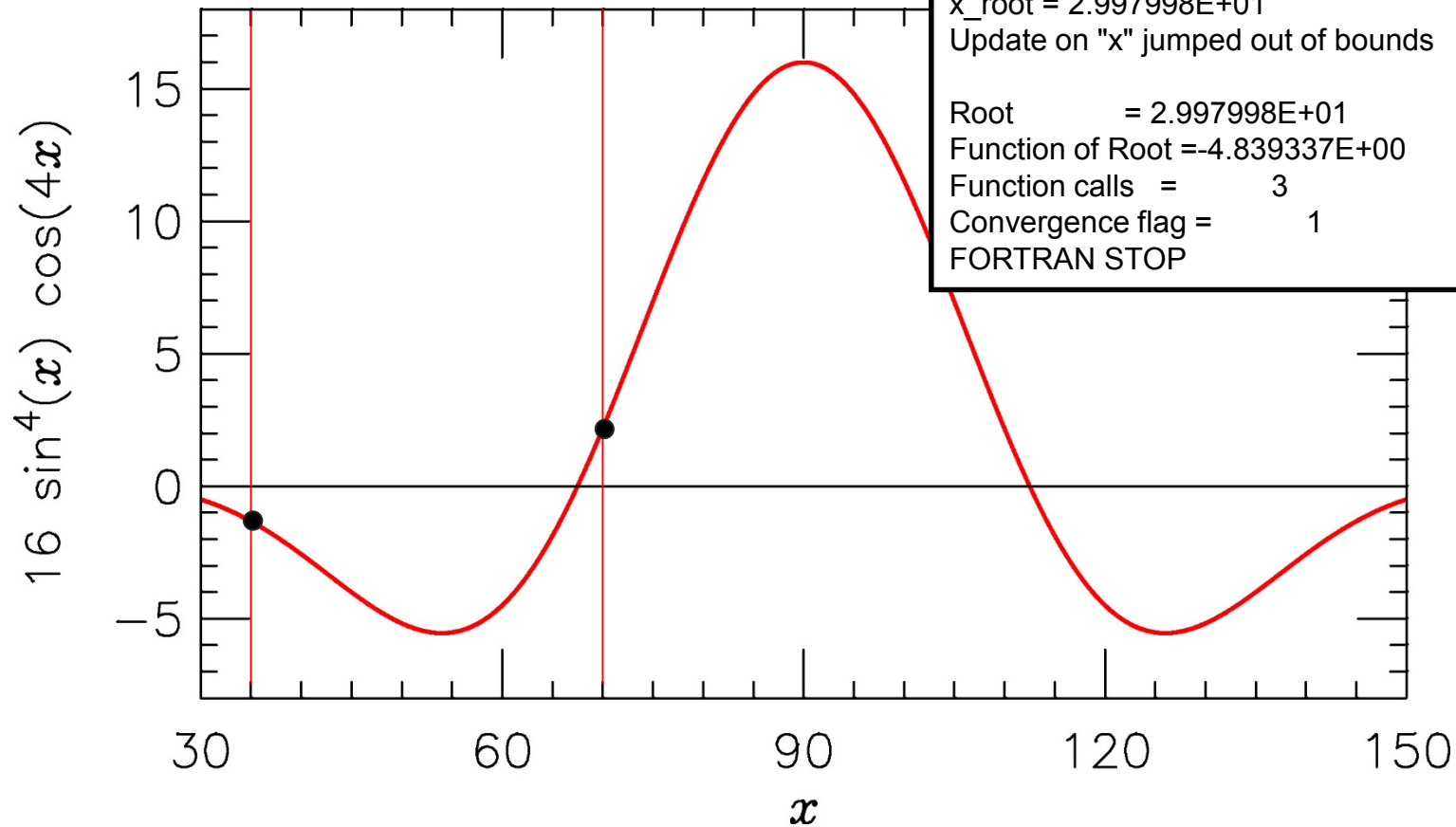
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# AOSC 652: Analysis Methods in AOSC

## Assignment #7



### Example of non-convergence:

```
Enter starting point for search interval : 35.  
Enter ending point for search interval  : 70.  
Enter error tolerance for roots (i.e., 1.E-2, 1.E-4) : 1.E-4  
x_root = 4.829267E+01  
x_root = 2.997998E+01  
Update on "x" jumped out of bounds
```

```
Root          = 2.997998E+01  
Function of Root = -4.839337E+00  
Function calls =      3  
Convergence flag =      1  
FORTRAN STOP
```

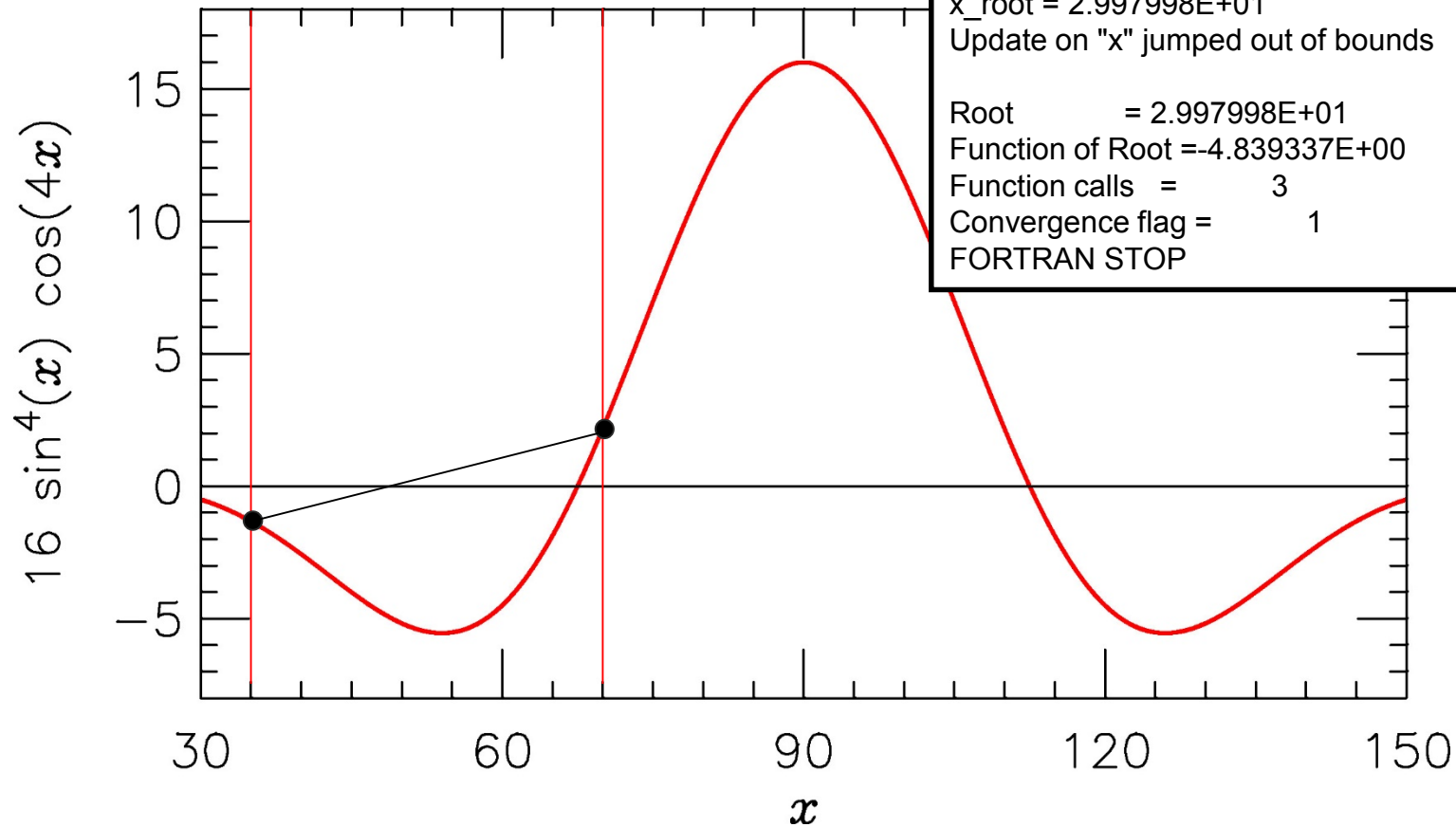
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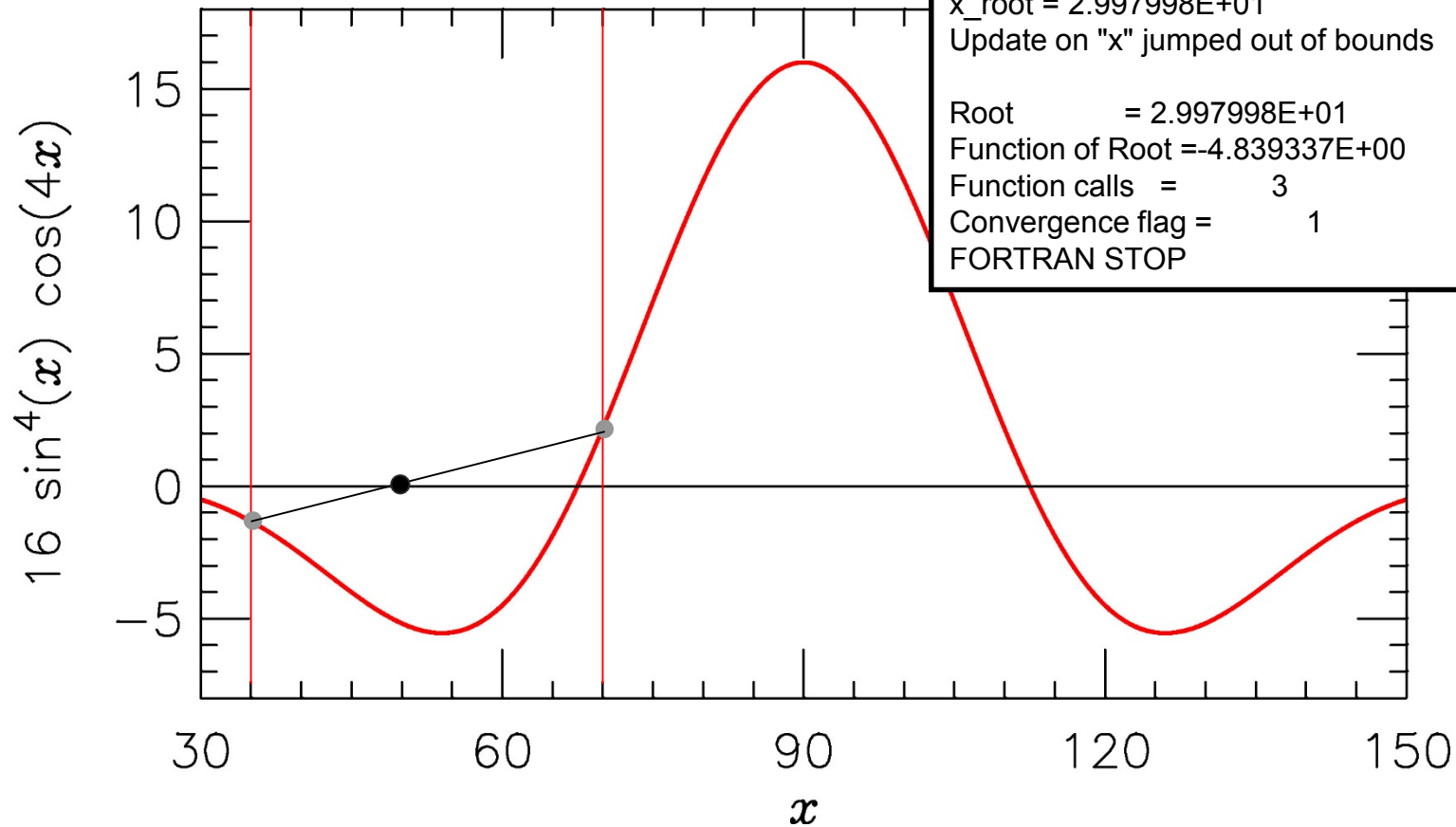
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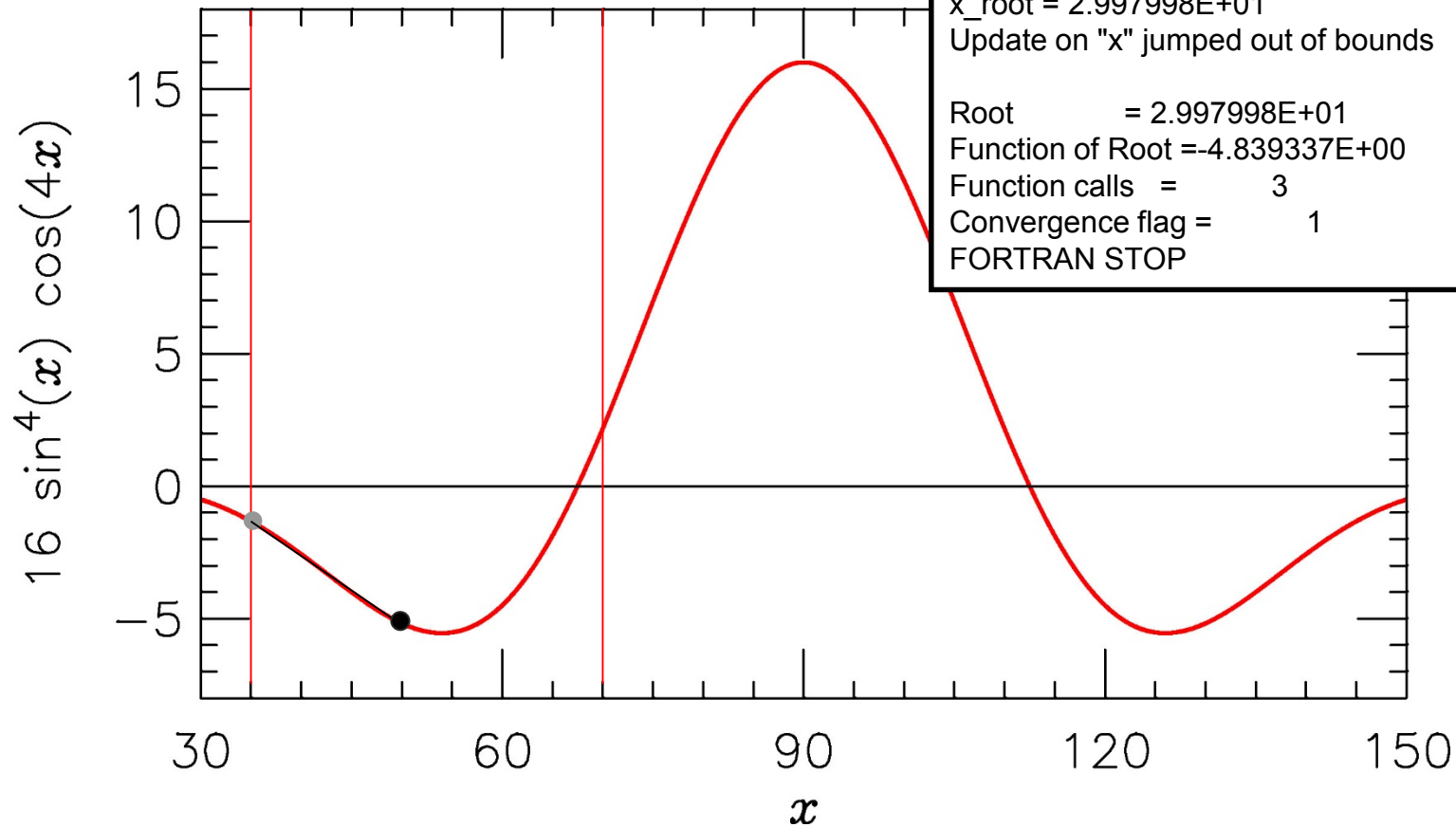
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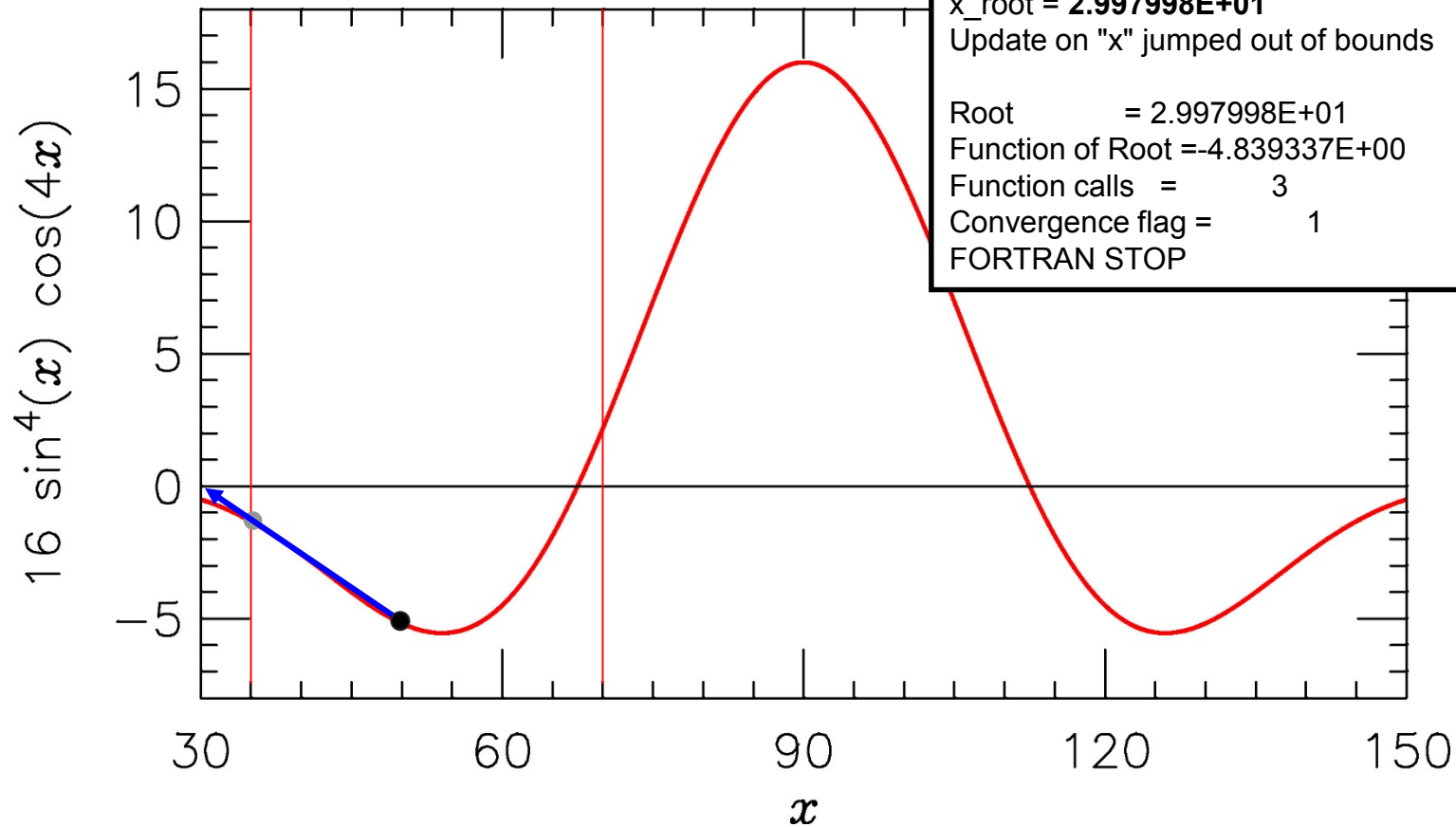
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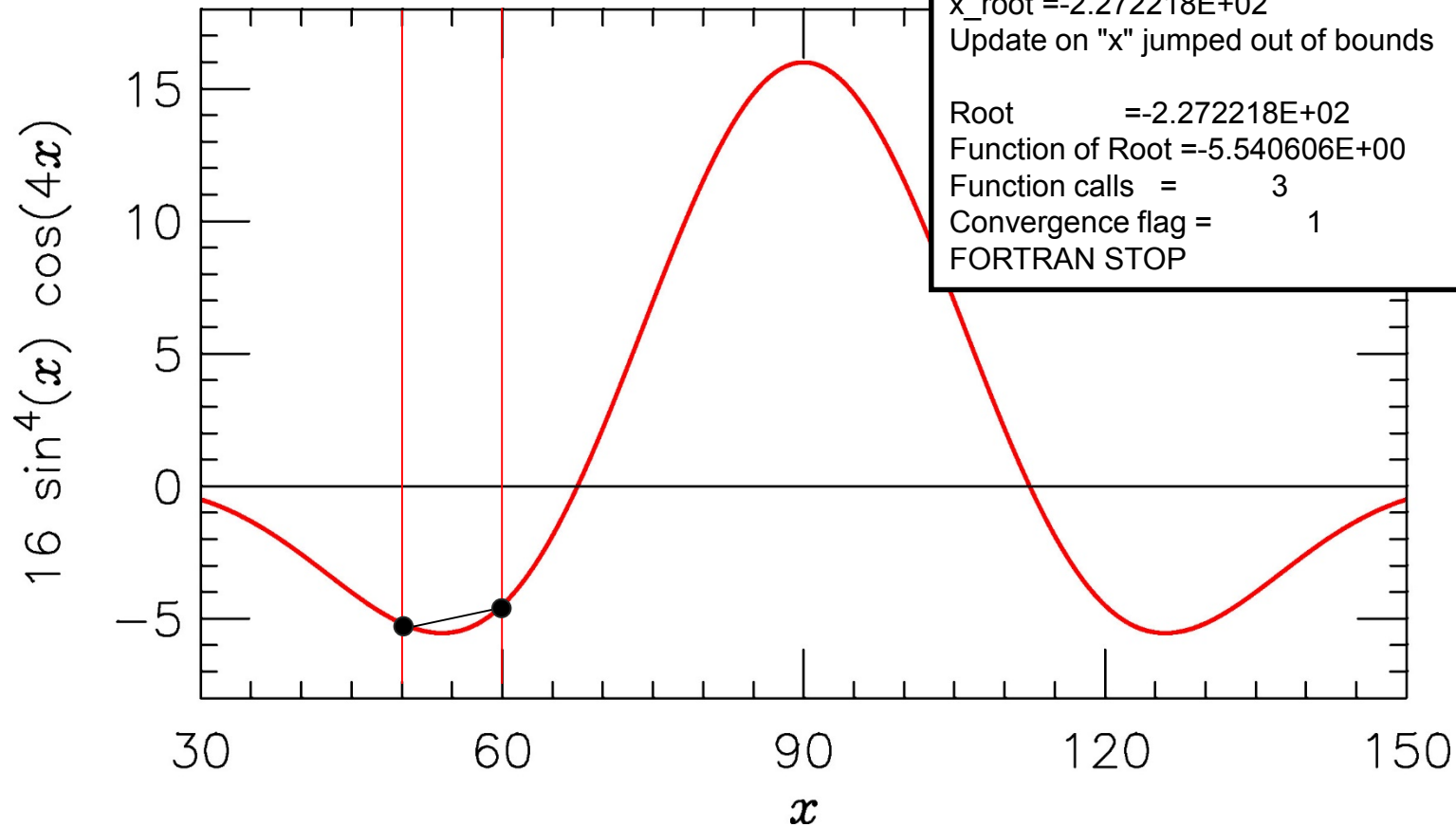
# AOSC 652: Analysis Methods in AOSC

## Assignment #7

### Example of non-convergence:

```
Enter starting point for search interval : 50.  
Enter ending point for search interval  : 60.  
Enter error tolerance for roots (i.e., 1.E-2, 1.E-4) : 1.E-4  
x_root = 1.264188E+02  
x_root = -2.272218E+02  
Update on "x" jumped out of bounds
```

```
Root          = -2.272218E+02  
Function of Root = -5.540606E+00  
Function calls =      3  
Convergence flag =      1  
FORTRAN STOP
```



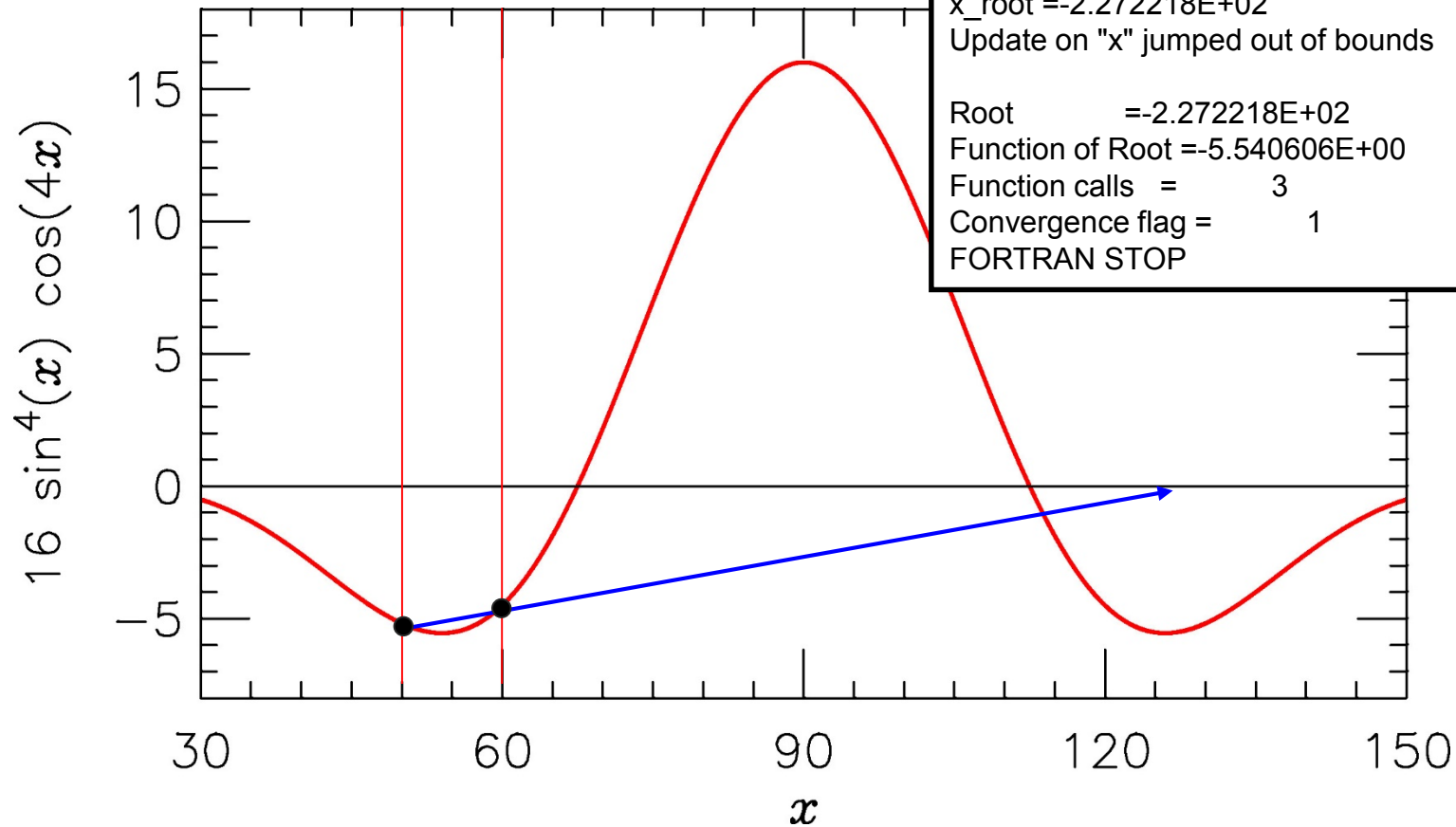
# AOSC 652: Analysis Methods in AOSC

## Assignment #7

### Another example of non-convergence:

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Enter ending point for search interval  : 60.  
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FORTRAN STOP
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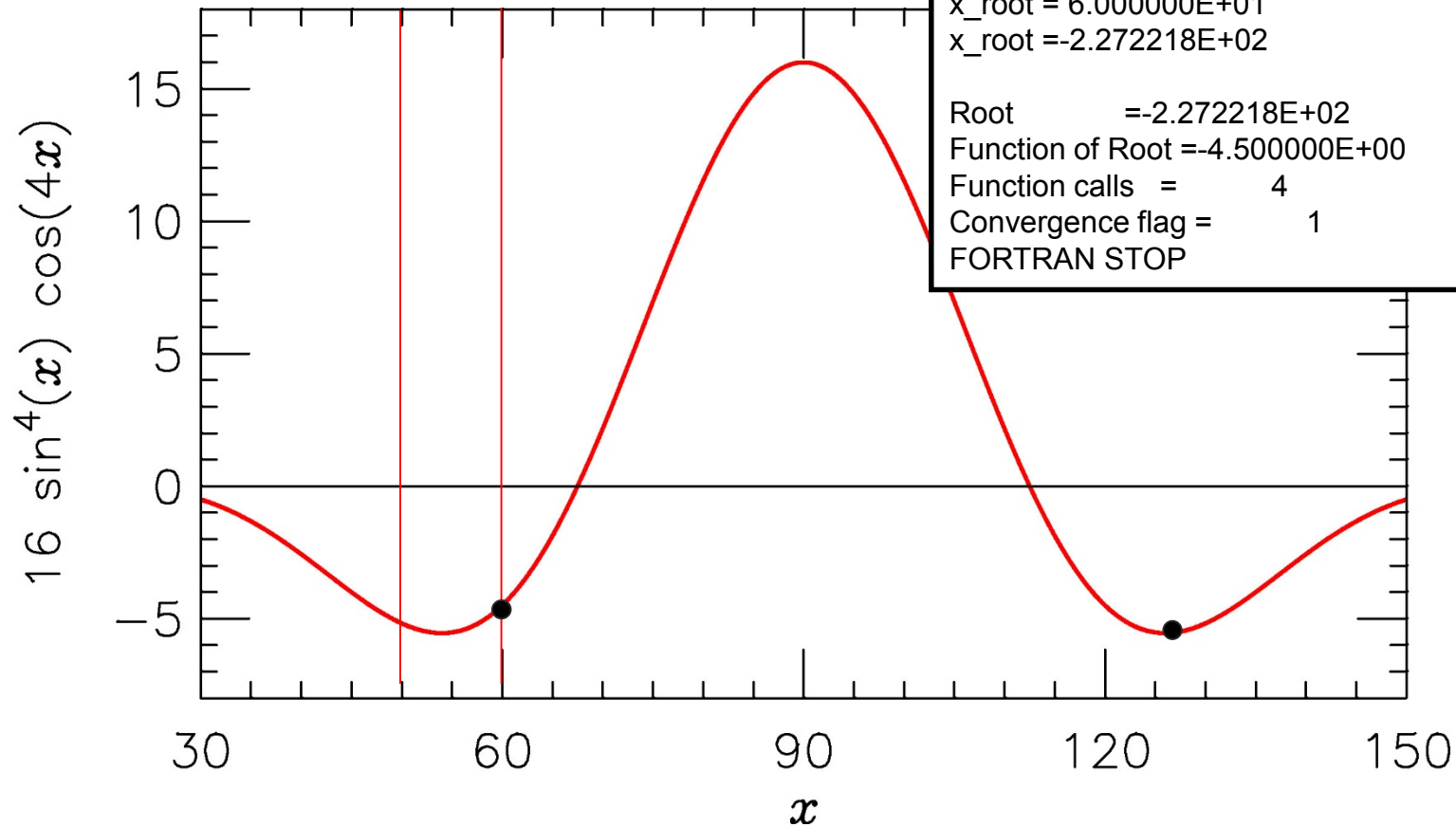
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x_root = 6.000000E+01  
x_root = -2.272218E+02
```

```
Root          = -2.272218E+02  
Function of Root = -4.500000E+00  
Function calls =      4  
Convergence flag =      1  
FORTRAN STOP
```



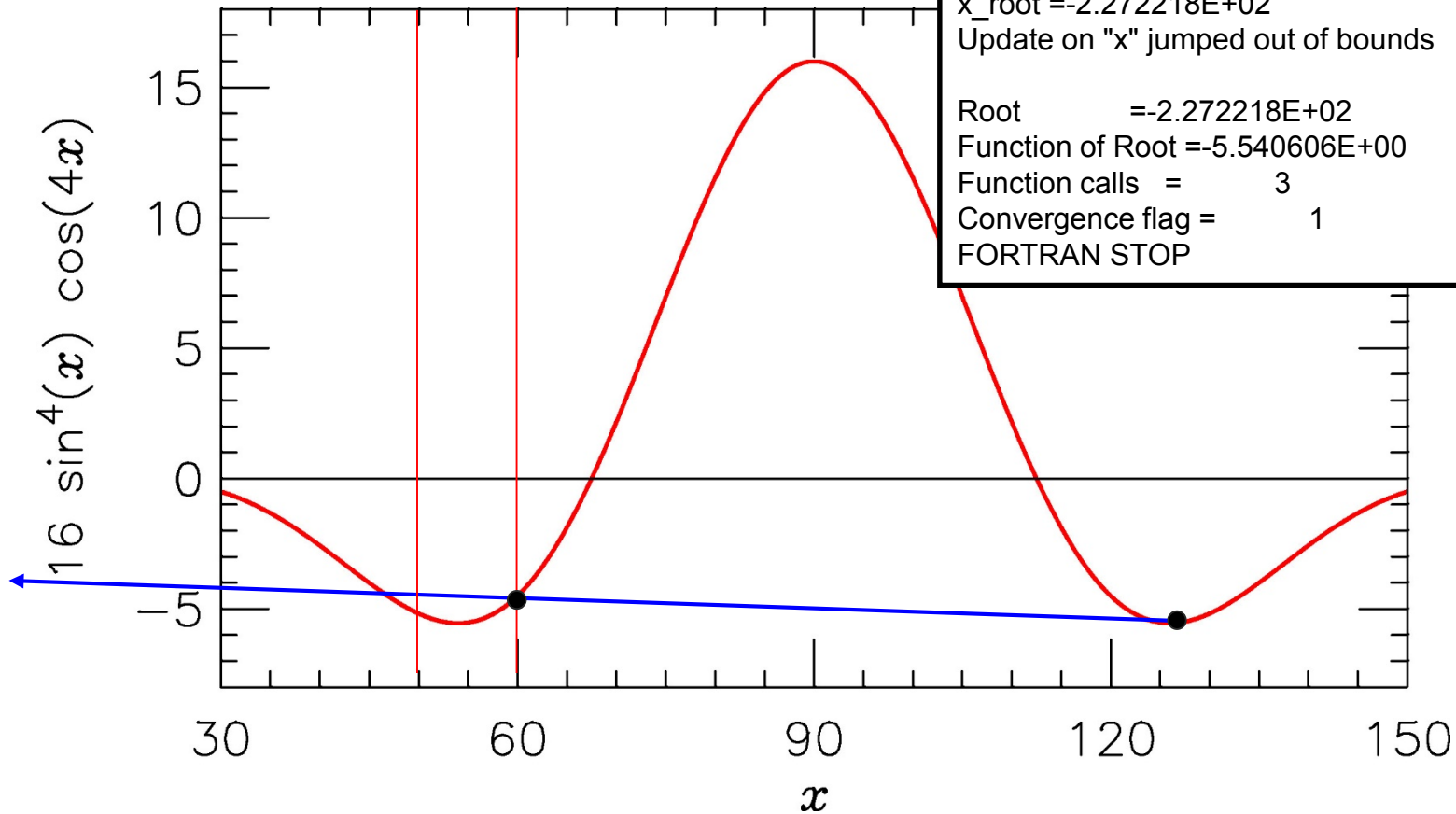
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## Readings for Monday:

Pages 47 to 53 of *Ice Ages and Astronomical Causes*  
by Richard A. Muller and Gordon J. MacDonald  
Sections 8.1, Chapter 8 (Fourier Analysis)  
on line Matlab book by Prof. Cleve Moler  
Section 12.0 of Press et al.: f12-0.pdf

~16 pages:  
please read prior to class

Team Python: have a look at Chapter 19, Miscellany, of DeCaria book