# Analysis Methods in Atmospheric and Oceanic Science

## AOSC 652

Fourier Analysis

Week 9, Day 2

26 Oct 2016

# AOSC 652: Analysis Methods in AOSC

## Fourier Series and Spectral Analysis

**Last lecture**: intro and examples using dial tones (time scale of **millisecs**)

**Today**: solar activity/sunspots (time scale of **decades**)

**Assignment #9**: spectral analysis of Vostok Ice Core Record (**millenia**)

## Fourier Series and Spectral Analysis

Our analysis will be based on the simple method to calculate the ***power spectrum*** of a time series introduced last lecture:

For a *discrete* time series represented by variable $signal(time)$:

$i.e.,$ there exists $signal(1), signal(2), ..., signal(N)$

and corresponding time elements $time(1),\ time(2),\ ...,\ time(N)$

the values of $H_{\text{REAL}}$ and $H_{\text{IMAGINARY}}$ at frequency $f$ are given by:

$$H_{\text{REAL}} = 0$$

$$H_{\text{IMAGINARY}} = 0$$

do j=1 to N by 1

$$H_{\text{REAL}} = H_{\text{REAL}} + signal(\text{j}) * \cos(2\pi f \ time(\text{j}))$$

$$H_{\text{IMAGINARY}} = H_{\text{IMAGINARY}} + signal(\text{j}) * \sin(2\pi f \ time(\text{j}))$$

enddo

$$\underline{Power = H_{\text{REAL}}{}^2 + H_{\text{IMAGINARY}}{}^2}$$ (see page 52, Muller & MacDonald)

## Fourier Series and Spectral Analysis

Solar activity/sunspots (time scale of decades)

Copy file ~rjs/aosc652/week_09/sunspot_number_monthly.dat to your work area.

This file contains *Sunspot Number* vs *Time*, from 1749 to present

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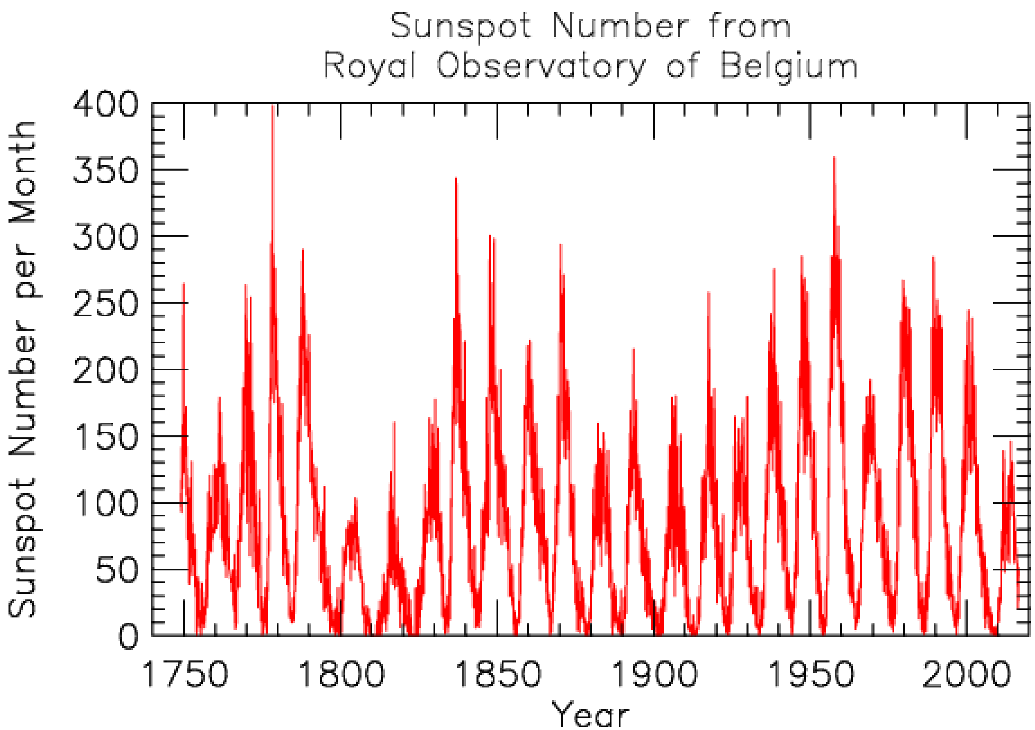


Data from http://www.sidc.be/silso/DATA/SN_m_tot_V2.0.txt

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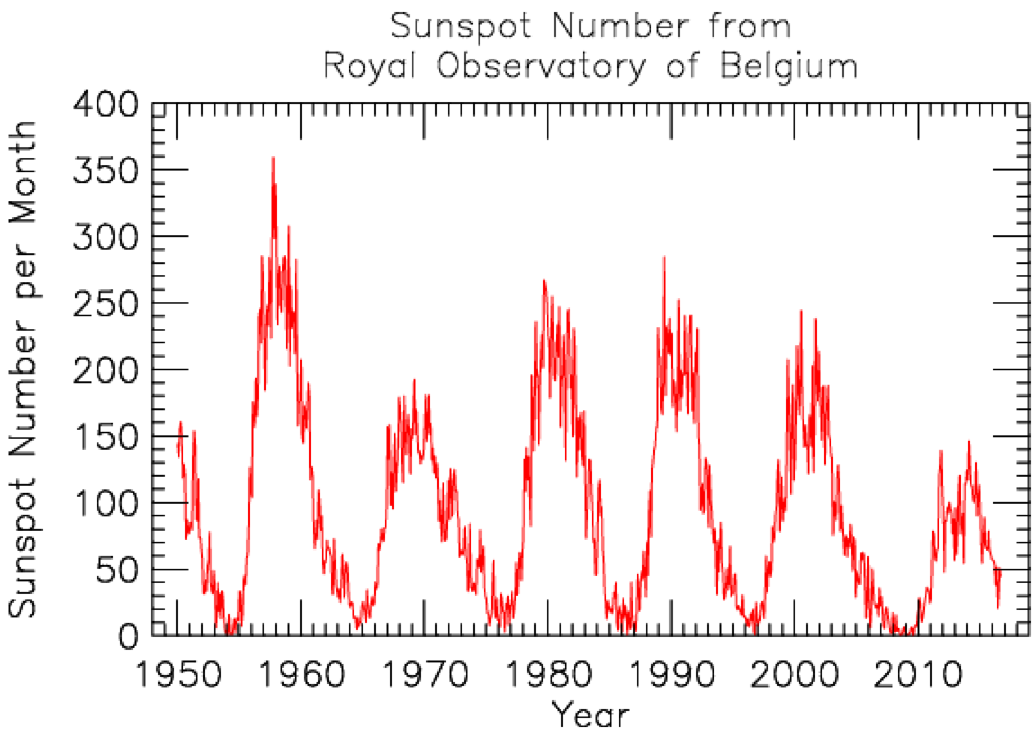


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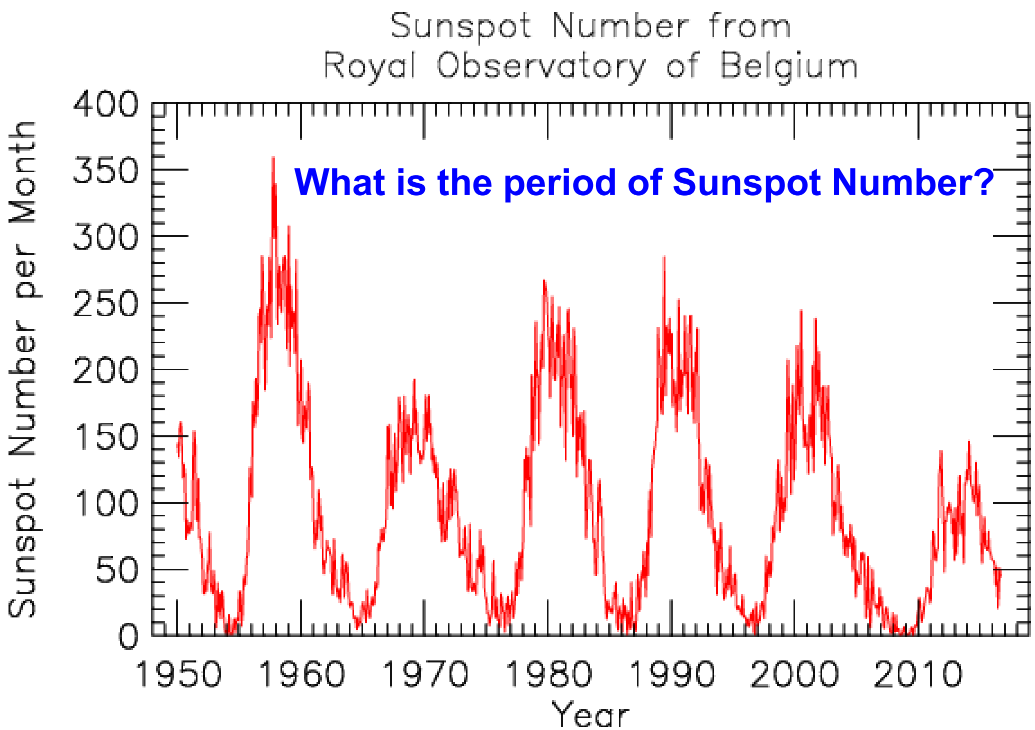


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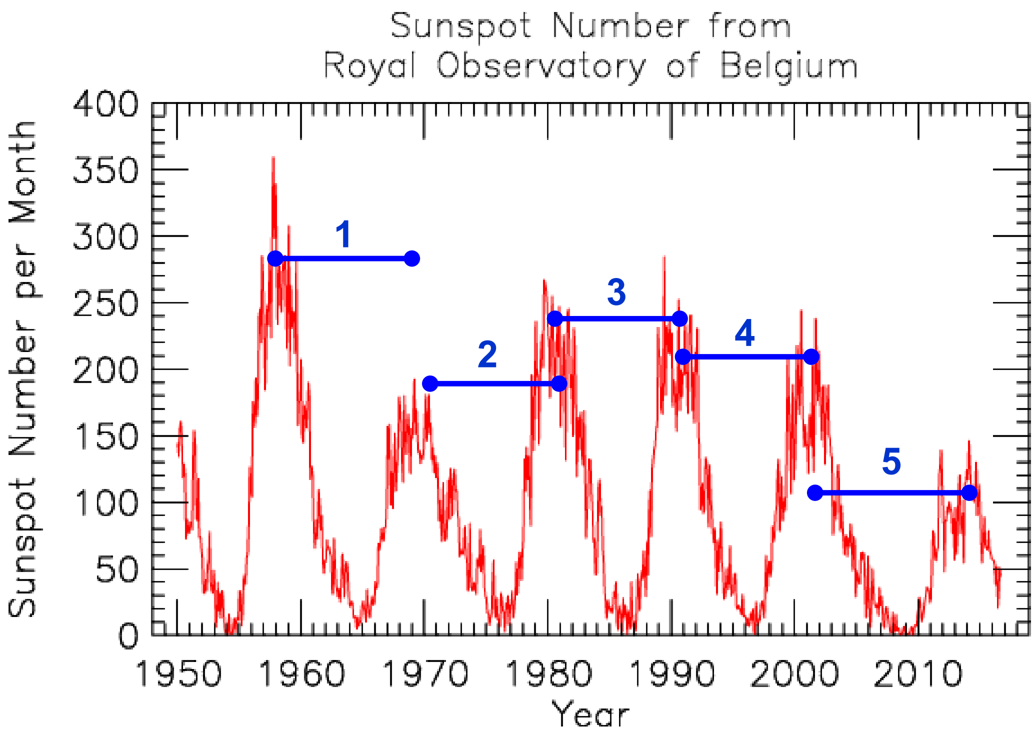


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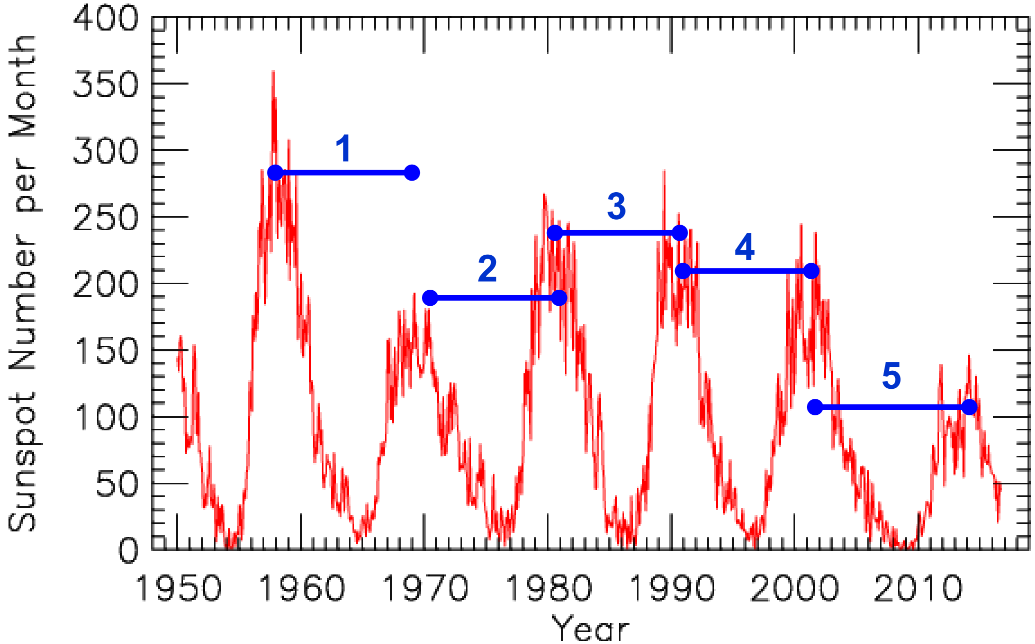


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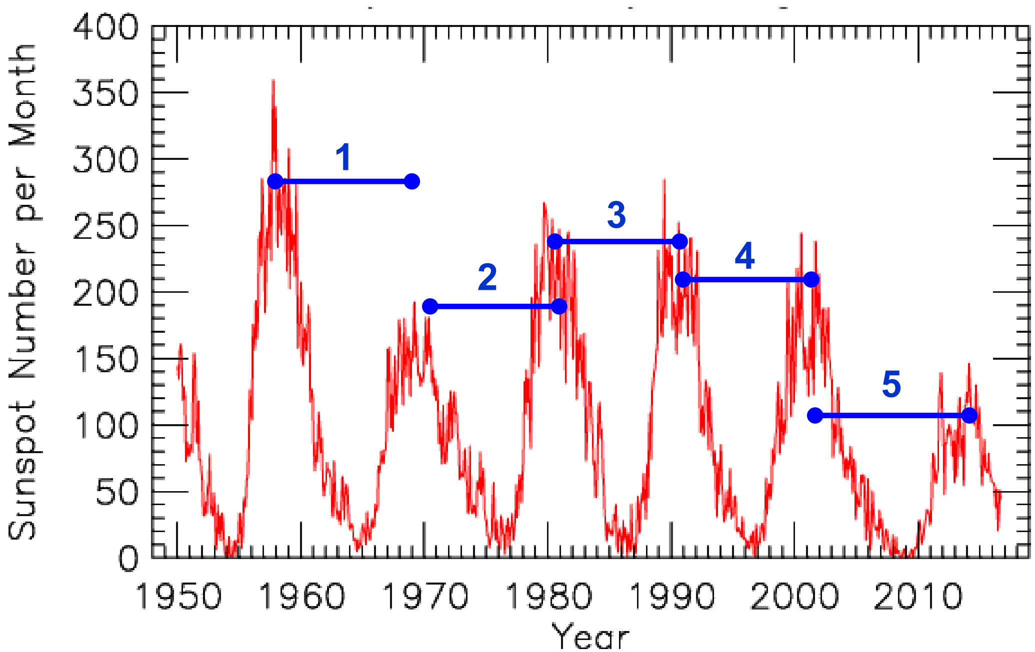


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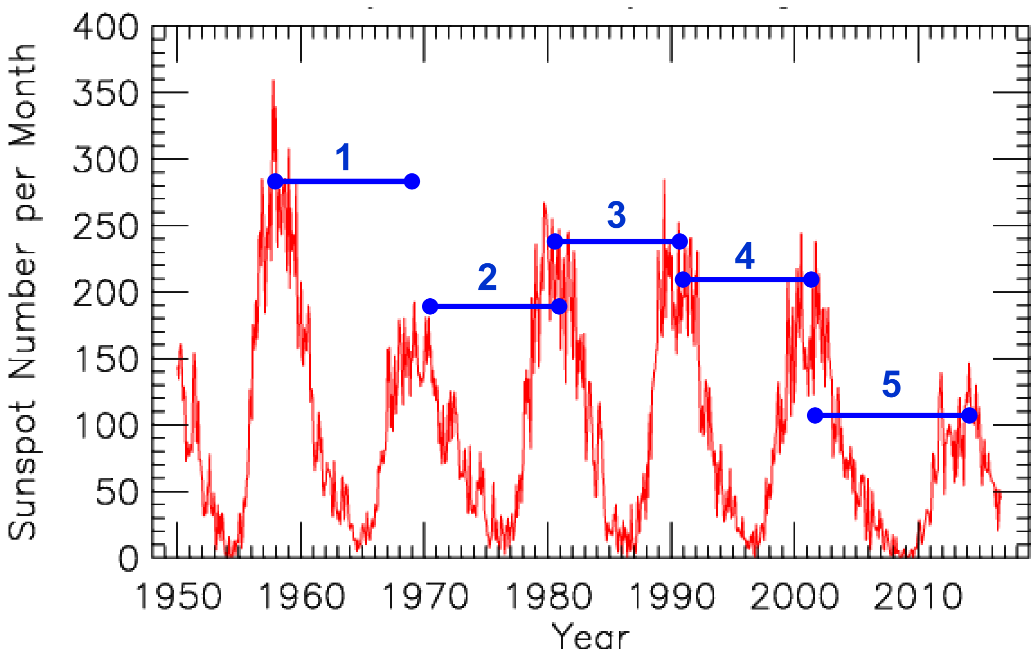


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## Fourier Series and Spectral Analysis

Using program fourier_analysis.f (from Monday), find the Power Spectrum of the time series in file **sunspot_number_monthly.dat**, for time periods from **7 to 15 years** every **0.1 years** and ***display the results*** :

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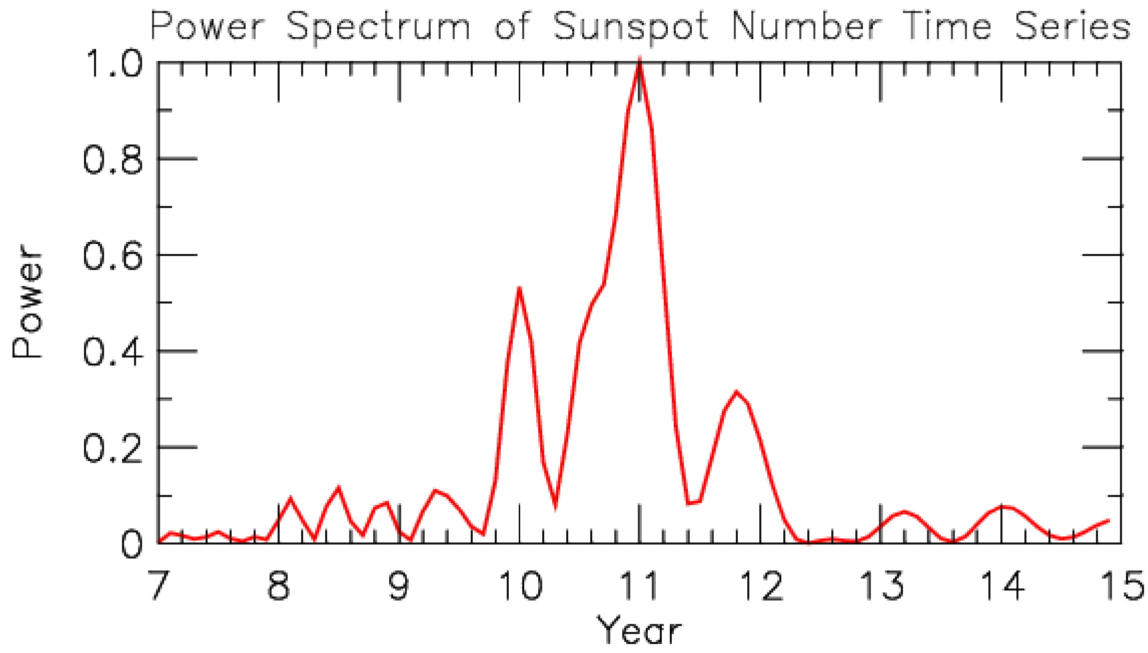
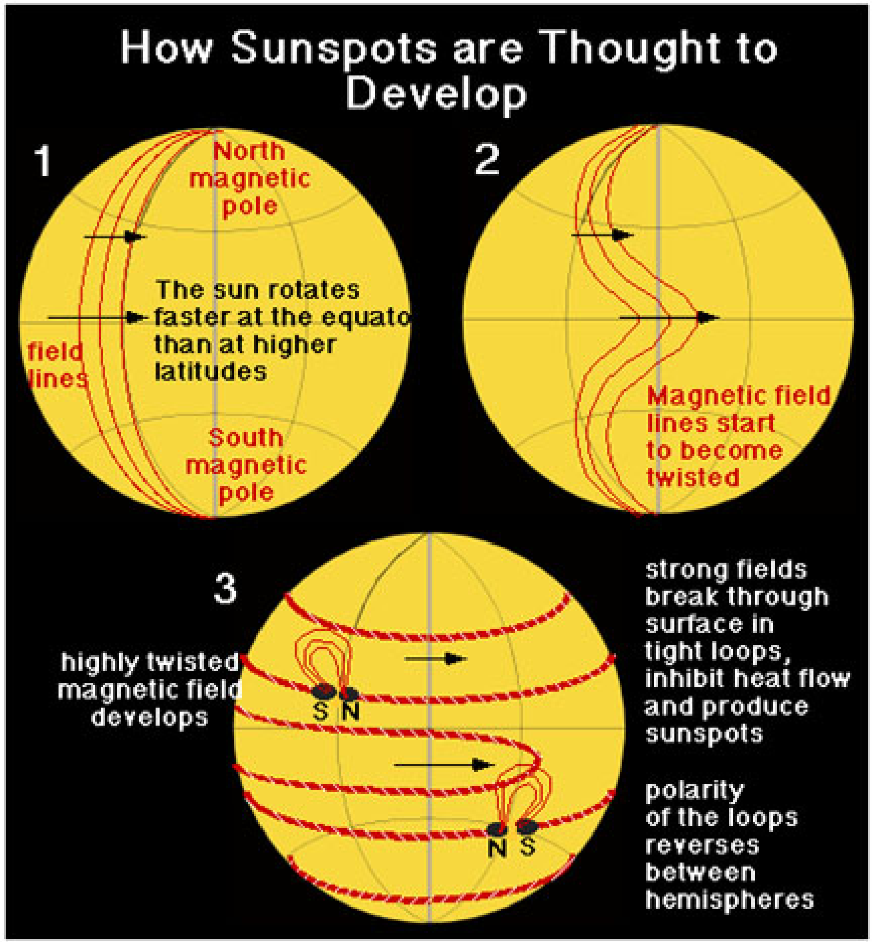


# AOSC 652: Analysis Methods in AOSC

**11 year solar cycle**




Differential rotation of the sun (equator rotates faster, ~25 days, than mid latitudes ~27 days) causes magnetic field lines to stretch and twist.

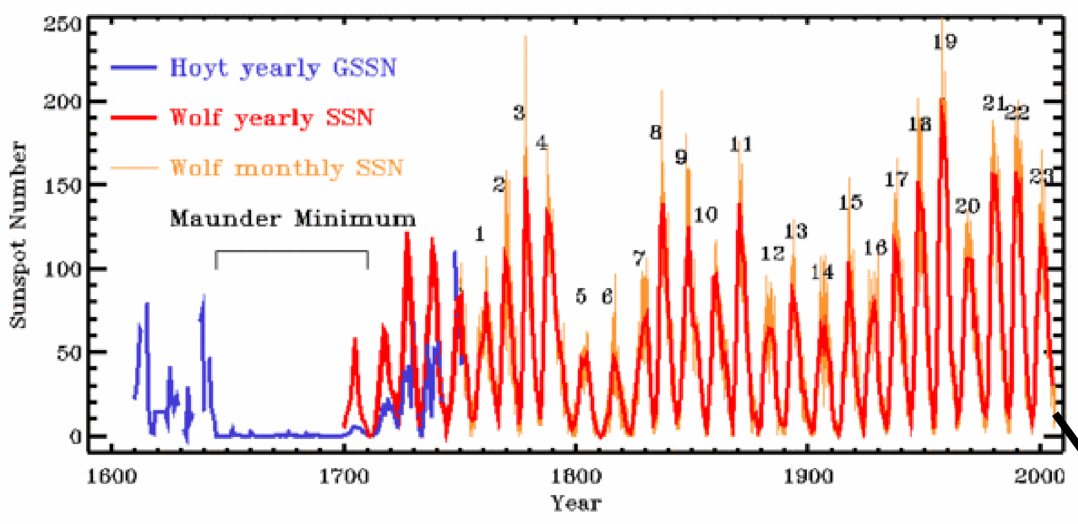
Eventually, the magnetic field lines loop around the sun and strong fields emerge from the surface.

Sun spots are regions where magnetic field lines leave or re-enter the surface.

Eventually (every ~11 years) magnetic anomalies wrap enough that they re-align with the North-South magnetic field lines.

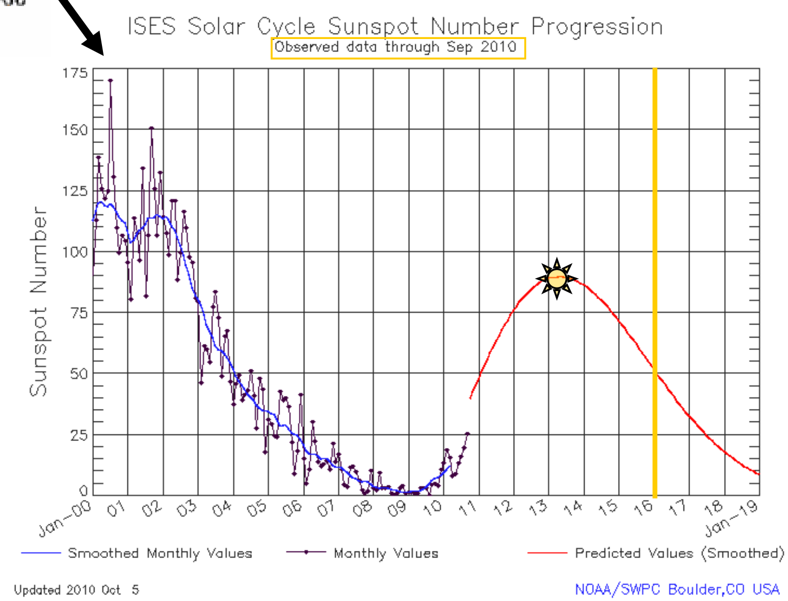
http://www.windows.ucar.edu/spaceweather/images/sunspot_form.jpg

# AOSC 652: Analysis Methods in AOSC




The cycle repeats itself every ~11 years

http://schools-wikipedia.org/wp/s/Solar_cycle.htm




Prior solar cycle #23 had an extended minimum. Predictions for cycle #24 suggest the next maximum will be the lowest in decades.

Also see http://www.windows2universe.org/sun/activity/sunspot_history.html

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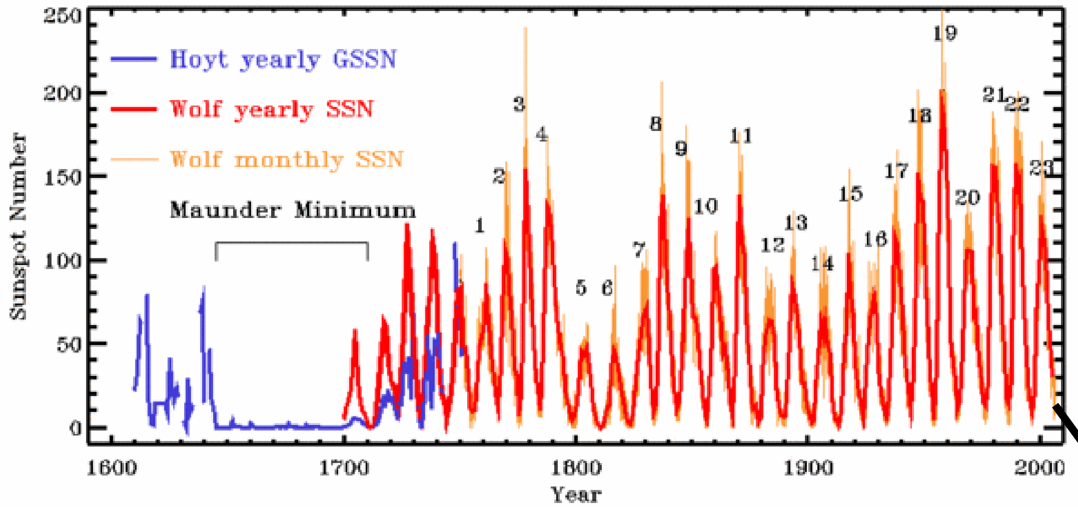


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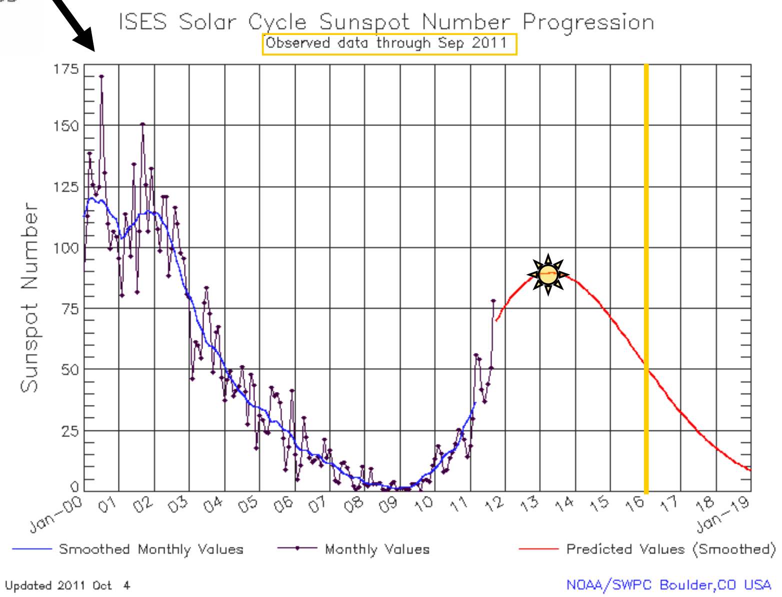


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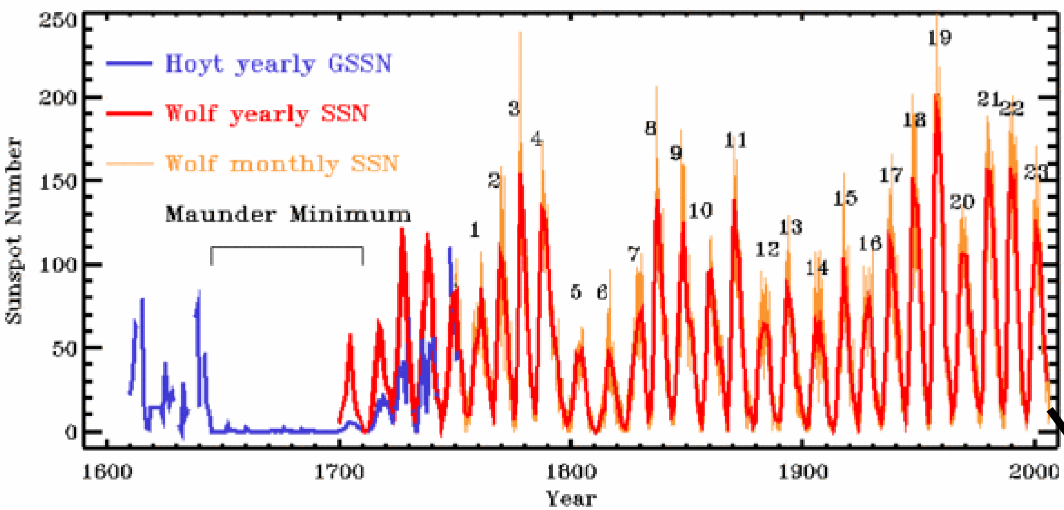


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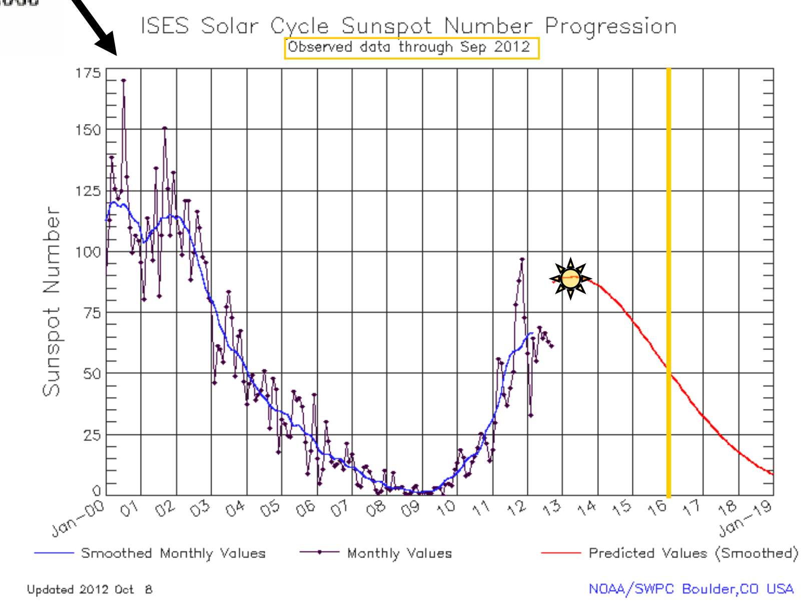


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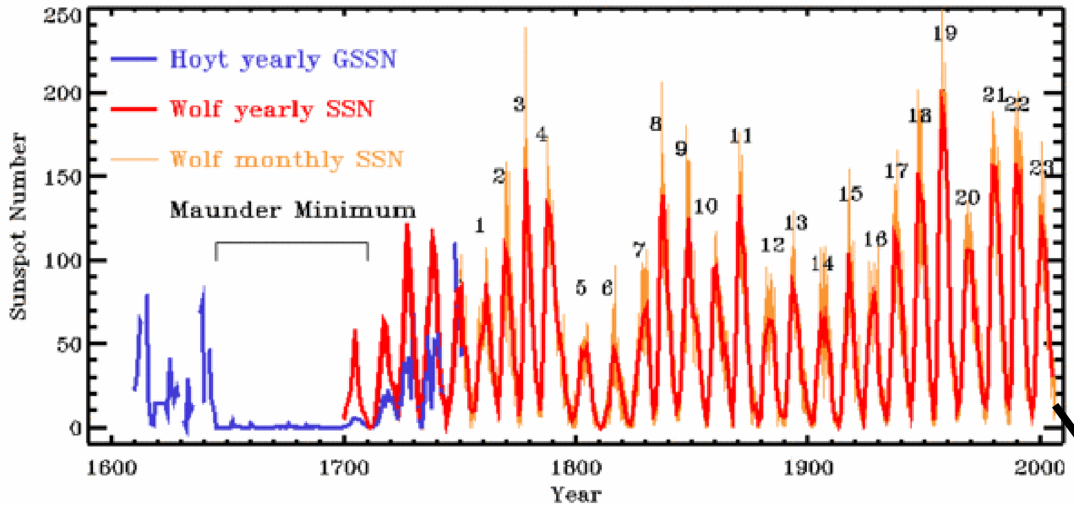


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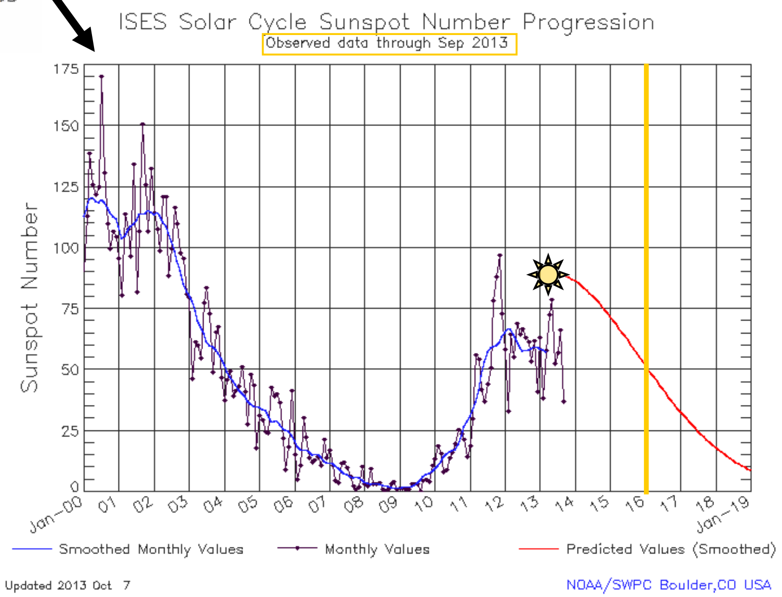


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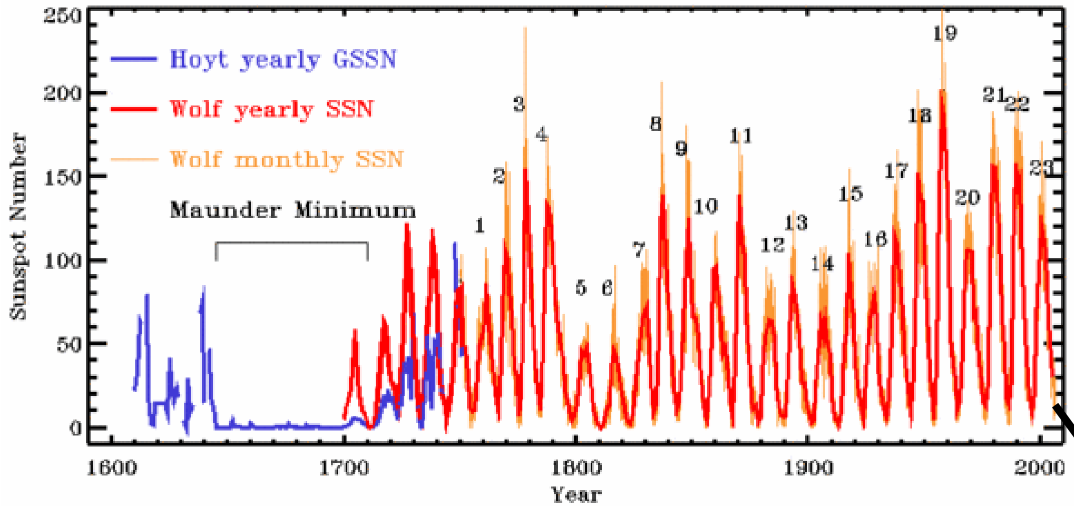


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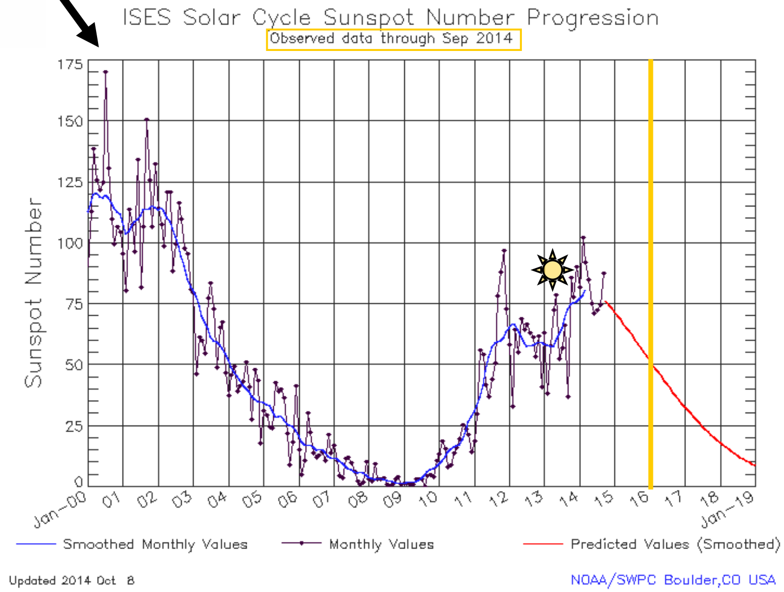


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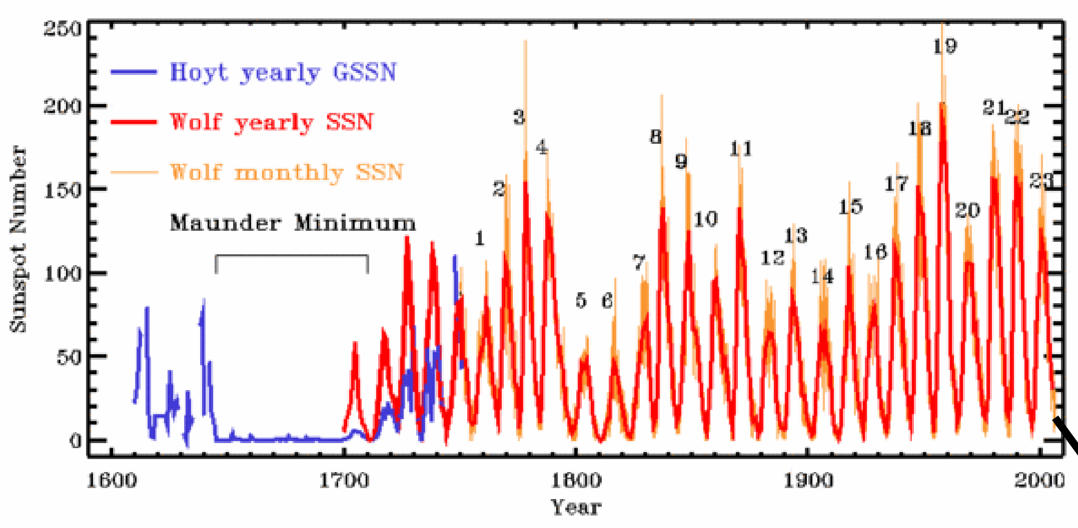


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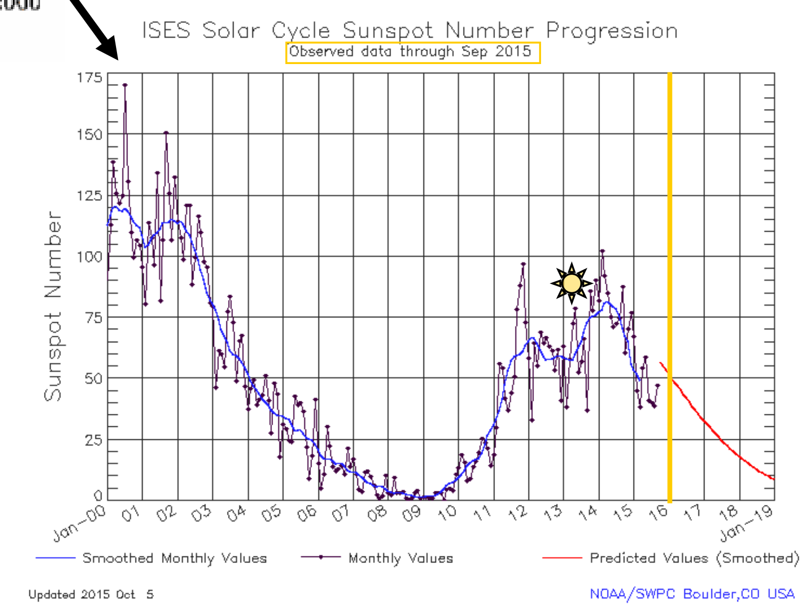


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26 Oct 2016

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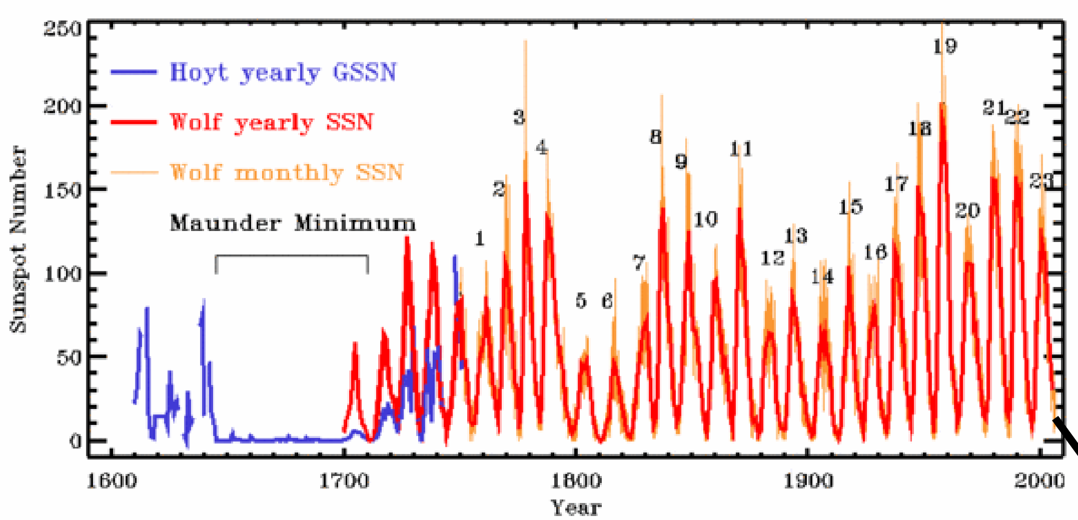


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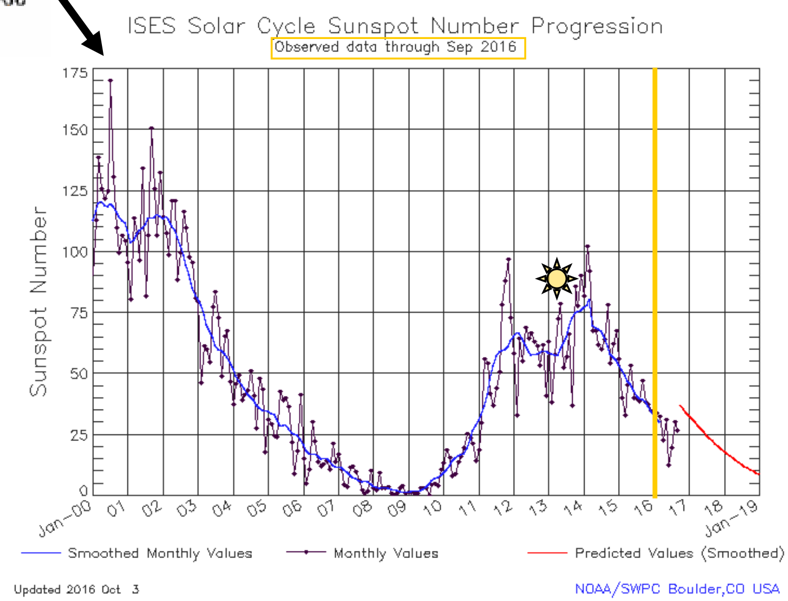


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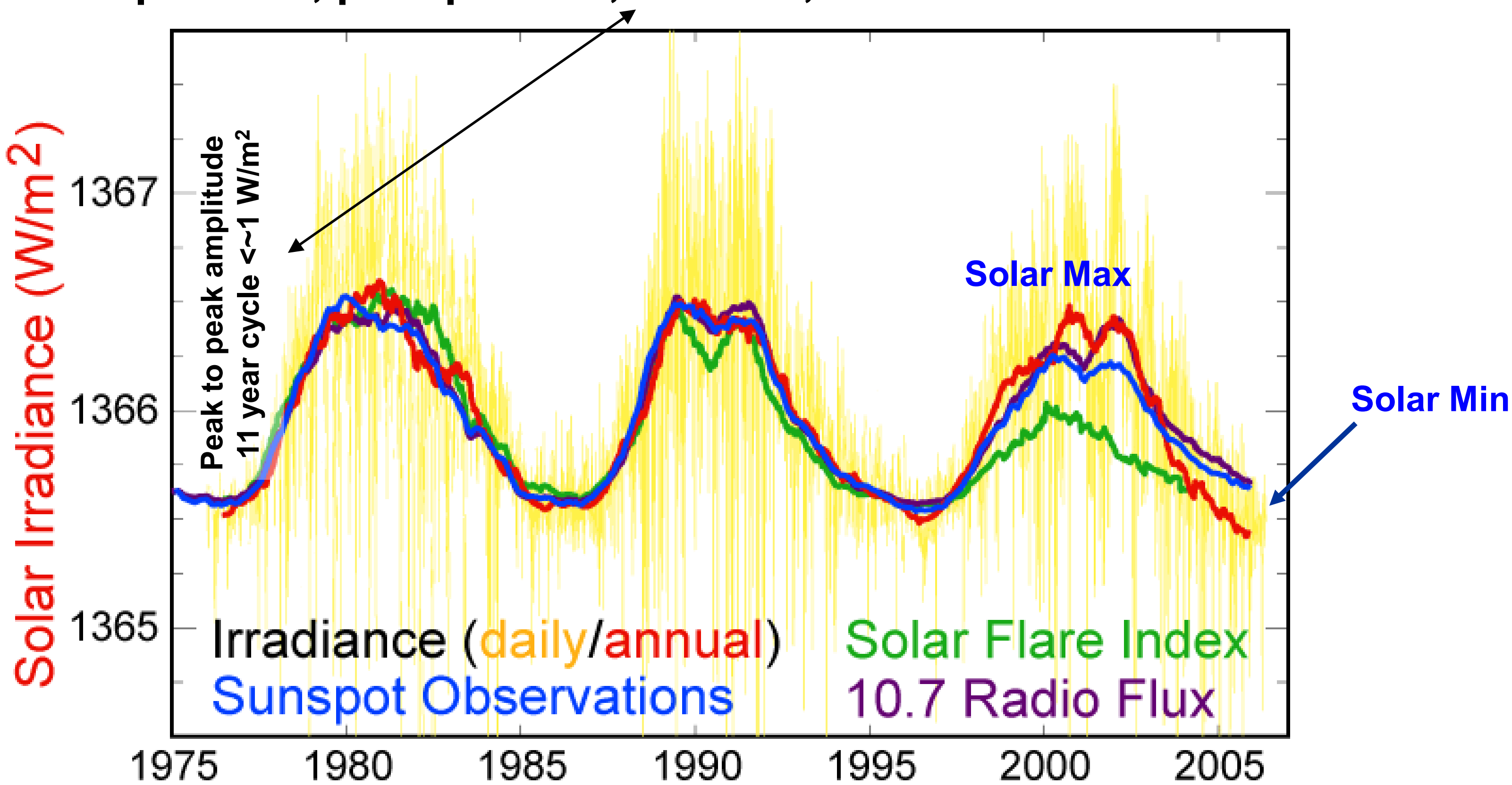
# AOSC 652: Analysis Methods in AOSC

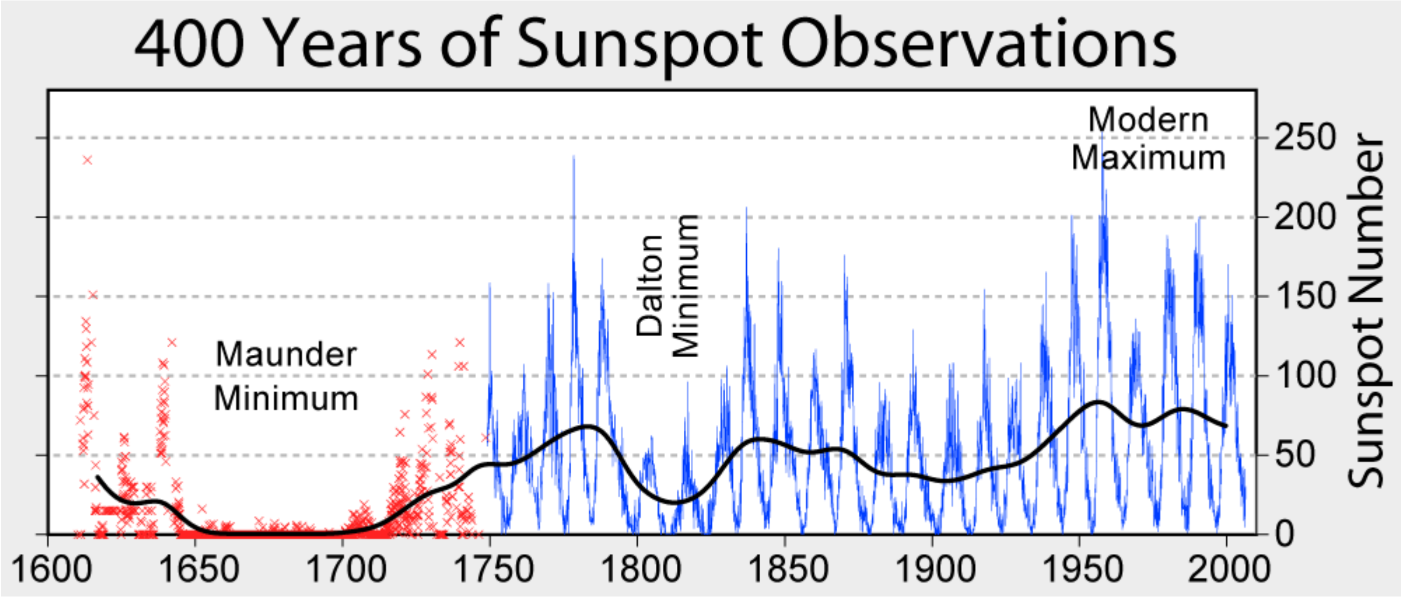
- **Total solar irradiance varies with sunspot number**
- **This variation has been linked to changes in atmospheric composition, precipitation, weather, and climate**




http://en.wikipedia.org/wiki/Solar_variation

# AOSC 652: Analysis Methods in AOSC

**Sunspot variations can be accurately reconstructed over past ~400 years:**




Source: http://commons.wikimedia.org/wiki/File:Sunspot_Numbers.png

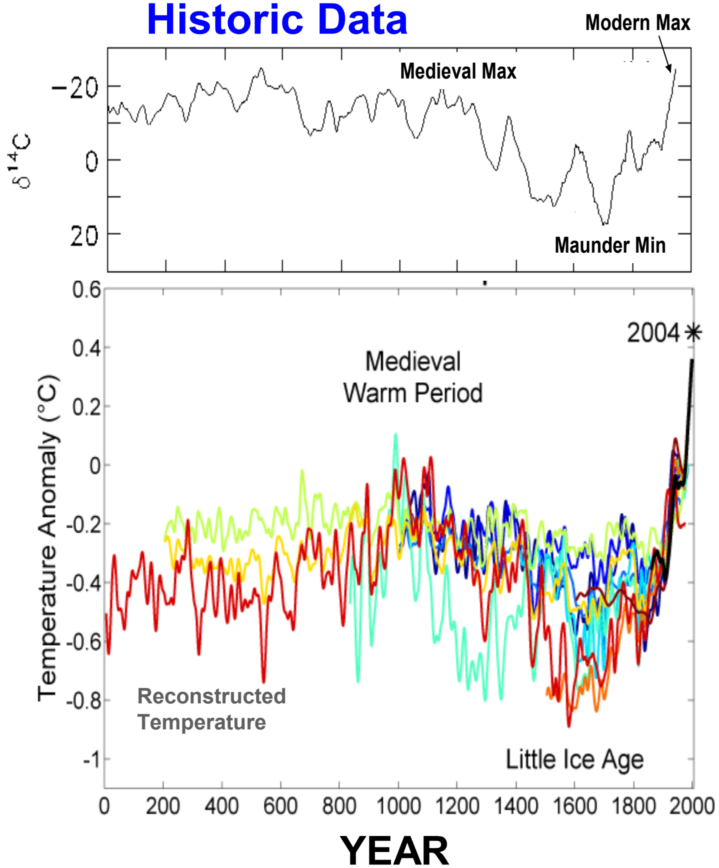
# AOSC 652: Analysis Methods in AOSC

## Solar variability:

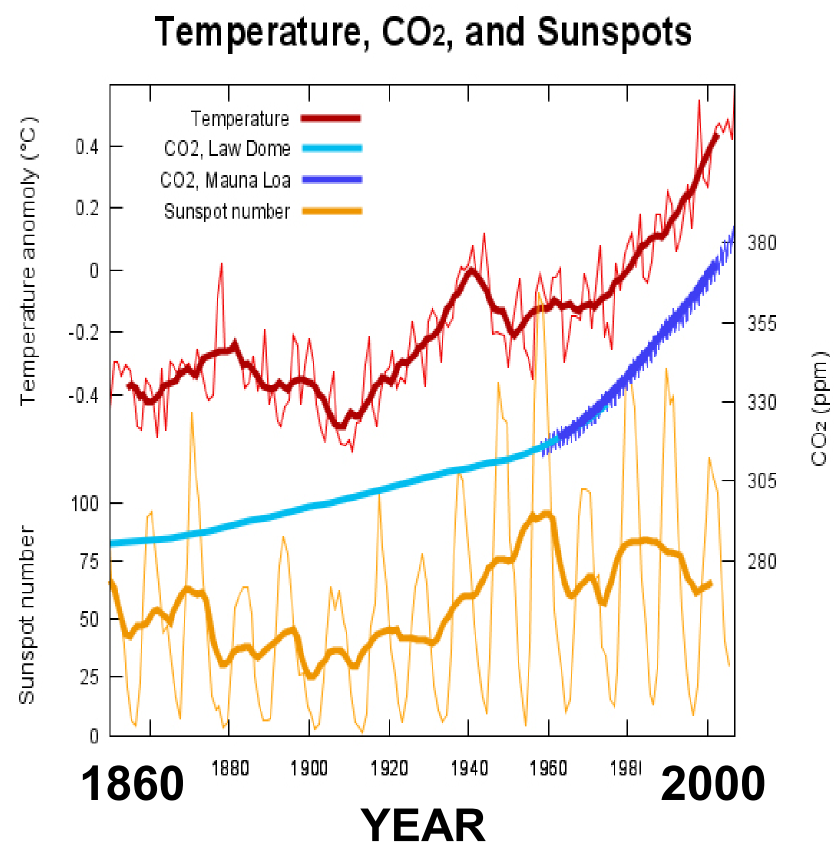
**Perhaps dominant forcing of Medieval Warming and Little Ice Age**

**Small effect on dT/dt since ~1900**

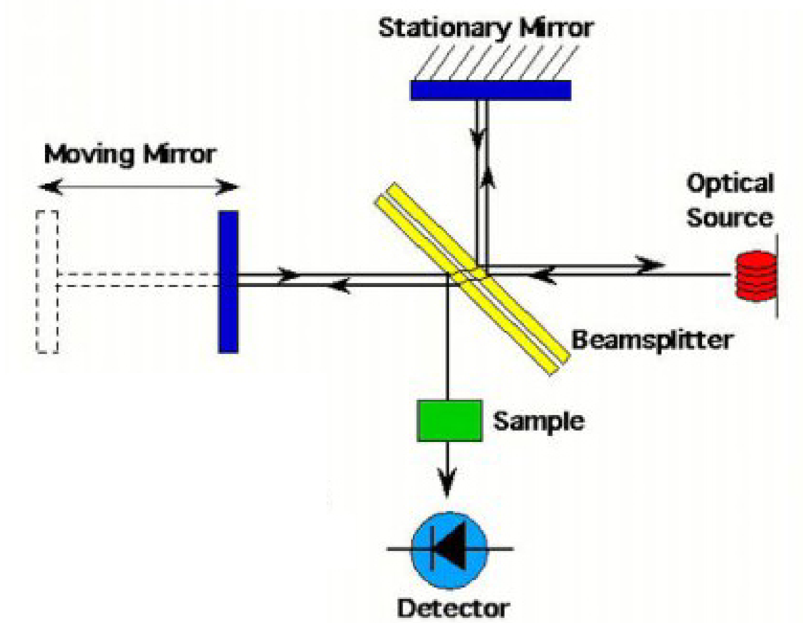
### Historic Data

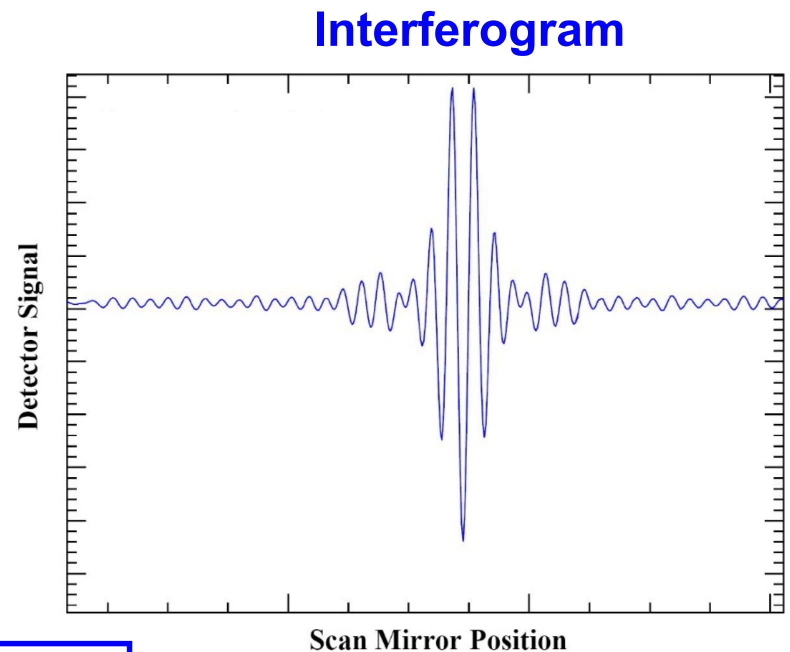



### Recent Data



## Fourier Transform Spectrometer




**The pictured device records an "Interferogram"**

**How do we go from an interferogram to a spectra?**

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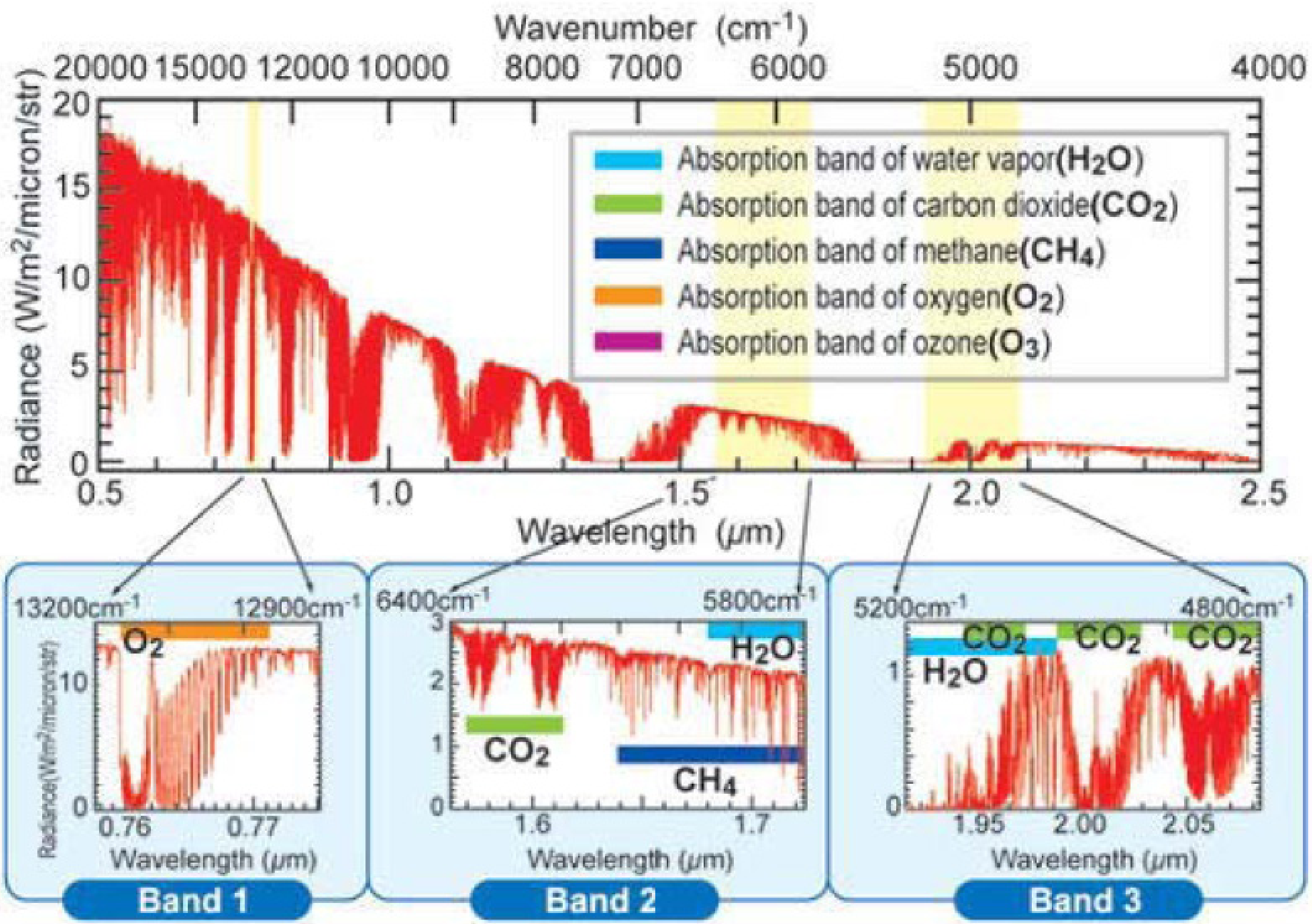
## Fourier Transform Spectrometer

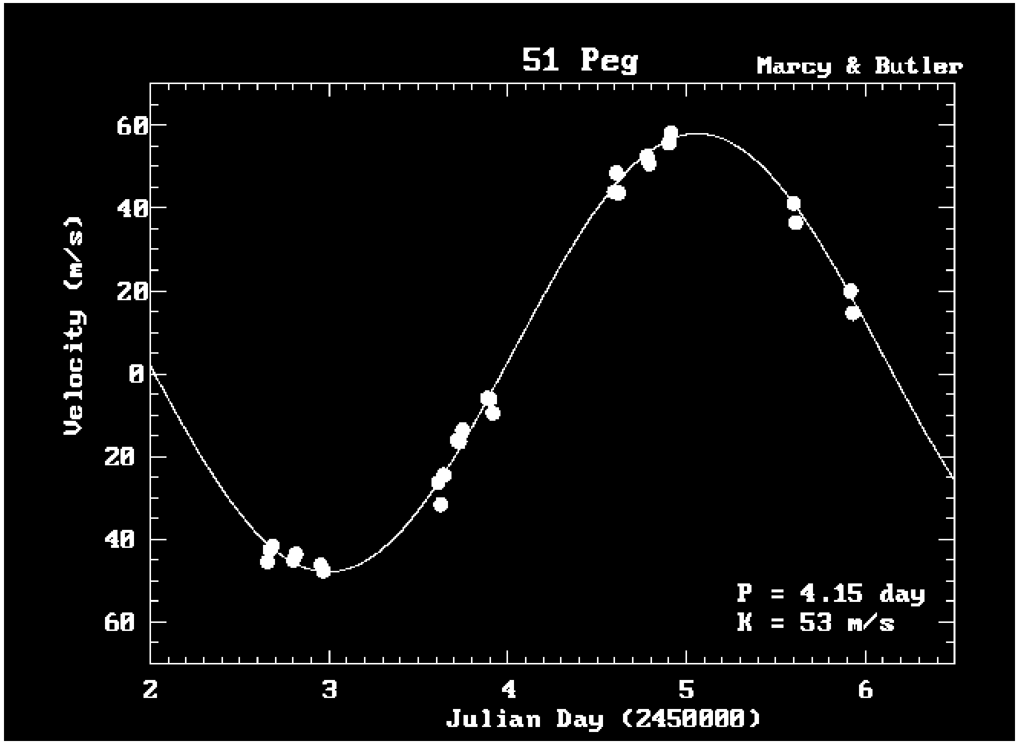
**Spectra (radiance vs wavelength) is the Fourier Transform of the interferogram !**




http://www.gosat.nies.go.jp/eng/gosat/zu8.htm

# AOSC 652: Analysis Methods in AOSC

## Discovery of Extra-solar Planets




First extra solar planet discovered by measuring small variations in the Doppler shift of light emitted by the parent star. These shifts are caused by the gravitational pull of the planet.

Amplitude and phase give planetary mass and distance from star,
in this case, ~0.5 x Jupiter and 0.05 Au

http://zebu.uoregon.edu/51peg.html

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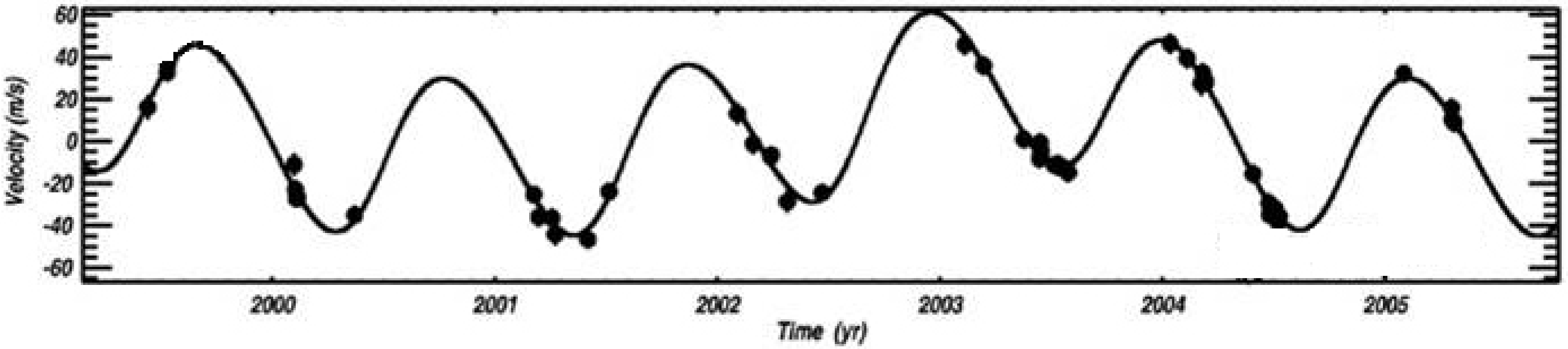


**What does this signal represent ?!?**

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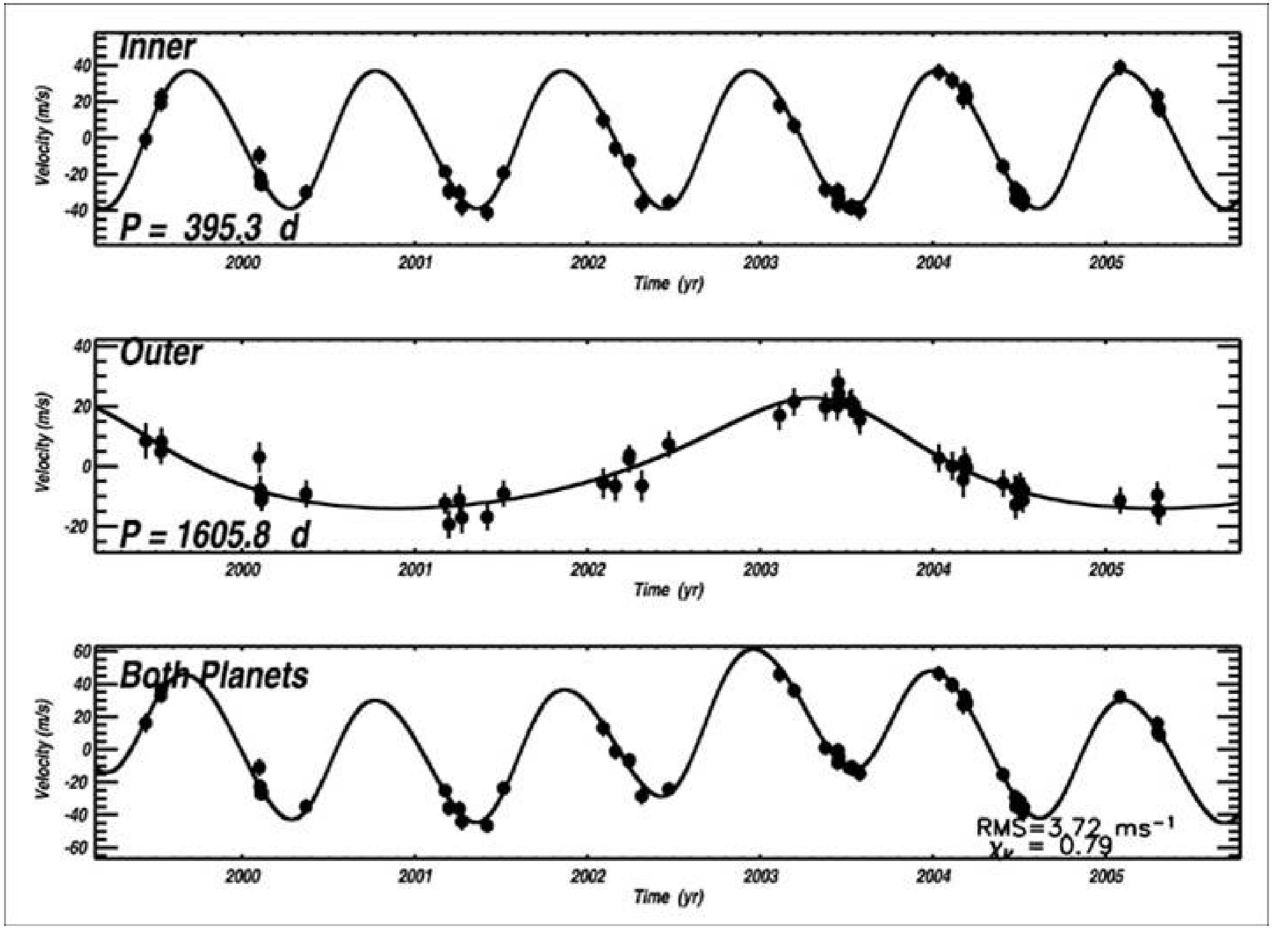


**Multiple planets : superposition of two sine waves !!!**

Vogt et al. Astrophysical Journal, 632: 638-658, 2005

## Let's digress to discuss FFT (Fast Fourier Transform):

# Historical Notes on the Fast Fourier Transform

JAMES W. COOLEY, PETER A. W. LEWIS, AND PETER D. WELCH

PROCEEDINGS OF THE IEEE, VOL. 55, NO. 10, OCTOBER 1967

THE fast Fourier transform (FFT) algorithm is a method for computing the finite Fourier transform of a series of $N$ (complex) data points in approximately $N \log_2 N$ operations. The algorithm has a fascinating history. When it was described by Cooley and Tukey[1] in 1965 it was regarded as new by many knowledgeable people who believed Fourier analysis to be a process requiring something proportional to $N^2$ operations with a proportionality factor which could be reduced by using the symmetries of the trigonometric functions. Computer programs using the $N^2$-operation methods were, in fact, using up hundreds of hours of machine time.

## Let’s digress to discuss FFT (Fast Fourier Transform):

**Fourier Transform**

Direct application of the definition requires $n$ multiplications and $n$ additions for each of the $n$ components of $Y$ for a total of $2n^2$ floating-point operations.

If $n$ is a power of 2, a one-dimensional FFT of length $n$ requires fewer than $3n \log_2 n$ floating-point operations. For $n = 2^{20}$, that's a factor of almost 35,000 faster than $2n^2$. Even if $n = 1024 = 2^{10}$, the factor is about 70.

A computer capable of doing one multiplication and addition every microsecond would require a million seconds, or about 11.5 days, to do a million-point ~~FFT~~. **Fourier Transform**

Chapter 8, Moler, Numerical Computing with MATLAB

## Let’s digress to discuss FFT (Fast Fourier Transform):

- **Numerous algorithm’s, all order N $\log_2$ N**
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- Moler (MATLAB) prefer algorithm of Frigo & Johnson (1998): speed

**MATLAB versions of Fast Fourier Transform include:**

- fft
- ffttx
- fftgui
- fftmatrix

See Moler reading

**MATLAB does not have a Fourier Transform (order $N^2$) operation in any library function**

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**User beware: computationally efficient FT algorithm may require data to be:**

- **regularly spaced**
- **have Npts be exact power of 2**
- **etc (see page 52, Muller and MacDonald)**

**for proper value of the FT to be returned**

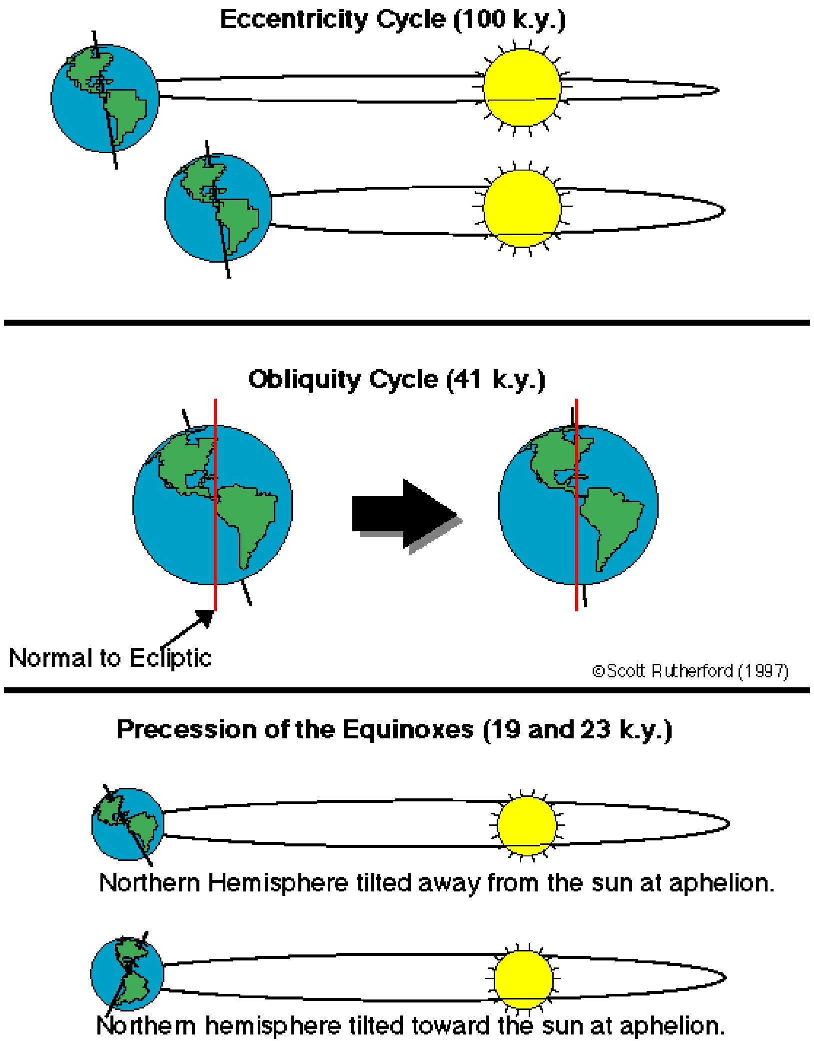
# AOSC 652: Analysis Methods in AOSC

## Fourier Series and Spectral Analysis

**The Earth's orbit varies on periods of:**

**100,000 years**
**41,000 years**
**23,000 years**
**19,000 years**

**due mainly to the gravitational pull of other planets**




http://ircamera.as.arizona.edu/NatSci102/NatSci102/lectures/climate.htm

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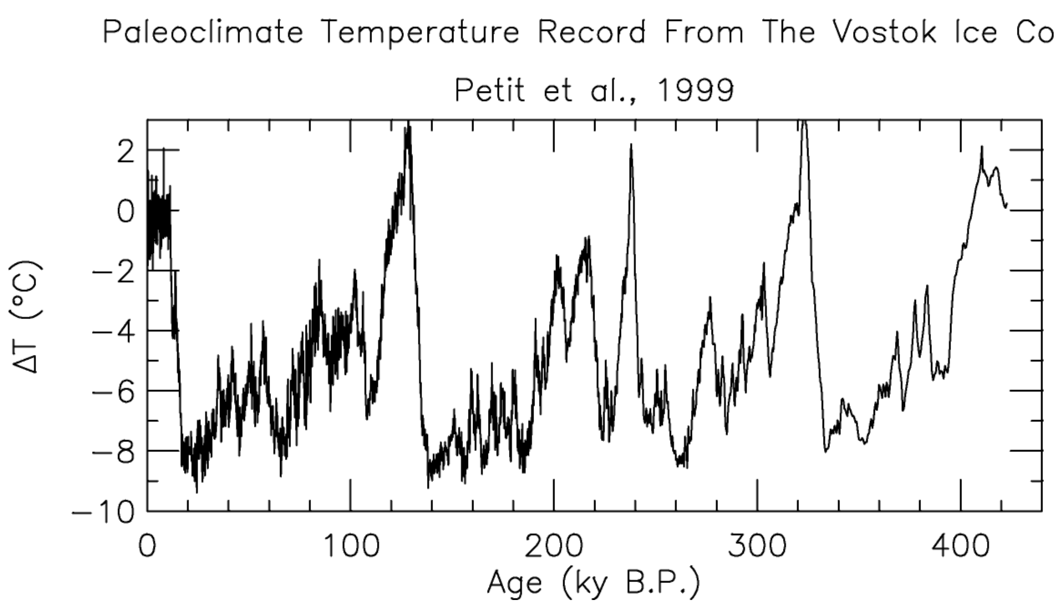
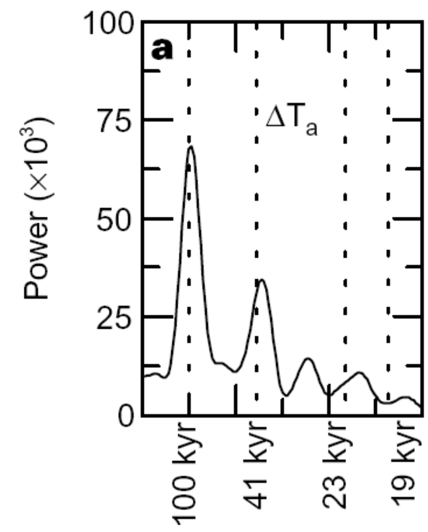
**Remarkably, these periods are embedded in Earth's long term climate record**




Petit *et al*., *Nature*, 1999.



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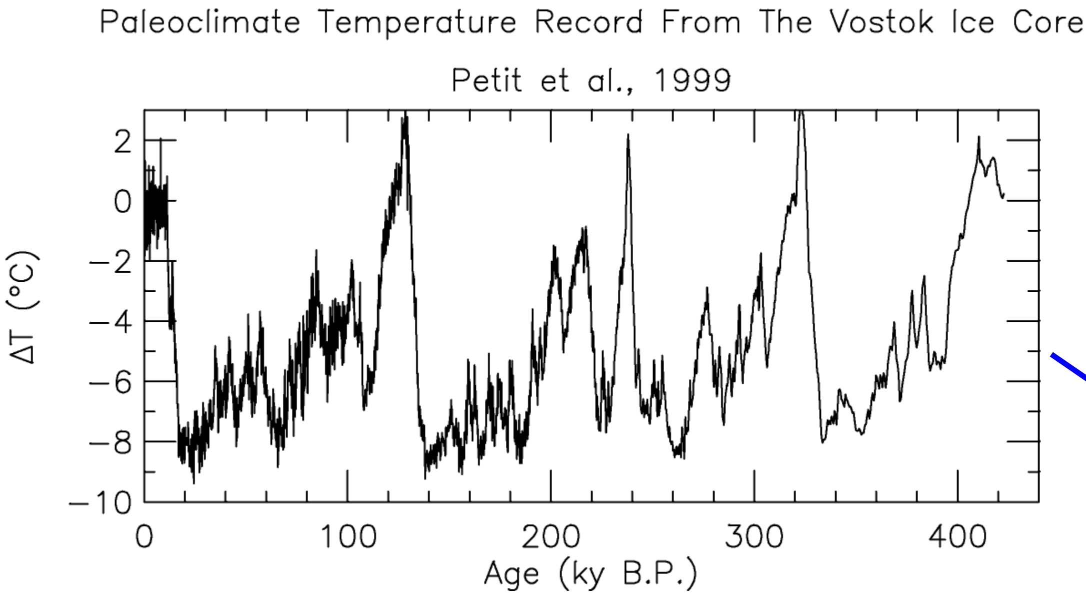
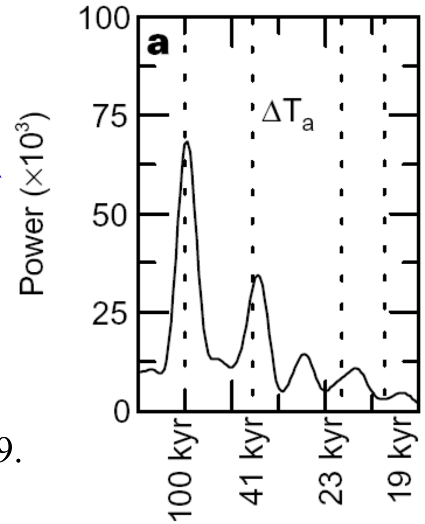
## Fourier Series and Spectral Analysis

**Assignment #9 involves producing the Power Spectrum of the actual Vostok Ice Core temperature record !**

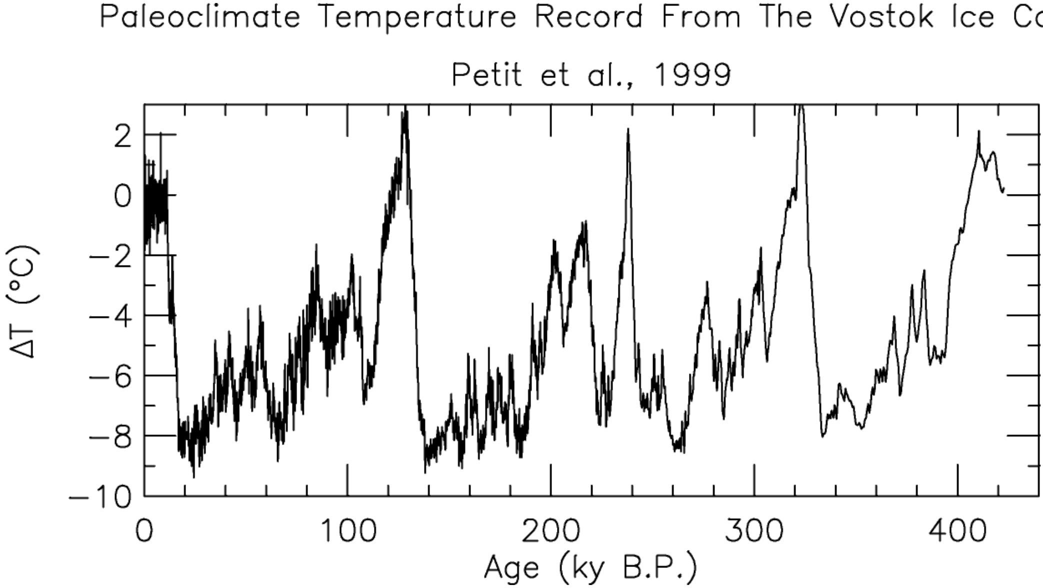


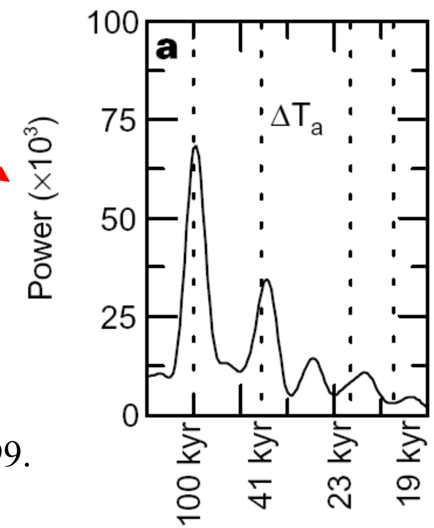




Petit *et al.*, *Nature*, 1999.

## Fourier Series and Spectral Analysis

**There is an important complication we must discuss**




Petit *et al.*, *Nature*, 1999.

## Fourier Series and Spectral Analysis

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**The calculation of Fourier Coefficients, upon which the Power Spectrum is based, assumes the input time series "repeats" indefinitely**

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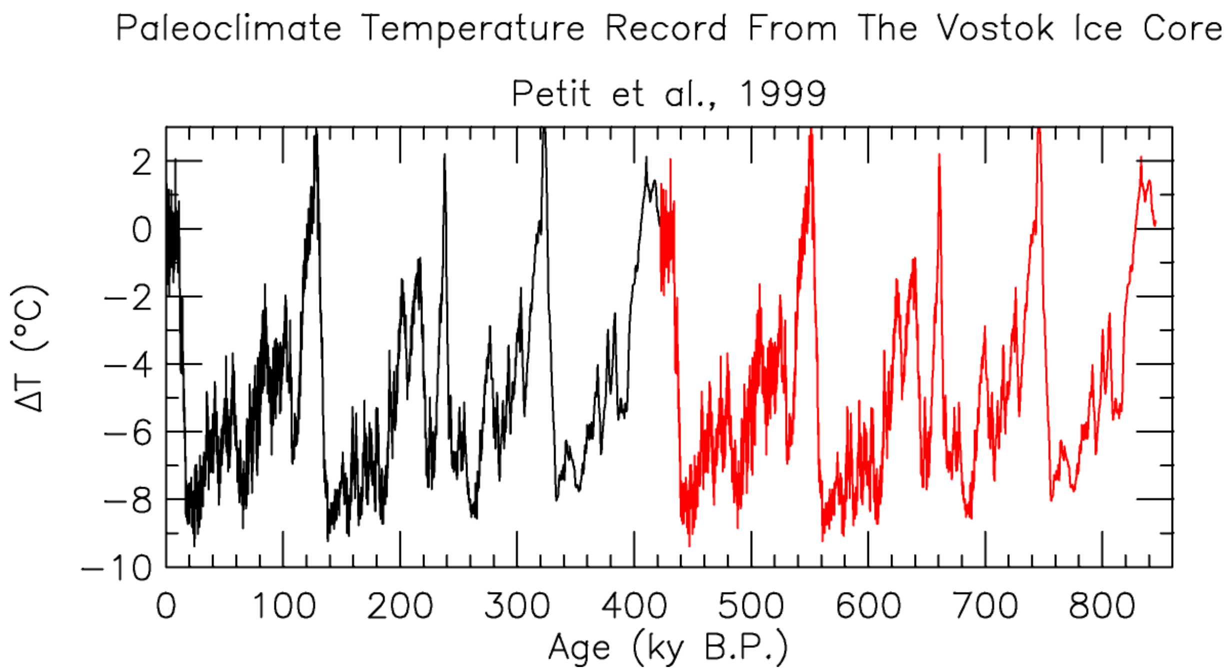

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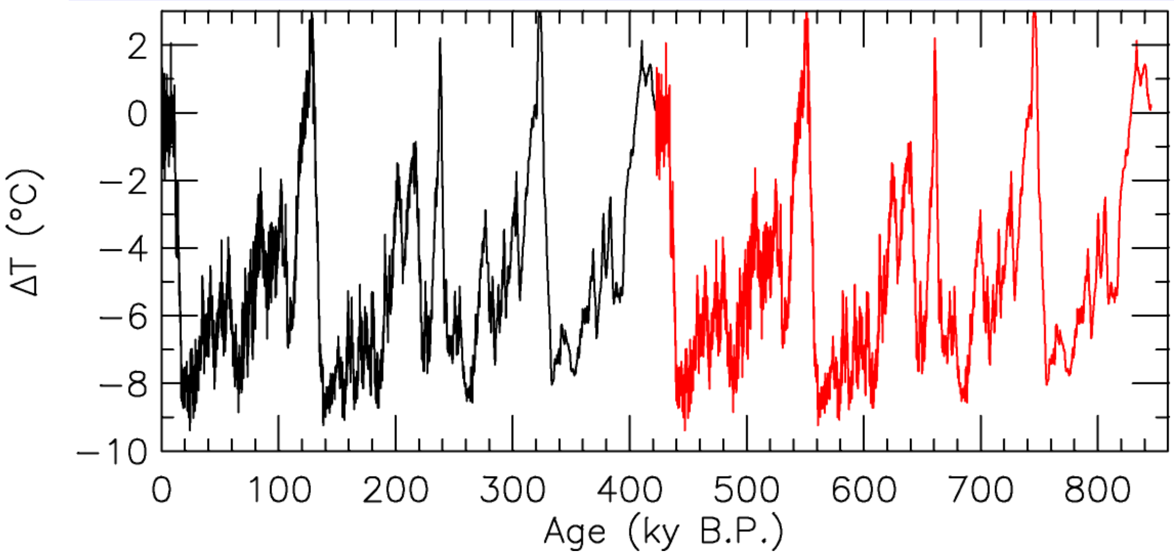
**Why did we neglect this complication for our analysis of the dial tone and sunspot time series?**



**Two ways to handle *Gibb's phenomenon* :**

**a) Blackman-Tukey method: involves taking an autocorrelation of the data, which "blurs" the power spectrum at high frequency.**

**b) Taper function: involves multiplying the raw data by a function that minimizes weight of data collected at beginning and end of the time series. Then, signal × taper is analyzed.**

**See pages 63 to 73, Muller and MacDonald**

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$$taper(t) = 0.5\,[\,1 - \cos(2\pi\, t / T)\,]$$

where $T$ = length of time interval

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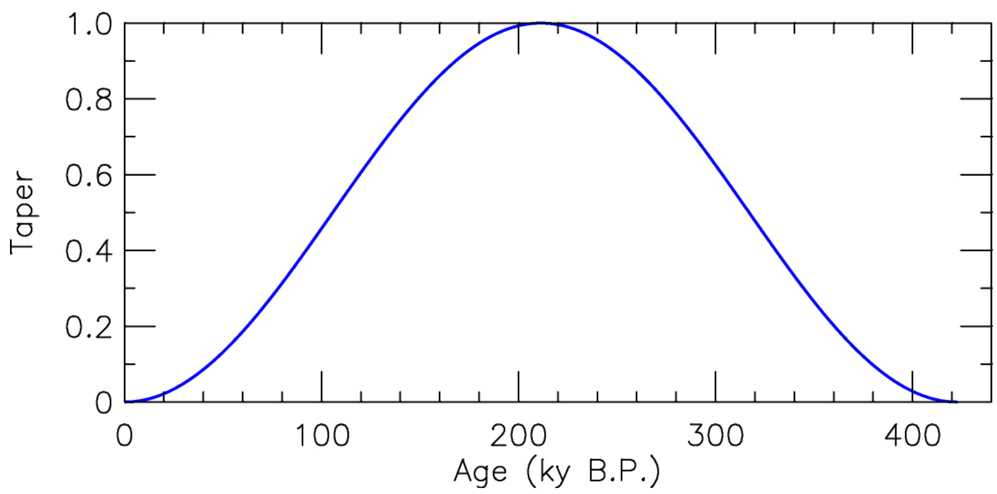

## Fourier Series and Spectral Analysis

**Will now guide you in the sunspot analysis using Python (here) and IDL (3408)**