Analysis Methods in Atmospheric and Oceanic Science

AOSC 652

- Today: Multiple Linear Regression
- Wed: Description of Projects; Python & IDL breakouts

Week 10, Day 1

31 Oct 2016

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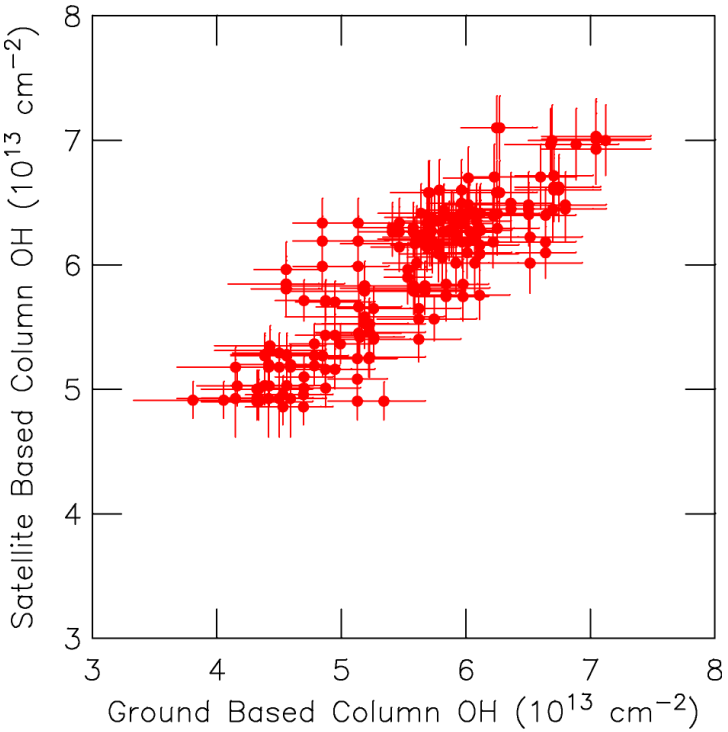
Correlation and Regression

Suppose you have two sets of measurements (or data and model) that you'd like to relate.



What are some aspects of the data that are typically examined?

Correlation and Regression

Linear Correlation Coefficient:

$$r = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N} (y_i - \bar{y})^2}}$$

r must lie between –1 and 1

If r = 1, the data are said to have a *complete positive correlation*

r = 0, the data are said to be *uncorrelated*

r^2 x 100 = percent of variance in common between x and y

See http://www.mega.nu/ampp/rummel/uc.htm#C2 for a nice explanation

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Correlation:

- Used to assess the “relationship” between two or more variables
- “Relationship” questions: **strong** or **weak** correlation ?
  linear and if not, **functional form** ?
- What else ?!?

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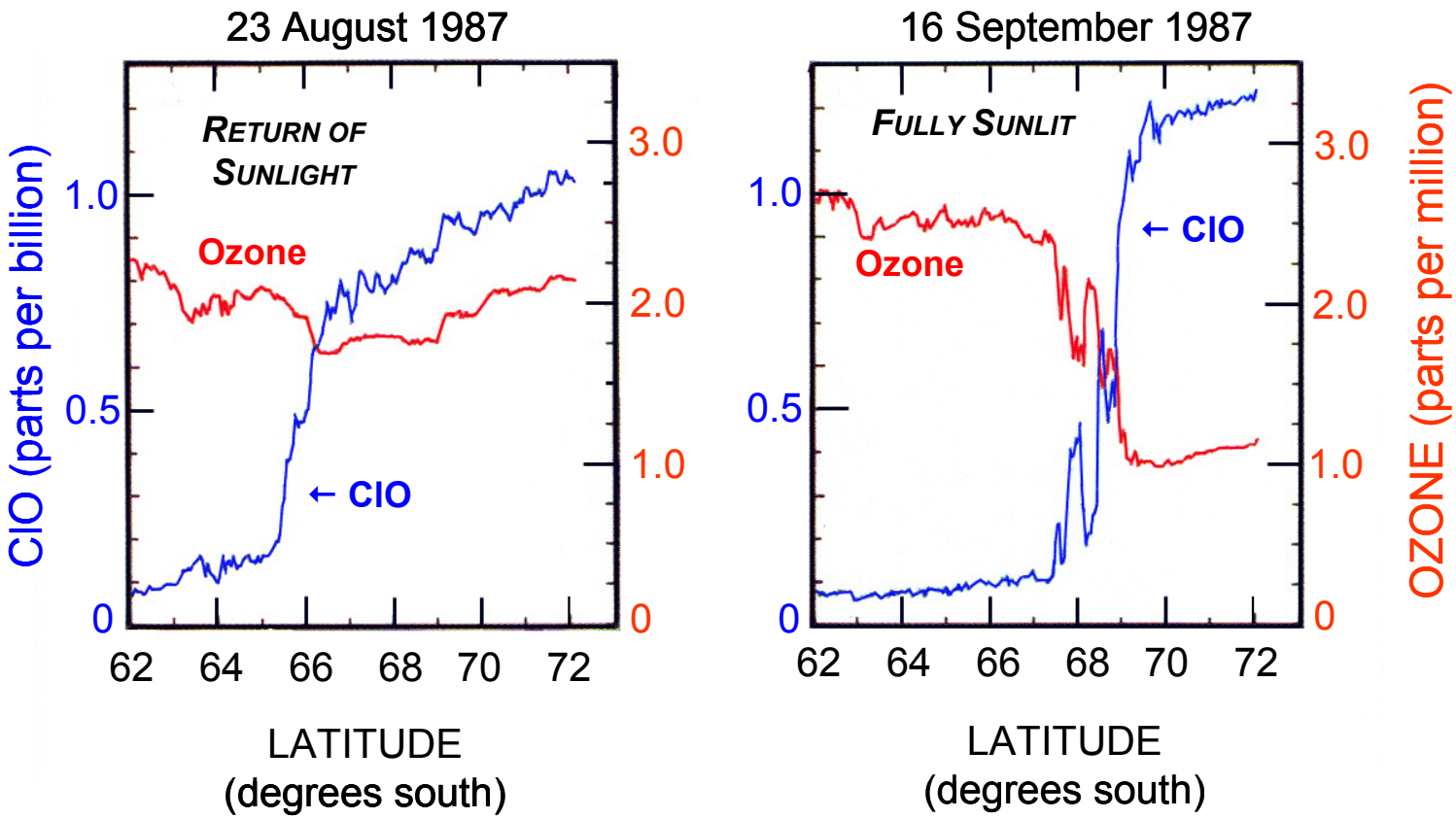
What other knowledge, in addition to these observations, was needed to demonstrate that ClO (from CFCs) causes the ozone hole ?!?

Airborne Antarctic Ozone Expedition:
Punta Arenas, Chile,1987



Anderson et al., *Science*, 1991

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Regression (as used in AOSC):

- **Development of a mathematical model between a set of observations (i.e., a time series or a spatial distribution) and one or more predictor variables (usually measured or inferred from a proxy)**
- **Regression analysis almost always is preceded by a correlation analysis**
- ***Strong science* involves understanding (or development) of the underlying, causal relations between observations and predictor variables**

Multiple Linear Regression

$$Y = c_0 + c_1 X_1 + c_2 X_2 + \ldots + c_p X_p$$

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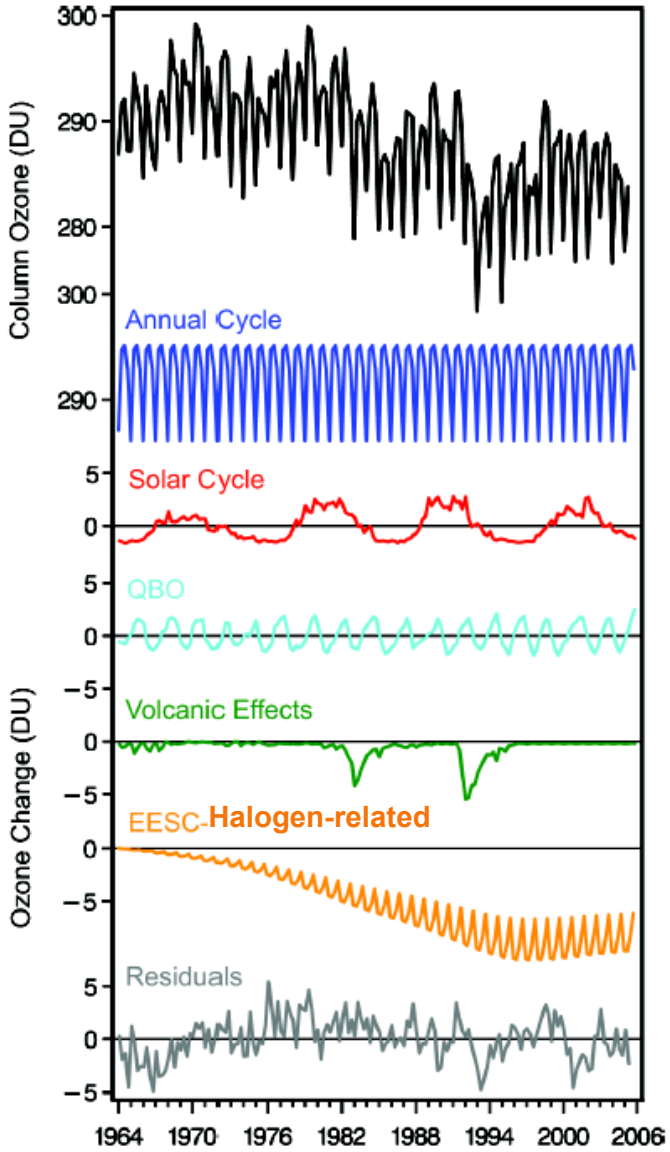

Variations in total column ozone (DU) for 60°S to 60°N (top panel) and individual components of a multiple linear regression of various explanatory variables times the regression coefficient. The residual between the data and the regression model is also shown (bottom panel).

Figure 3-1, 2006 WMO/UNEP Ozone Assessment Report

http://www.esrl.noaa.gov/csd/assessments/ozone/2006/images/Fig3-01.jpg

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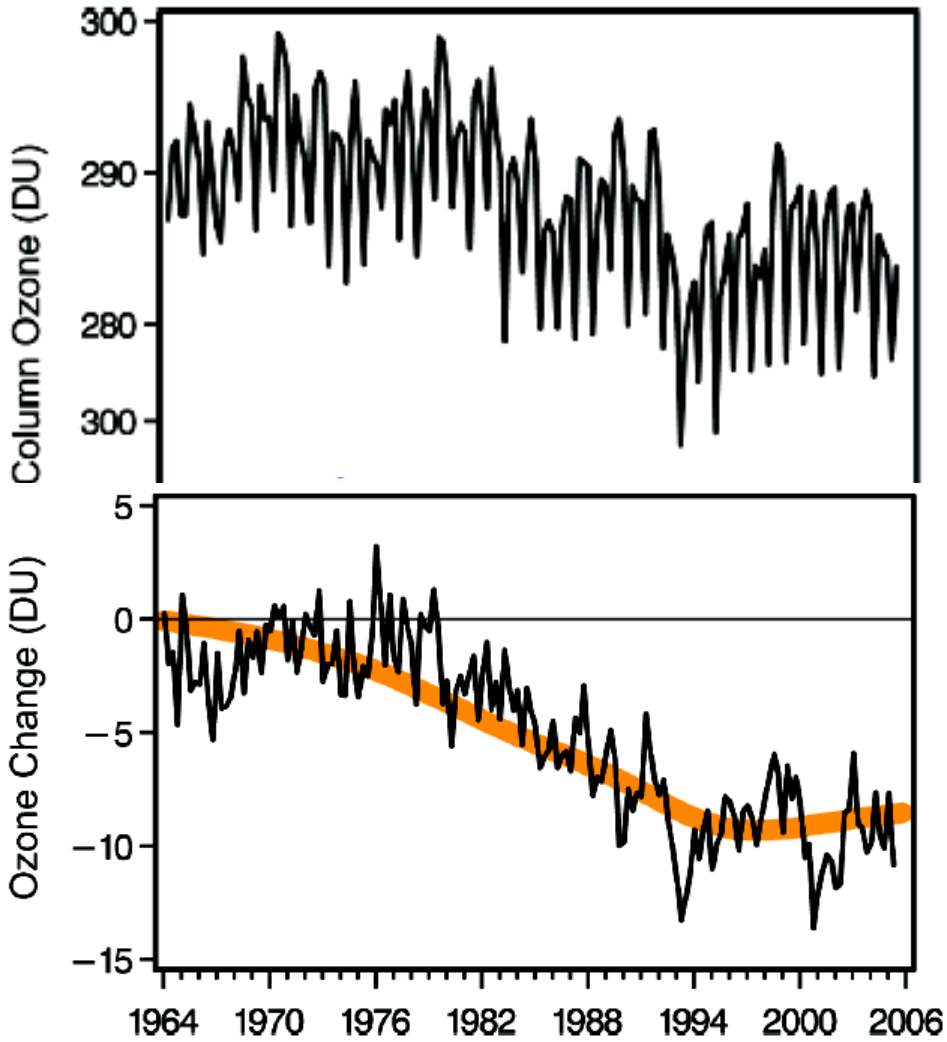

Raw Data (total ozone, 60°S to 60°N)

Adjusted data:
Deseasonalized total ozone deviations from 1964 value adjusted for effects of solar, volcanic, and QBO forcings (BLACK) compared to time series of stratospheric halogens (ORANGE) scaled to fit the data.

Figure 3-1, 2006 WMO/UNEP Ozone Assessment Report

http://www.esrl.noaa.gov/csd/assessments/ozone/2006/images/Fig3-01.jpg

Multiple Linear Regression

$$Y = c_0 + c_1 X_1 + c_2 X_2 + \ \ldots + c_p X_p$$

How do we solve for the regression coefficients ?

Multiple Linear Regression

$$\text{Cost Function} \Rightarrow \mathrm{F} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(c_0 + \sum_{j=1}^{p} c_j X_{i,j} - Y_i \right)^2$$

$$\frac{\partial F}{\partial c_0} = 2 \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(c_0 + \sum_{j=1}^{p} c_j X_{i,j} - Y_i \right) = 0$$

$$\frac{\partial F}{\partial c_1} = 2 \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(c_0 + \sum_{j=1}^{p} c_j X_{i,j} - Y_i \right) \left(X_{i,1} \right) = 0$$

$$\frac{\partial F}{\partial c_2} = 2 \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(c_0 + \sum_{j=1}^{p} c_j X_{i,j} - Y_i \right) \left(X_{i,2} \right) = 0$$

$$\vdots$$

$$\frac{\partial F}{\partial c_p} = 2 \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(c_0 + \sum_{j=1}^{p} c_j X_{i,j} - Y_i \right) \left(X_{i,p} \right) = 0$$

Multiple Linear Regression

$$c_0\ (n) + c_1 \sum_{i=1}^{n} X_{i,1} + c_2 \sum_{i=1}^{n} X_{i,2} + \ \ldots + c_p \sum_{i=1}^{n} X_{i,p} = \sum_{i=1}^{n} Y_i$$

$$c_0 \sum_{i=1}^{n} X_{i,1} + c_1 \sum_{i=1}^{n} (X_{i,1})^2 + c_2 \sum_{i=1}^{n} X_{i,1} X_{i,2} + \ \ldots + c_p \sum_{i=1}^{n} X_{i,1} X_{i,p} = \sum_{i=1}^{n} X_{i,1} Y_i$$

$$c_0 \sum_{i=1}^{n} X_{i,2} + c_1 \sum_{i=1}^{n} X_{i,1} X_{i,2} + c_2 \sum_{i=1}^{n} (X_{i,2})^2 + \ \ldots + c_p \sum_{i=1}^{n} X_{i,2} X_{i,p} = \sum_{i=1}^{n} X_{i,2} Y_i$$

$$\vdots$$

$$c_0 \sum_{i=1}^{n} X_{i,p} + c_1 \sum_{i=1}^{n} X_{i,1} X_{i,p} + c_2 \sum_{i=1}^{n} X_{i,2} X_{i,p} + \ \ldots + c_p \sum_{i=1}^{n} (X_{i,p})^2 = \sum_{i=1}^{n} X_{i,p} Y_i$$

Multiple Linear Regression

$$\begin{pmatrix} n & \sum_{i=1}^{n} X_{i,1} & \sum_{i=1}^{n} X_{i,2} & \cdots & \sum_{i=1}^{n} X_{i,p} \\ \sum_{i=1}^{n} X_{i,1} & \sum_{i=1}^{n} (X_{i,1})^2 & \sum_{i=1}^{n} X_{i,1}X_{i,2} & \cdots & \sum_{i=1}^{n} X_{i,1}X_{i,p} \\ \sum_{i=1}^{n} X_{i,2} & \sum_{i=1}^{n} X_{i,1}X_{i,2} & \sum_{i=1}^{n} (X_{i,2})^2 & \cdots & \sum_{i=1}^{n} X_{i,2}X_{i,p} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum_{i=1}^{n} X_{i,p} & \sum_{i=1}^{n} X_{i,1}X_{i,p} & \sum_{i=1}^{n} X_{i,2}X_{i,p} & \cdots & \sum_{i=1}^{n} (X_{i,p})^2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_p \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} Y_i \\ \sum_{i=1}^{n} X_{i,1}Y_i \\ \sum_{i=1}^{n} X_{i,2}Y_i \\ \vdots \\ \sum_{i=1}^{n} X_{i,p}Y_i \end{pmatrix}$$

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Multiple Linear Regression

$$\overline{\overline{A}} \times \vec{c} = \vec{b} \qquad \text{or} \qquad \vec{c} = \overline{\overline{A}}^{-1} \times \vec{b}$$

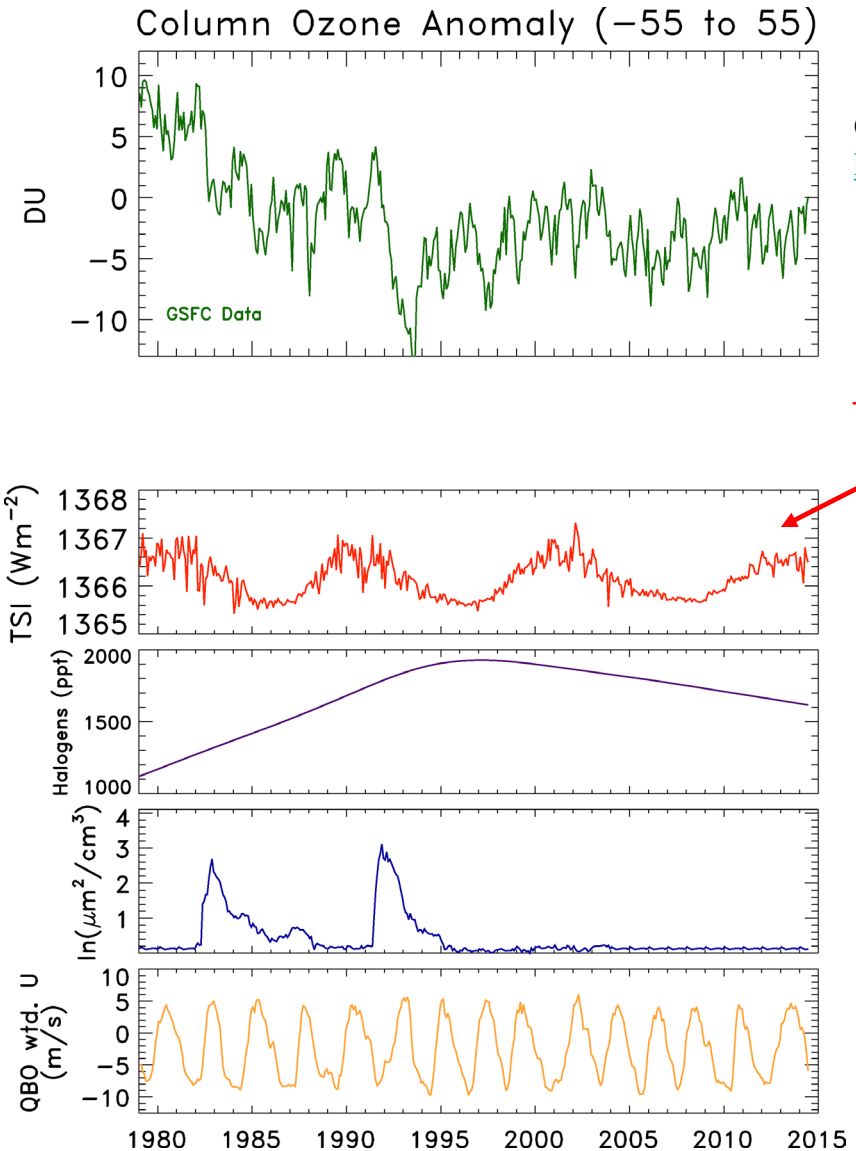
To regress an array of data points (e.g., global mean temperature) versus a set of predictor variables (e.g., ENSO, Volcanic Aerosols, Solar Irradiance, & Annual Avg CO_2) can read the data into a FORTRAN program and compute elements of matrix *A* and array *b*.

Then, can find the inverse of array *A.*

Then, can multiply the inverse of *A* by array *b* to find the array *c*.

Or, can use the "regress" function in IDL or MATLAB !

Global ozone anomaly versus time and 4 other, related quantities

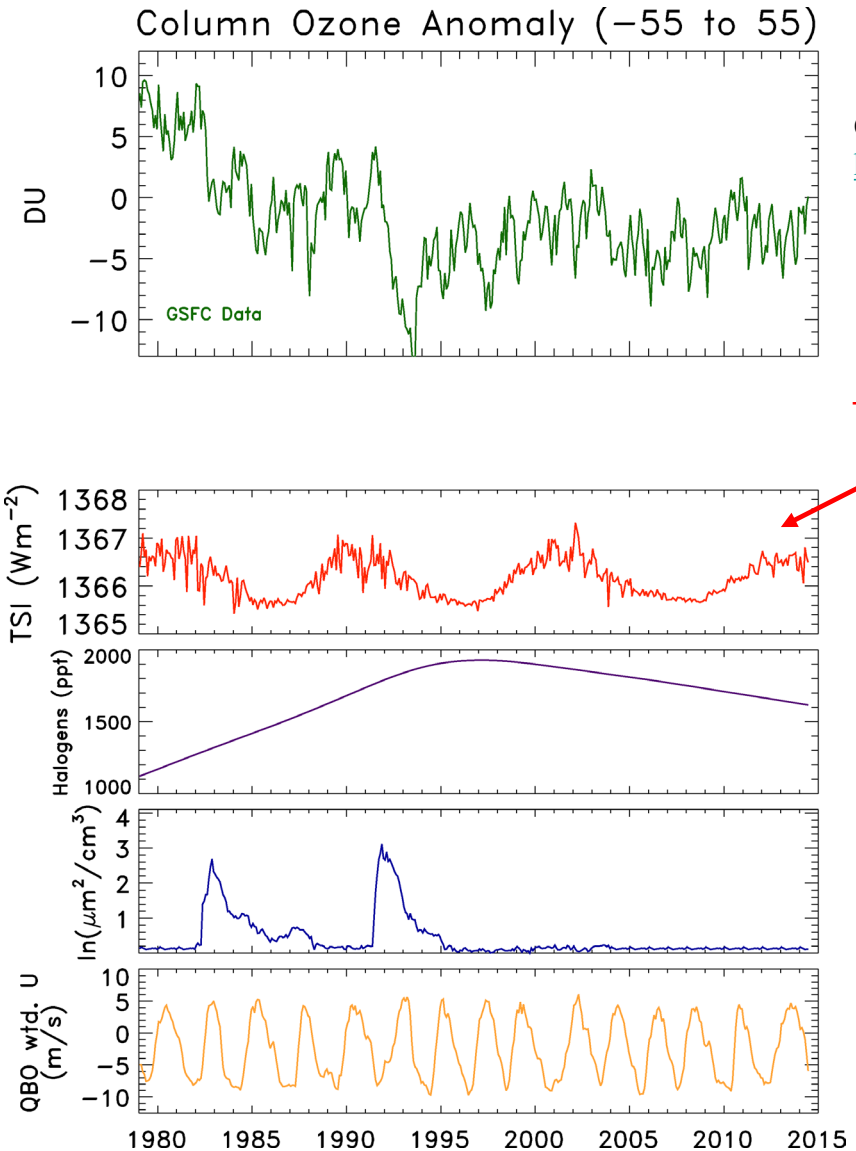


Ozone data from
http://acdb-ext.gsfc.nasa.gov/Data_services/merged

TSI : Total Solar Irradiance

The amount of energy Earth receives from the sun varies, with a periodicity of __ years

Global ozone anomaly versus time and 4 other, related quantities



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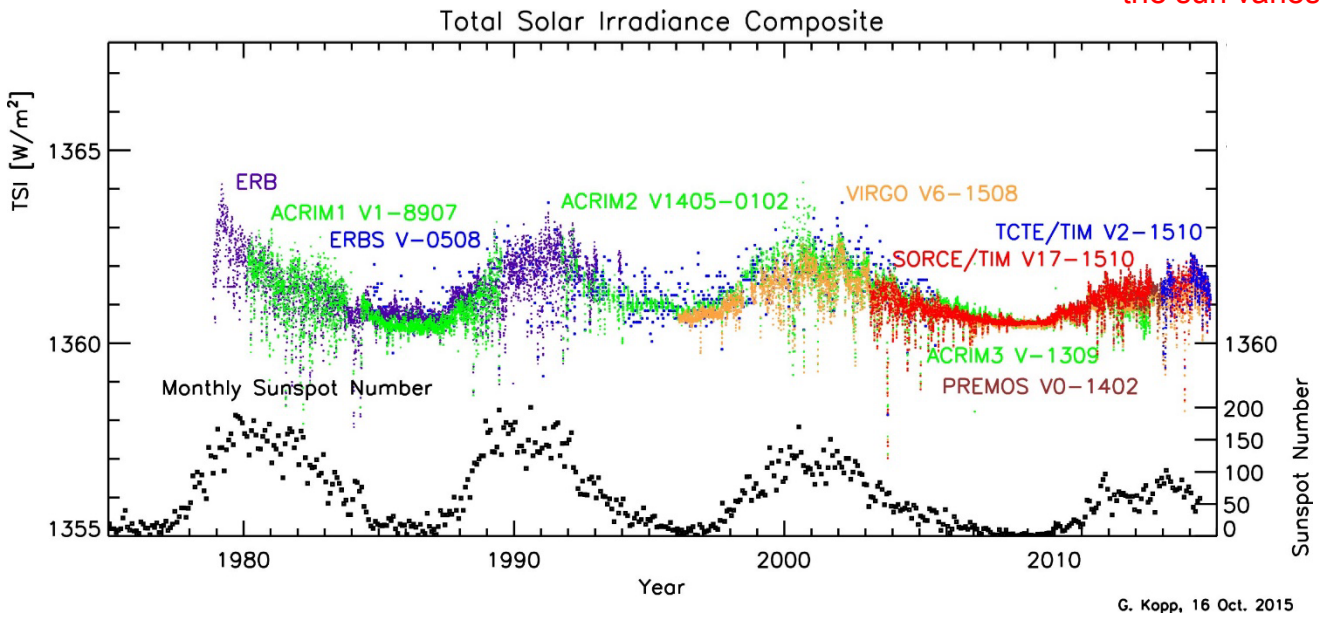
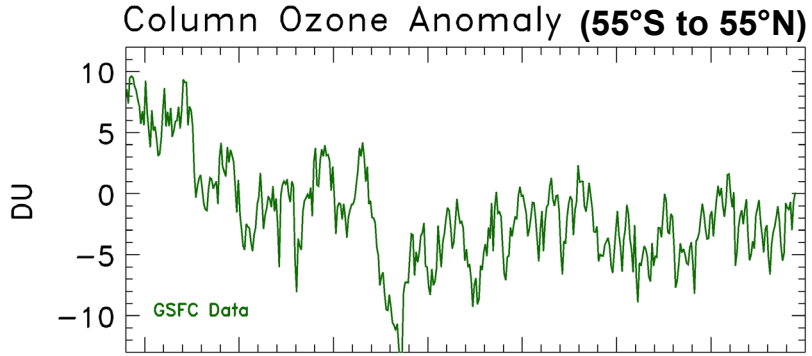


http://spot.colorado.edu/~koppg/TSI/TSI_Composite.jpg

Global ozone anomaly versus time and 4 other, related quantities



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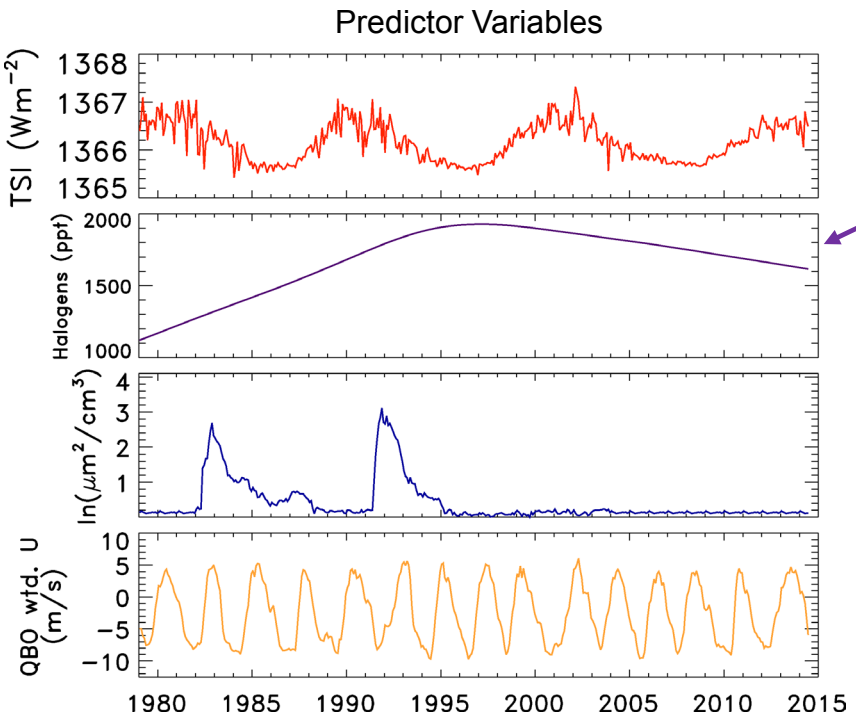
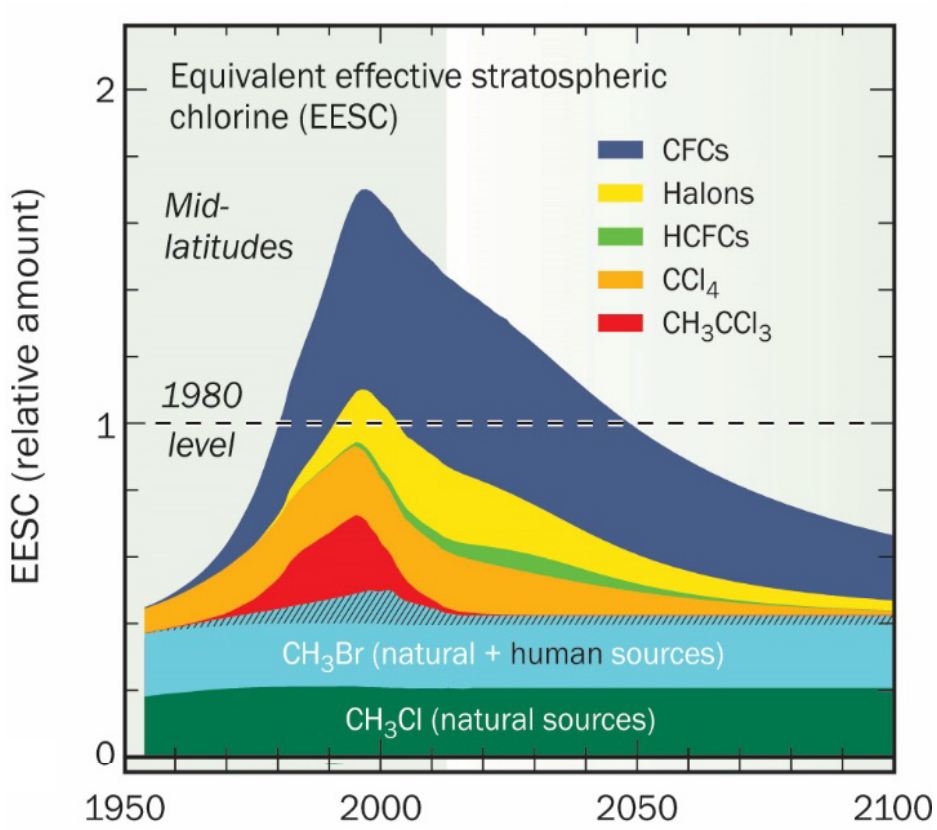


Stratospheric Halogen Loading:

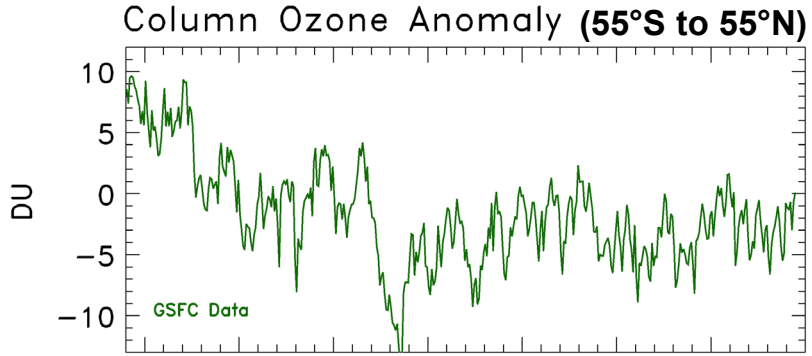
Global ozone anomaly versus time and 4 other, related quantities



Stratospheric Halogen Loading:

http://www.esrl.noaa.gov/csd/assessments/ozone/2014/twentyquestions/images/Q16-1.jpg

Global ozone anomaly versus time and 4 other, related quantities



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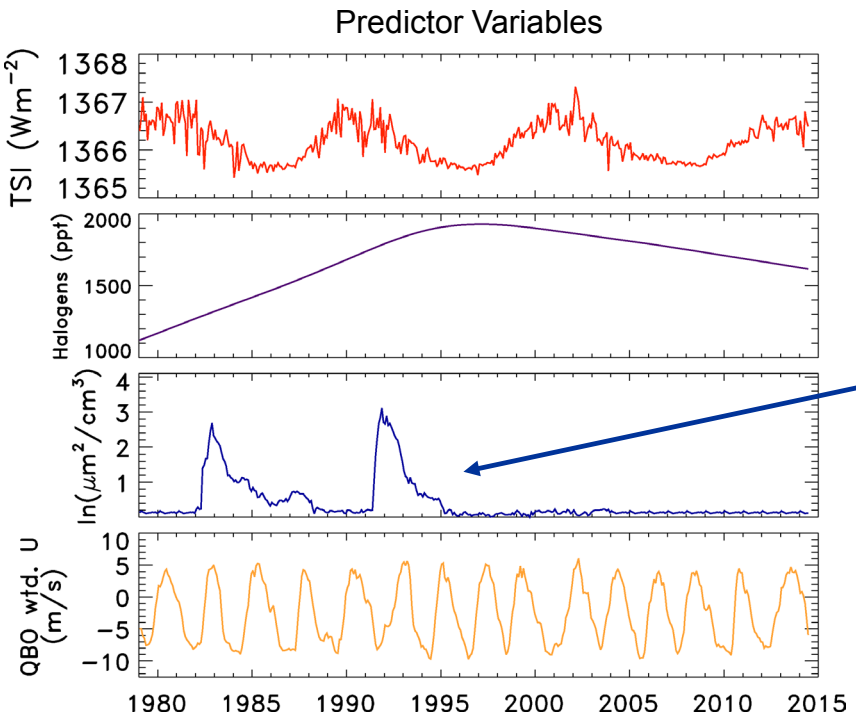
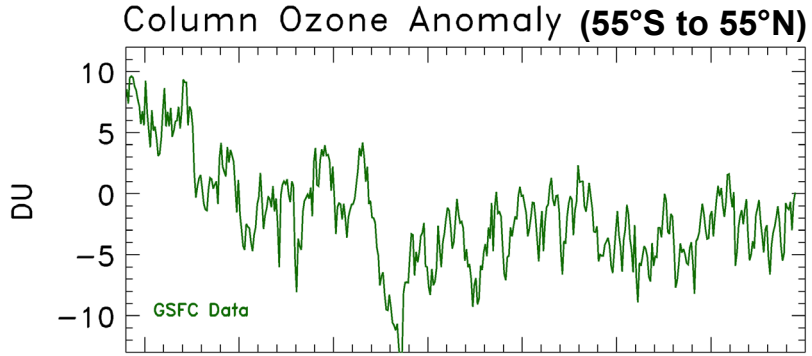


Volcanic Aerosol

Global ozone anomaly versus time and 4 other, related quantities



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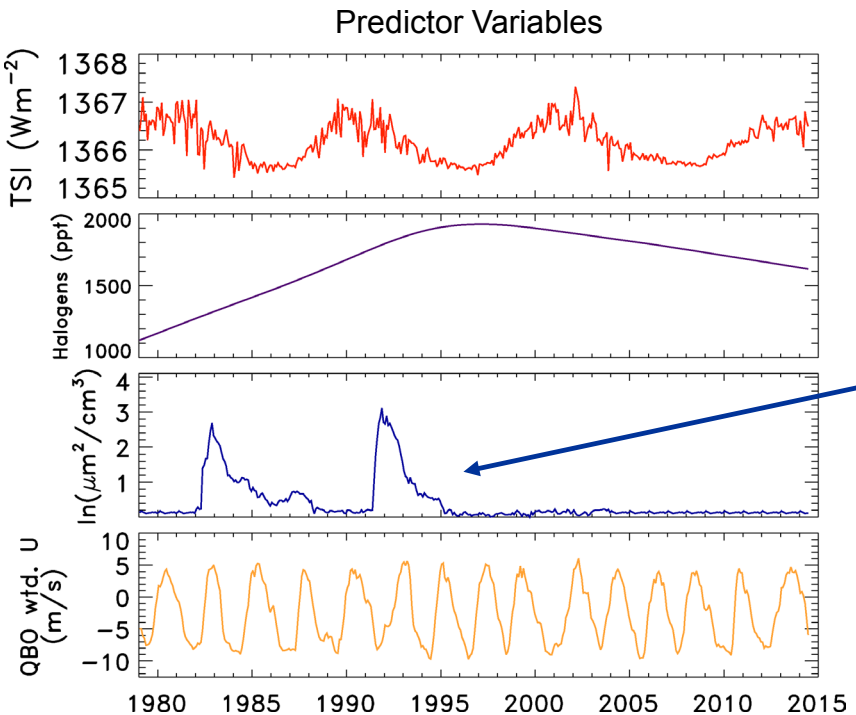


Volcanic Aerosol

Global ozone anomaly versus time and 4 other, related quantities

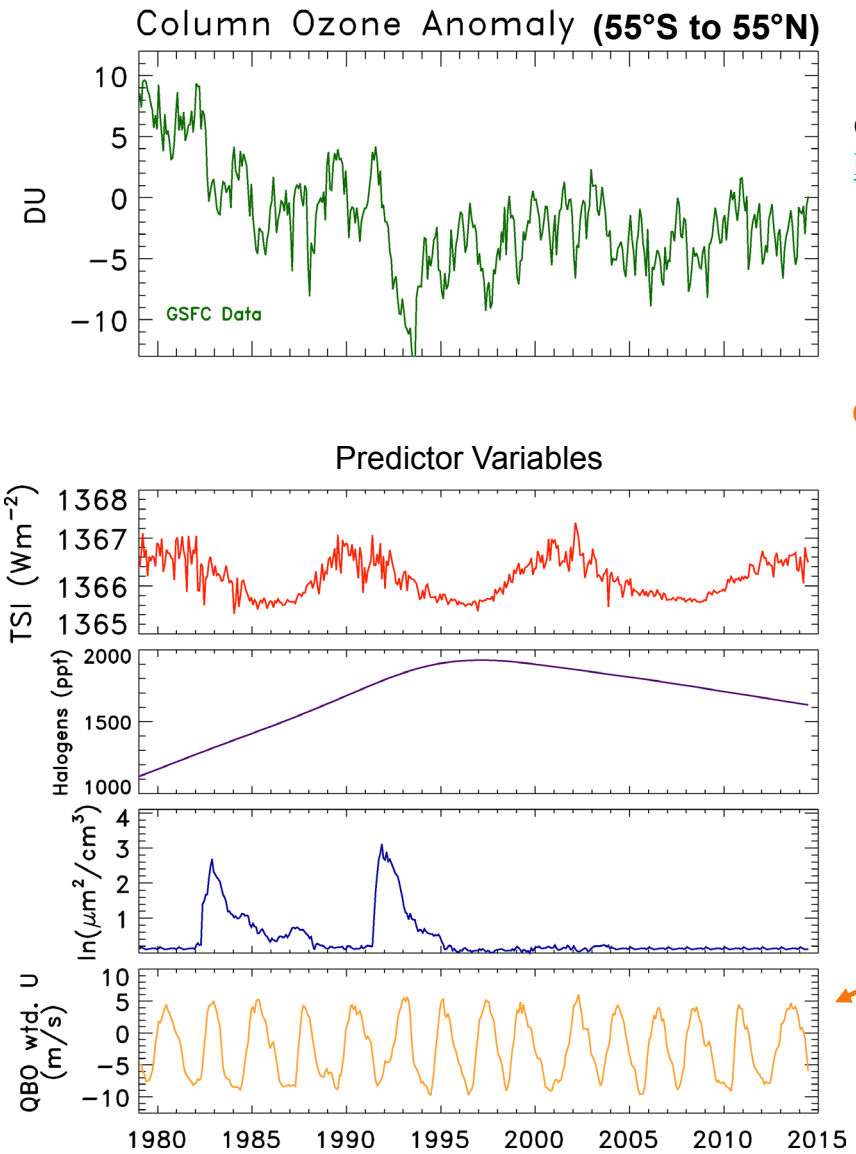
El Chichon, March 1982

https://sites.google.com/site/hesbearcat/fuego.jpg

Mt Pinatubo, June 1991

http://images.publicradio.org/content/2008/01/30/20080130_mount_pinatubo_23.jpg

Global ozone anomaly versus time and 4 other, related quantities



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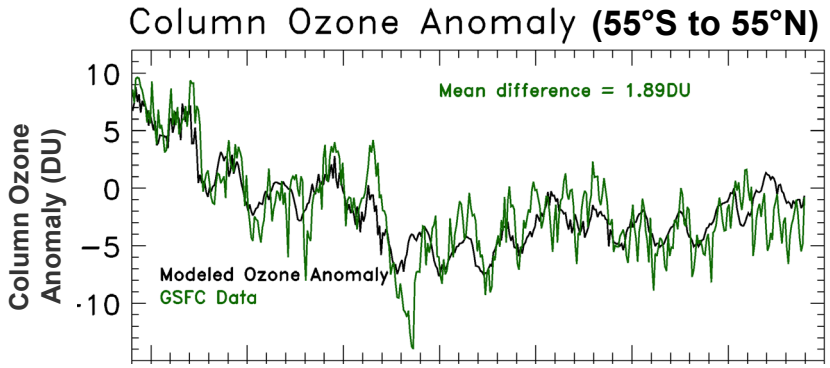
QBO : Quasi-Biennial Oscillation of direction of zonal wind in the tropical lower stratosphere

The zonal wind varies from prevailing easterlies to prevailing westerlies every ~2 yrs

The direction of the wind affects propagation of waves and transport of ozone from source region (tropics) to accumulation region (mid-lats)

Many hundreds of papers have examined theory of QBO, effect of QBO on O_3, and generation of QBO in models

Multiple Linear Regression of Global Total Column Ozone Anomaly



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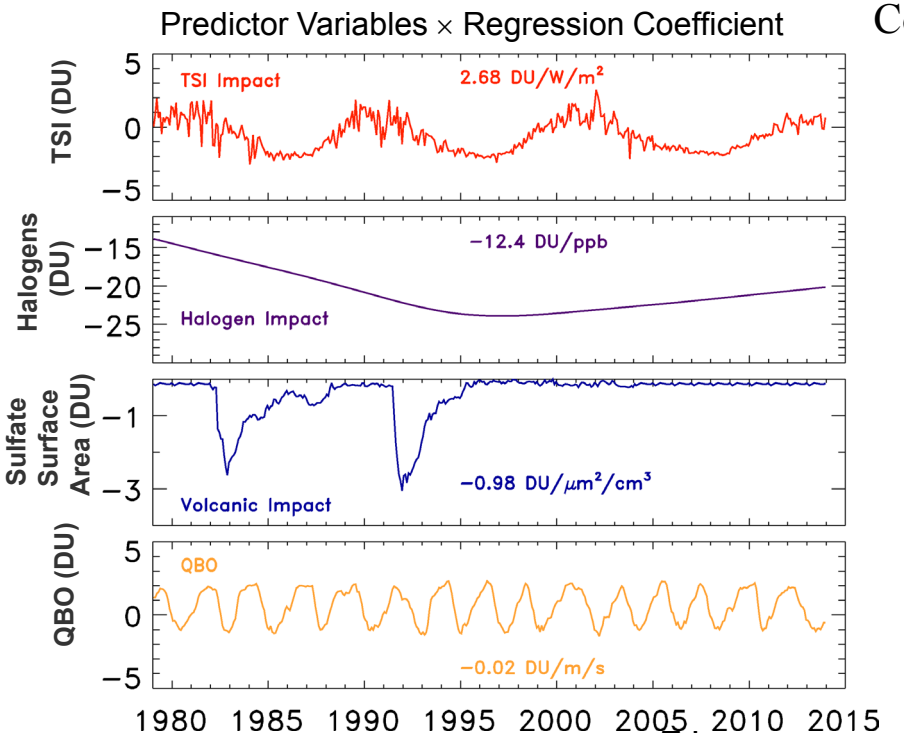

Column Ozone Anomaly (DU) =

19.5	DU +	
2.68	DU / W m^{-2}	× TSI +
−12.4	DU / ppb	× Halogens +
−0.98	DU	× ln (SSA) +
−0.20	DU / m s^{-1}	× QBO

where
TSI = total solar irradiance
Halogens=stratospheric chlorine & bromine loading
SSA = Sulfate Surface Area
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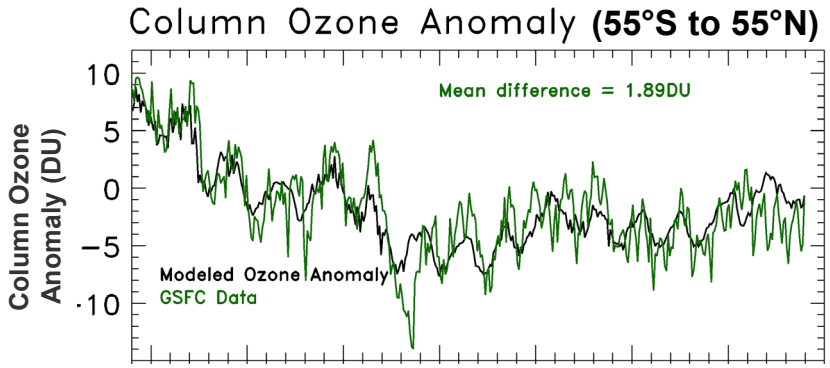
Multiple Linear Regression of Global Total Column Ozone Anomaly



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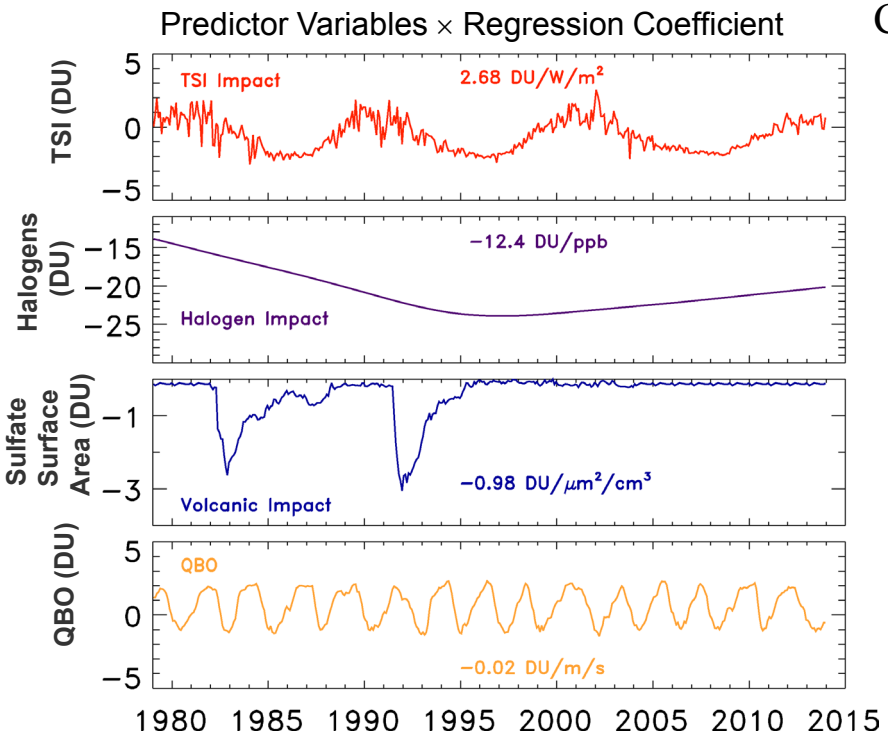

Regression Coefficients

Predictor Variables

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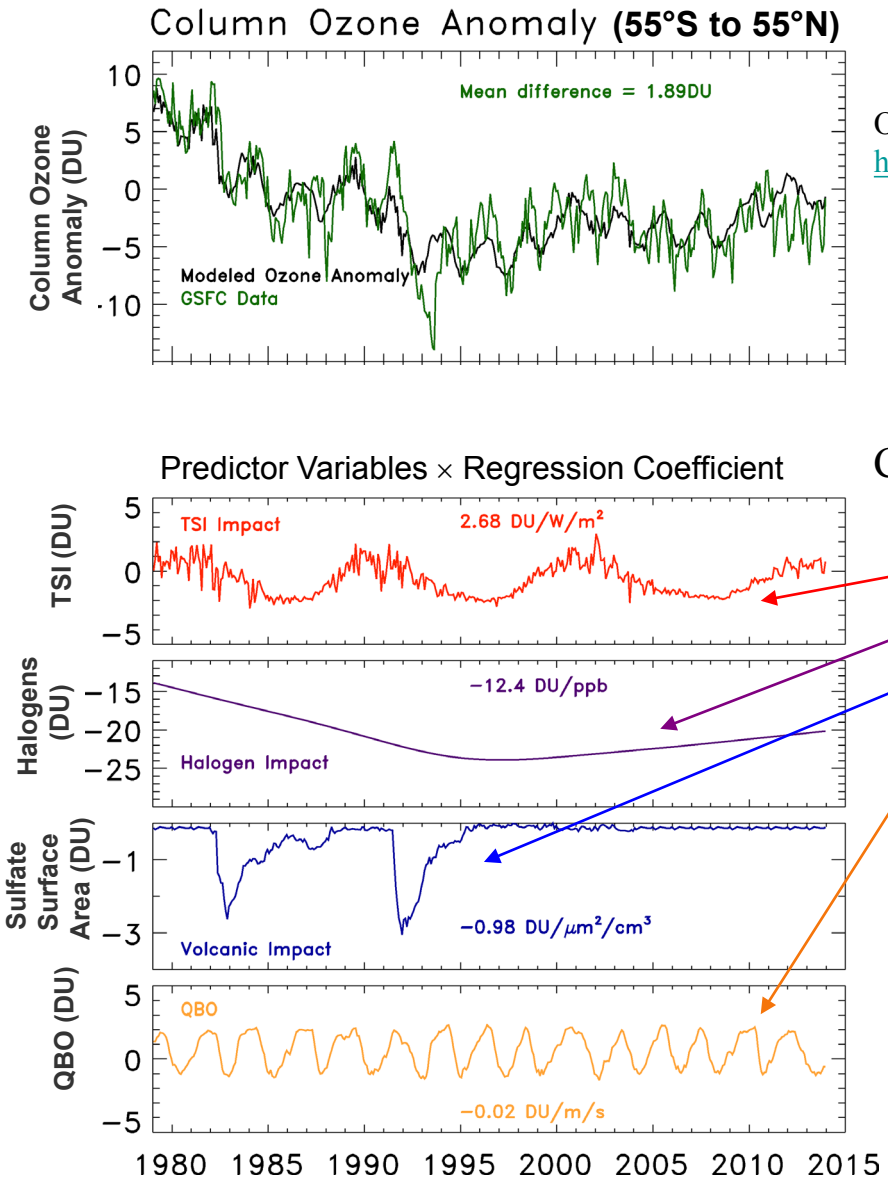
Multiple Linear Regression of Global Total Column Ozone Anomaly



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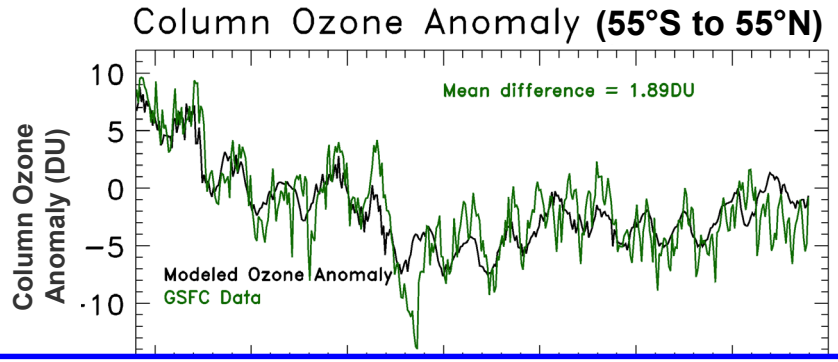

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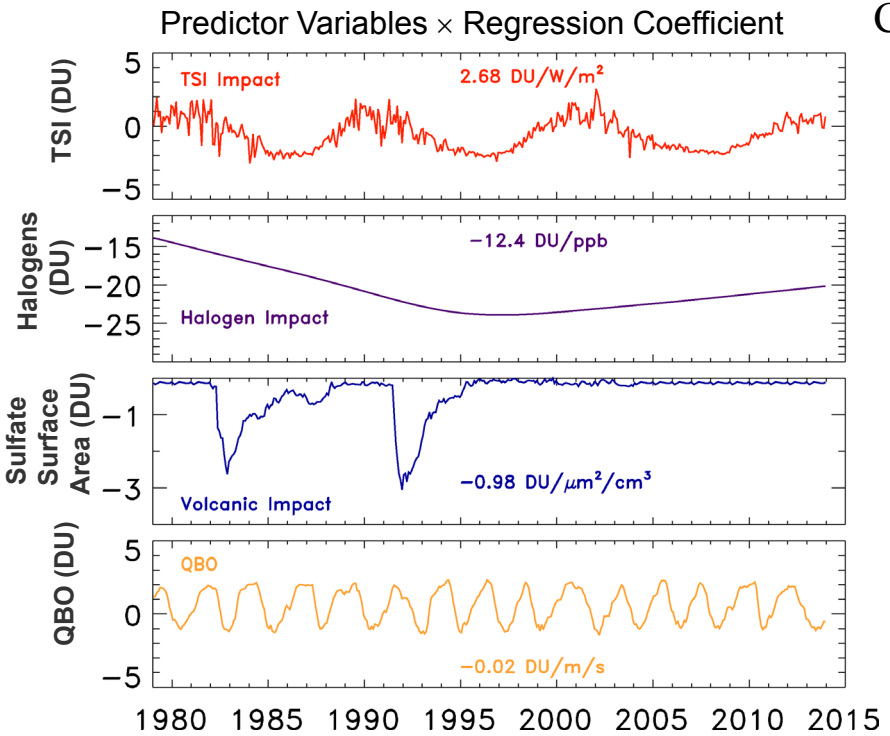
What other quantity should we examine ?



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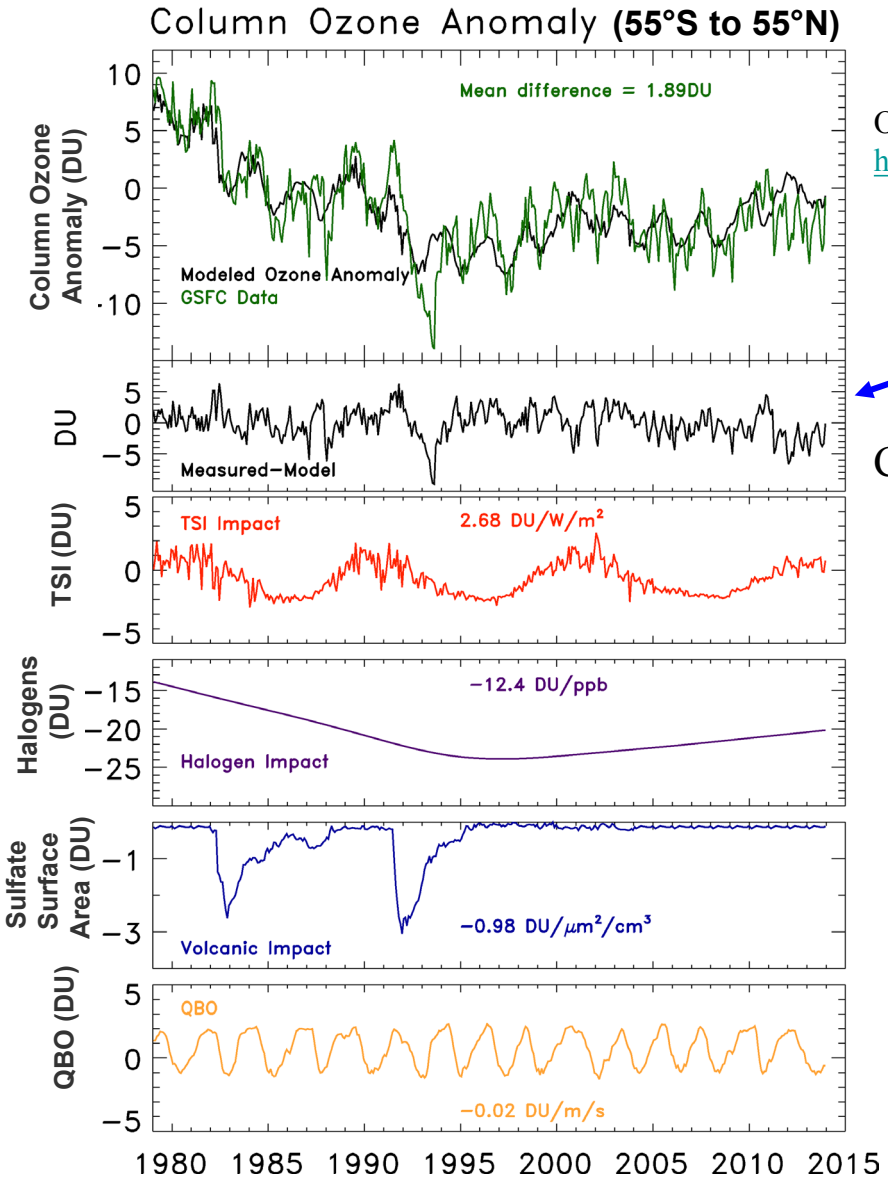
Global ozone anomaly versus time and 4 other, related quantites



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The residual !

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Multiple Linear Regression

$$\overline{\overline{A}} \times \vec{c} = \vec{b} \qquad \text{or} \qquad \vec{c} = \overline{\overline{A}}^{-1} \times \vec{b}$$

What are some of the mathematical concerns one must address when conducting regression analysis ?

What is the difference between multiple linear regression and multivariate regression?

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Multiple Linear Regression

$$\overline{\overline{A}} \times \vec{c} = \vec{b} \qquad \text{or} \qquad \vec{c} = \overline{\overline{A}}^{-1} \times \vec{b}$$

What is the difference between multiple linear regression and multivariate regression?

"Multiple linear regression" is a model for one response variable ("y") and one or more predictor variables ("X").

"Multivariate linear regression" broadens that to more than one response variable ("Y"). The idea is that the response variables may be correlated for each "observation"

For example: a set of stocks, all reacting simultaneously to market factors.

MATLAB: Multiple Linear Regression ⇒ REGRESS
Multivariate Linear Regression ⇒ MVREGRESS
See http://www.mathworks.de/matlabcentral/newsreader/view_thread/154512

IDL: Multiple Linear Regression ⇒ REGRESS
Multivariate Linear Regression ⇒ No native version; many user defined versions available on line

Multiple Linear Regression

$$\overline{\overline{A}} \times \vec{c} = \vec{b} \qquad \text{or} \qquad \vec{c} = \overline{\overline{A}}^{-1} \times \vec{b}$$

Can we perform a regression using non-linear, multiple functions ?

Sure! The math is a bit more difficult, but not too bad. See Section 10.9 of Ayyub and McCuen for a description.