Analysis Methods in Atmospheric and Oceanic Science

AOSC 652

Ordinary Differential Equations

Week 12, Day 2

16 Nov 2016

Numerical Solution to Differential Equations

Suppose: $\frac{dy}{dx} = f(x, y)$ with boundary conditions of y_0 and x_0

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Euler's Method:

$$\Delta y_1 = \Delta x\, f(x_0, y_0)$$

$$y_{\Delta x} = y_0 + \Delta y_1$$

$$\Delta y_2 = \Delta x\, f(x_0 + \Delta x, y_{\Delta x})$$

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etc.

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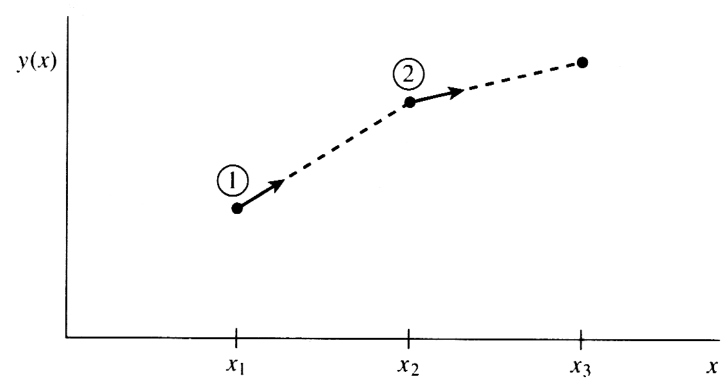

Figure 16.1.1. Euler's method. In this simplest (and least accurate) method for integrating an ODE, the derivative at the starting point of each interval is extrapolated to find the next function value. The method has first-order accuracy.

Press *et al.*

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etc.

The underlying concept for solving differential equations is to rewrite dy/dx as finite steps Δx and Δy, and multiply the eqn by Δx. In the limit of small step sizes, a good approximation to the underlying differential equation can be achieved.

Euler's method, while conceptually important, is *not recommended* for practical use.

Press *et al*., page 702

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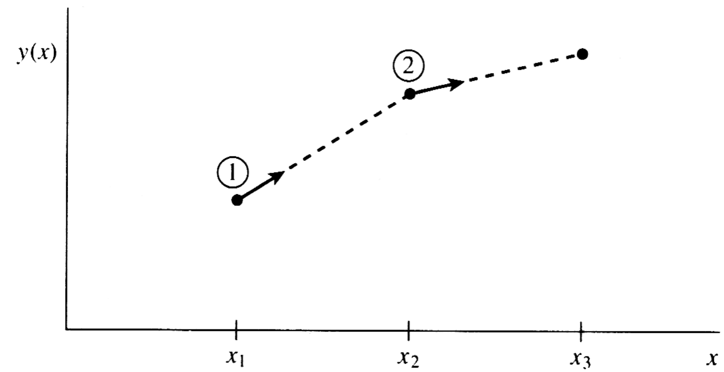

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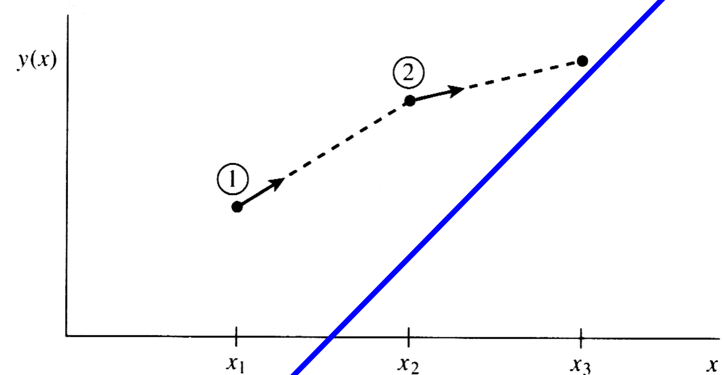

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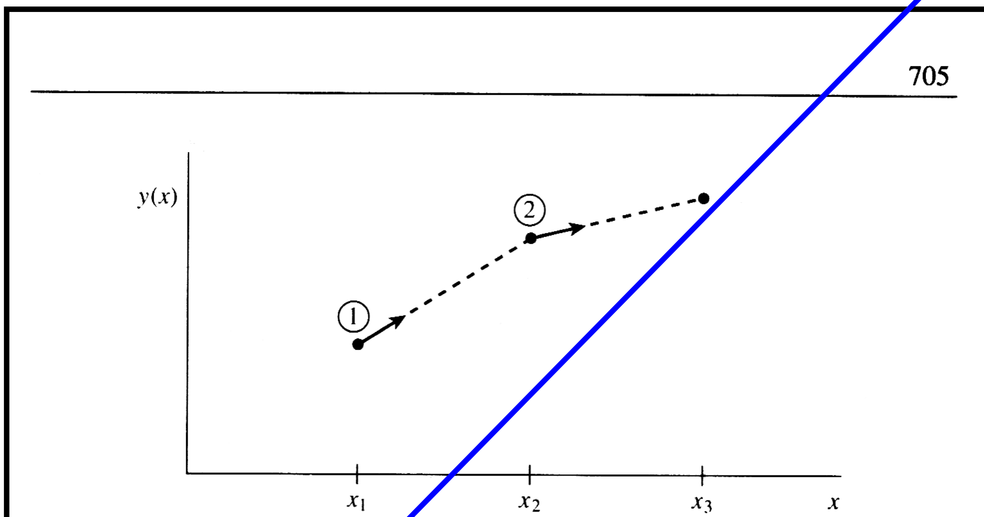
etc.

Euler's method is very simple., very intuitive, and …
THE WORST POSSIBLE METHOD WE COULD USE
to solve and ODE, in part because the error grows
as the solution progresses

Storey, page 9



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etc.

Consider a Taylor Series expansion of $f(x,y)$

$$f(x,y) = f(x_0, y_0) + (x - x_0) f'(x,y) + \frac{(x-x_0)^2}{2} f''(x,y) + \ldots$$

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Can rewrite $\Delta y_{\Delta x}$ as:

$$y_{\Delta x} = y_0 + \Delta x\, f(x_0, y_0) + error$$

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What does *error* equal?

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Suppose: $\frac{dy}{dx} = f(x,y)$ with boundary conditions of y_0 and x_0

Error is "second order":
i.e., varies as Δx^2

Note: Press reading uses
h rather than Δx

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As we shall see, as Δx becomes smaller, solution becomes more accurate.

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However, in many geophysical applications, the model grid size (or timestep) can not be made infinitesimally small

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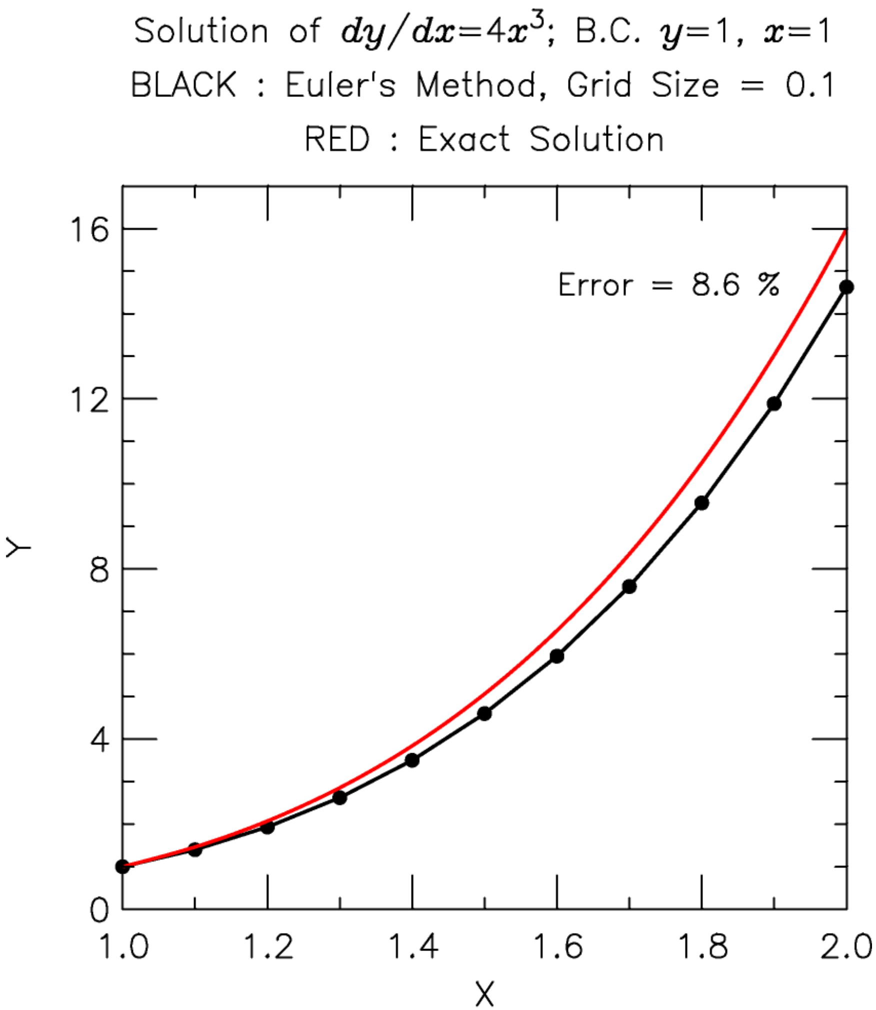
Numerical Solution to Differential Equations

Let's examine: $\frac{dy}{dx} = 4\,x^3$ with boundary condition $y = 1\;,\; x = 1$



$$y_{\Delta x} = y_0 + \Delta x\, f(x_0, y_0) + error$$

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Calculations done using FORTRAN code:
~rjs/aosc652/week_12/simple_eqn.f
Please have a look at code.

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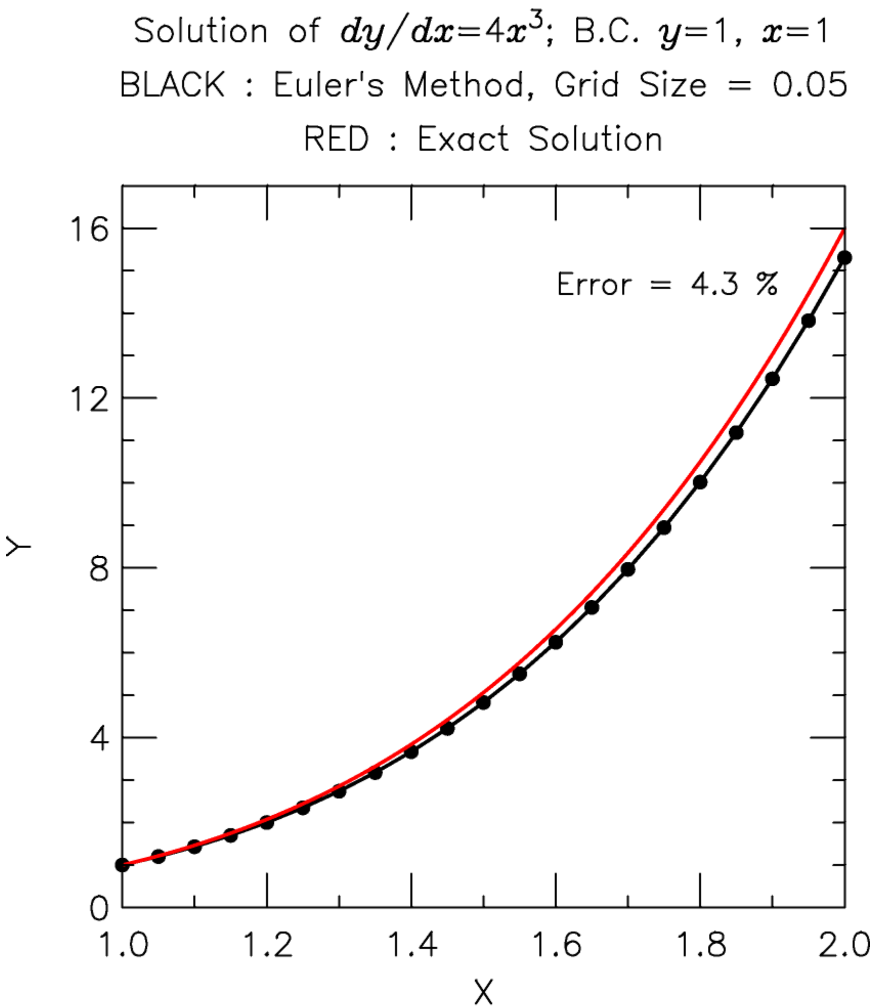

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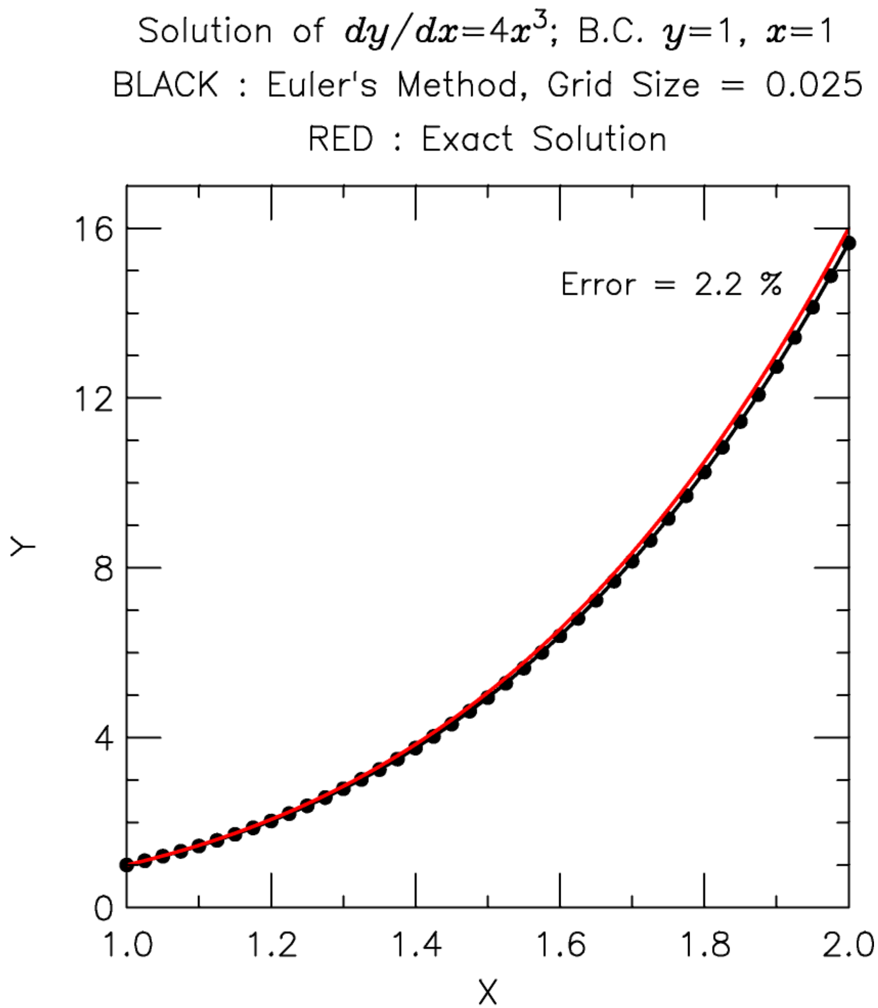

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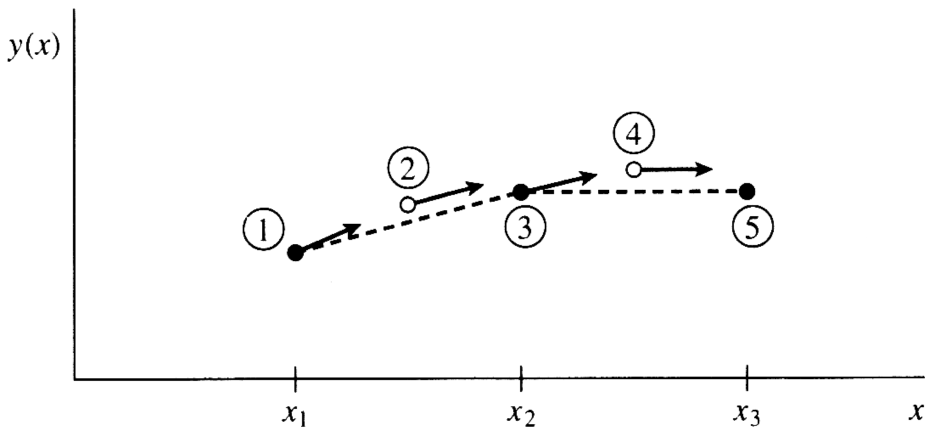
Modified Euler's Method (Storey reading):

- Evaluate slope at start of interval
- Estimate value of dependent variable (*y*) at end of interval using Euler's method
- Evaluate slope at end of interval
- Average two slopes
- Compute revised value of dependent variable at end of interval

Mid-point Method (Press reading):

- Evaluate slope at start of interval
- Rather than stepping to end of interval, take a half step (0.5 ***h*** in Press)
- Evaluate slope at halfway point
- Use slope at halfway point to step forward full step (***h***)



Press *et al.*, page 705

Figure 16.1.2. Midpoint method. Second-order accuracy is obtained by using the initial derivative at each step to find a point halfway across the interval, then using the midpoint derivative across the full width of the interval. In the figure, filled dots represent final function values, while open dots represent function values that are discarded once their derivatives have been calculated and used.

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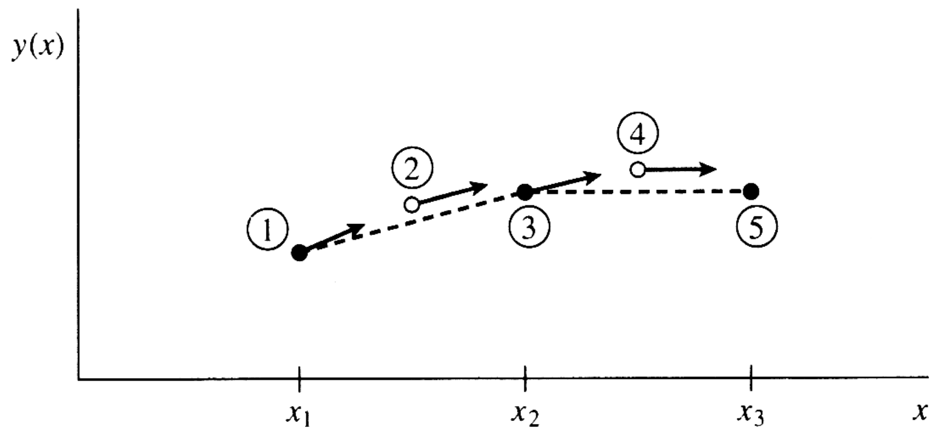

Press *et al.*, page 705

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- Can show using Taylor series expansion that error is order h^3

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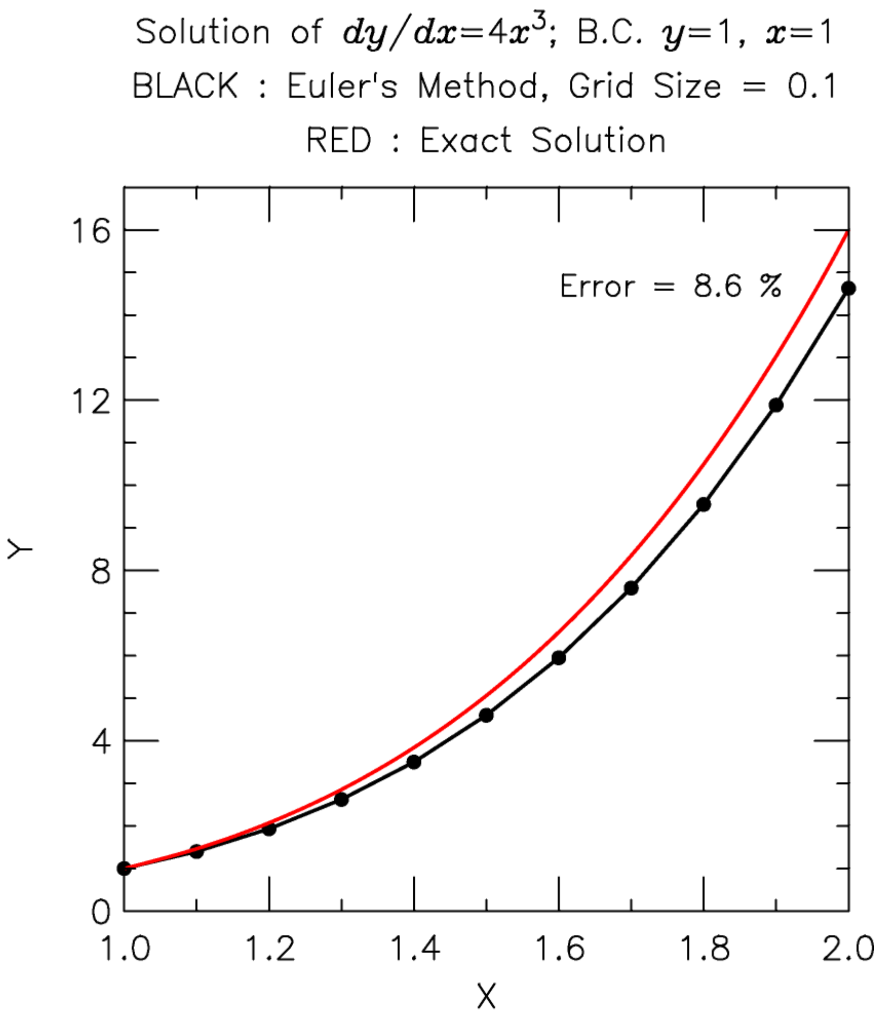

$$y_{\Delta x} = y_0 + \Delta x\, f(x_0, y_0) + error$$

$$error = \Delta x^2\, f'(x_0, y_0)$$

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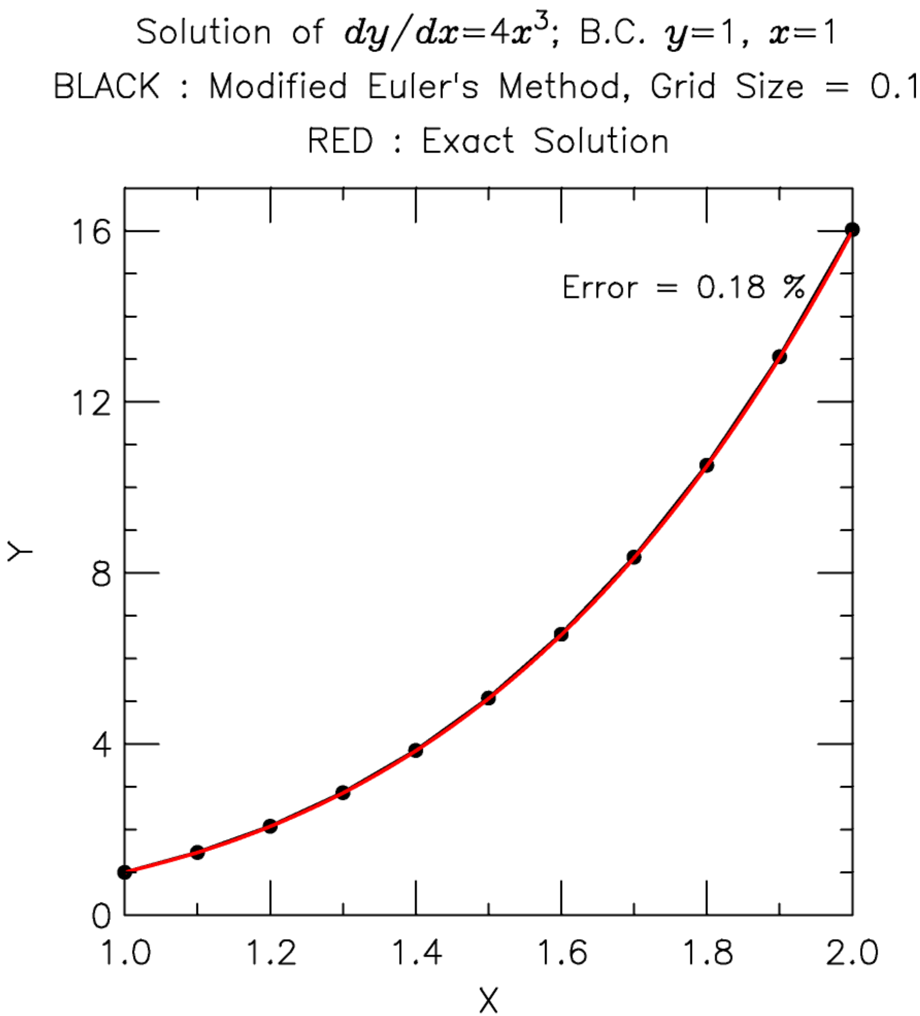

$$y_{\Delta x} = y_0 + \Delta x\; 0.5\left[f(x_0, y_0) + f(x_0 + \Delta x, y_0 + \Gamma)\right] + error,$$

where $\Gamma = \Delta x\, f(x_0, y_0)$

$$error \;\approx\; \frac{\Delta x^3}{2} f''(x_0, y_0)$$

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Runge-Kutta Method:

a) Evaluate slope at start of interval (S_1)

b) Compute value of dependent variable at mid-point of interval ($y_{\Delta x//2}$)

c) Compute slope at mid-point of interval (S_2)

d) Revise value of dependent variable at mid-point using S_2

e) Revise value of slope at mid-point (S_3) using S_2 and $y_{\Delta x/2}$

f) Compute value of dependent variable at end of interval using y_0 and S_3

g) Compute value of slope at end of interval using S_3 and value of y at end of interval from step f

h) Compute value of dependent variable at end of interval ($y_{\Delta x}$):

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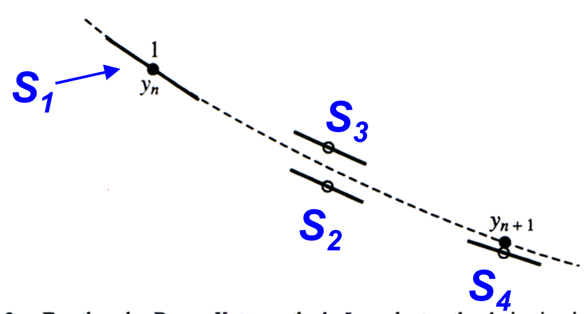

Figure 16.1.3. Fourth-order Runge-Kutta method. In each step the derivative is evaluated four times: once at the initial point, twice at trial midpoints, and once at a trial endpoint. From these derivatives the final function value (shown as a filled dot) is calculated. (See text for details.)

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e) Revise value of slope at mid-point (S_3) using S_2 and $y_{\Delta x/2}$

f) Compute value of dependent variable at end of interval using y and S_3

g) Compute value of slope at end of interval using S_3 and value of y at end of interval from step f

h) Compute value of dependent variable at end of interval ($y_{\Delta x}$):

$$y_{\Delta x} = y_0 + \frac{\Delta x}{6} f(S_1 + 2S_2 + 2S_3 + S_4) + error$$

$$error \approx \frac{\Delta x^5}{4!} f''''(x_0, y_0)$$

Runge-Kutta Method:

Please see pages 704 to 708 of Press for erudite description of benefits and possible pitfalls of the Runge-Kutta method, for two Runge-Kutta subroutines, and for the following succinct description of the method:

By far the most often used is the classical *fourth-order Runge-Kutta formula*, which has a certain sleekness of organization about it:

$$k_1 = hf(x_n, y_n) \leftarrow \textit{this f is } S_1$$

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \leftarrow \textit{this f is } S_2$$

$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \leftarrow \textit{this f is } S_3$$

$$k_4 = hf(x_n + h, y_n + k_3) \leftarrow \textit{this f is } S_4$$

$$y_{n+1} = y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5) \qquad (16.1.3)$$

$$error \approx \frac{\Delta x^5}{4!} f''''(x_0, y_0)$$

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Runge-Kutta Method:

In fact you can get pretty far on this old workhorse … RK is for ploughing the fields. Even the old workhorse is more nimble with new horseshoes (adaptive stepsize) Press *et al.*, page 706

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Runge-Kutta Method:

The other methods (Bulirsch-Stoer or predictor-corrector) can be very efficient when high accuracy is required … but these methods are the high-strung racehorses of ODE solvers. Press *et al*., page 706

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One of the standard workhorses for solving ODEs is called the Runge-Kutta method, which is a higher order approximation to the midpoint method.

Storey, page 19

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$$k_4 = hf(x_n + h, y_n + k_3) \leftarrow \textit{this f is } S_4$$

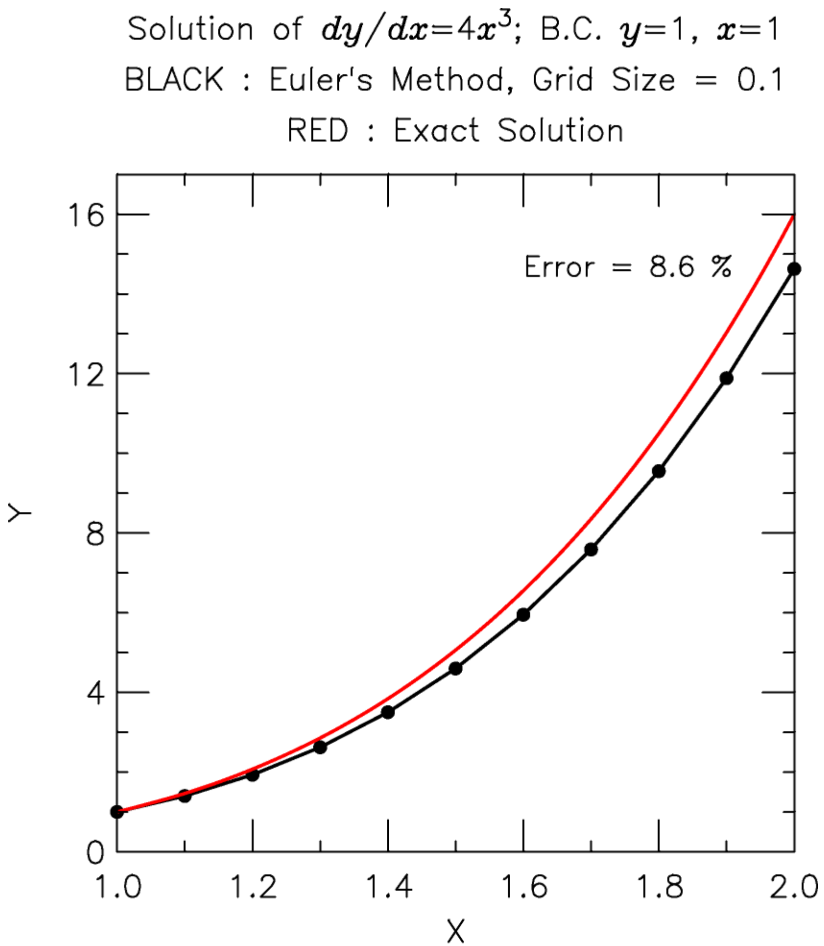
$$y_{n+1} = y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5) \qquad (16.1.3)$$

$$error \approx \frac{\Delta x^5}{4!} f''''(x_0, y_0)$$

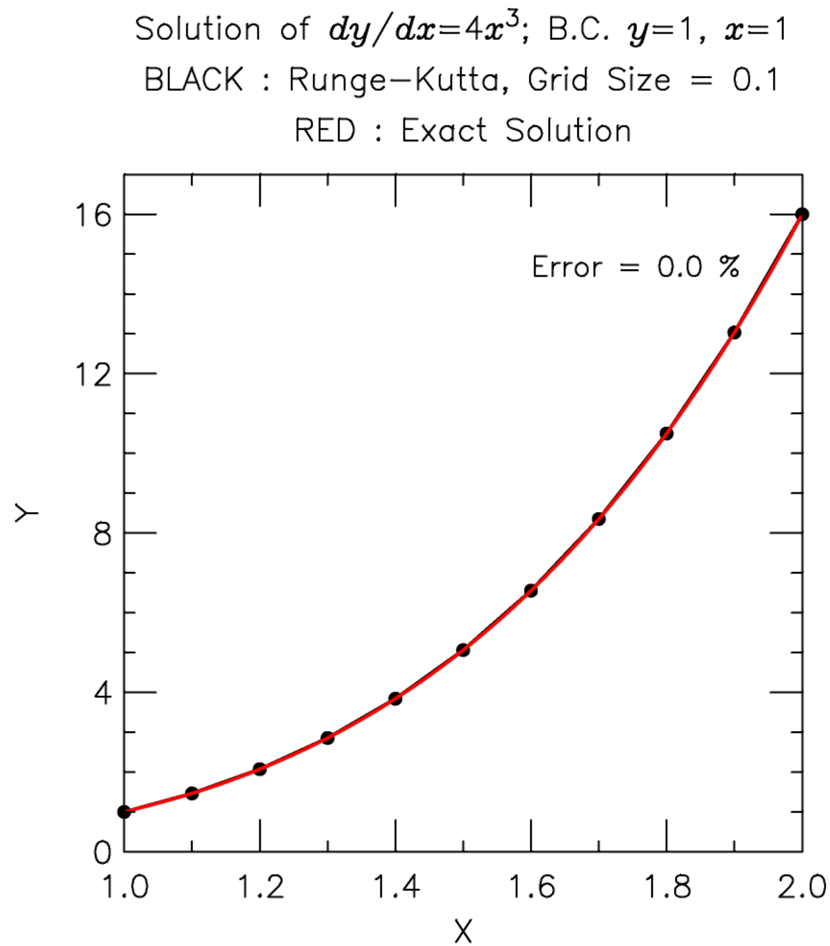
What should happen when we apply the Runge-Kutta method to: $\frac{dy}{dx} = 4x^3$?

Numerical Solution to Differential Equations

Euler's Method



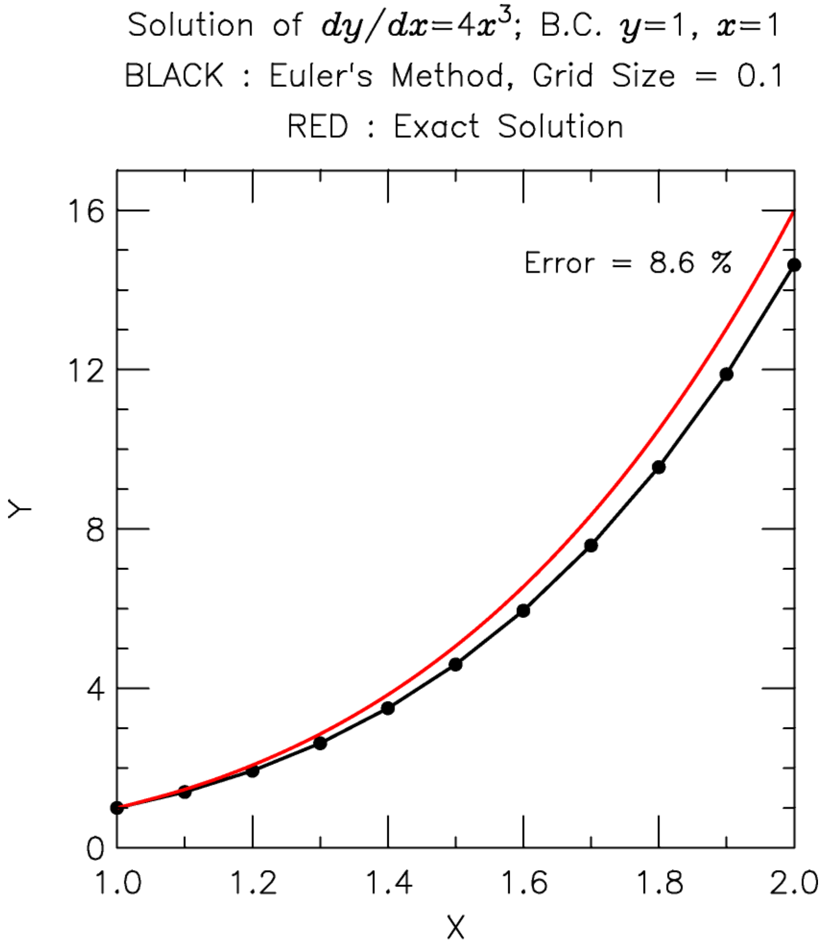
Runge-Kutta Method



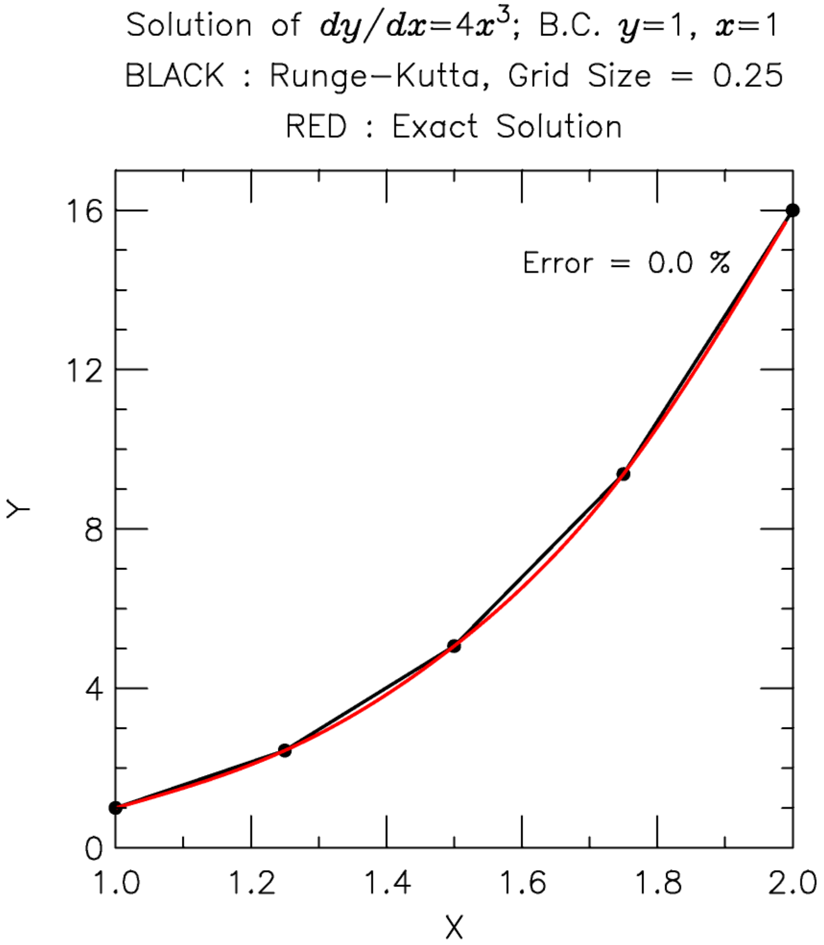
AOSC 652: Analysis Methods in AOSC

Numerical Solution to Differential Equations

Euler's Method

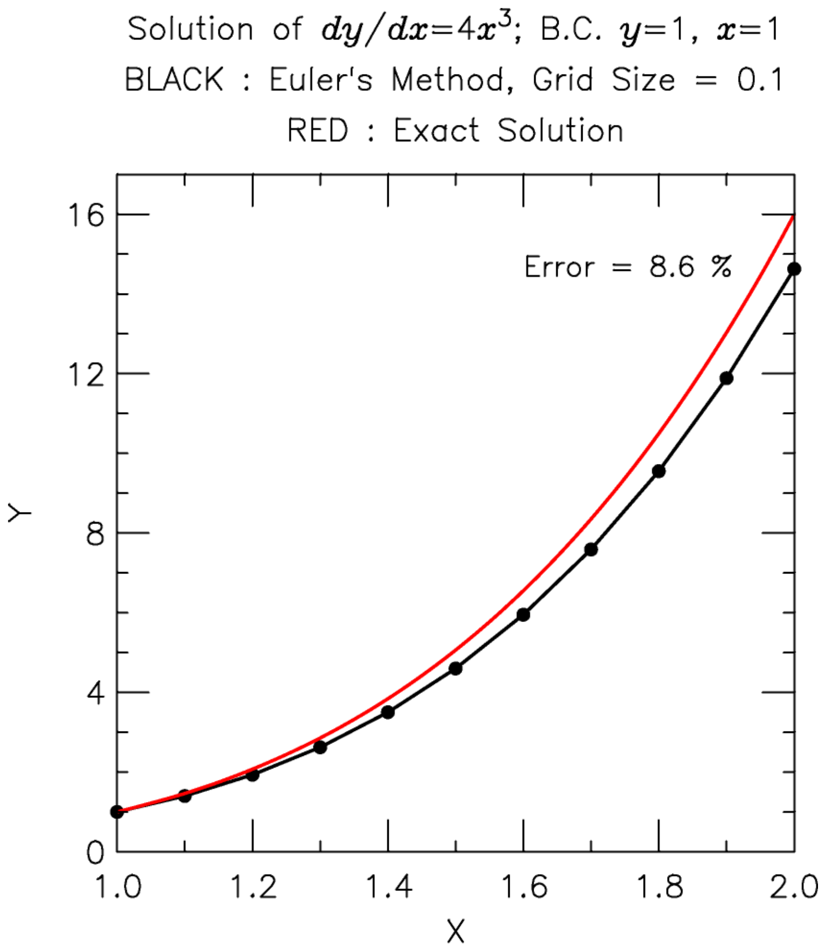


Runge-Kutta Method

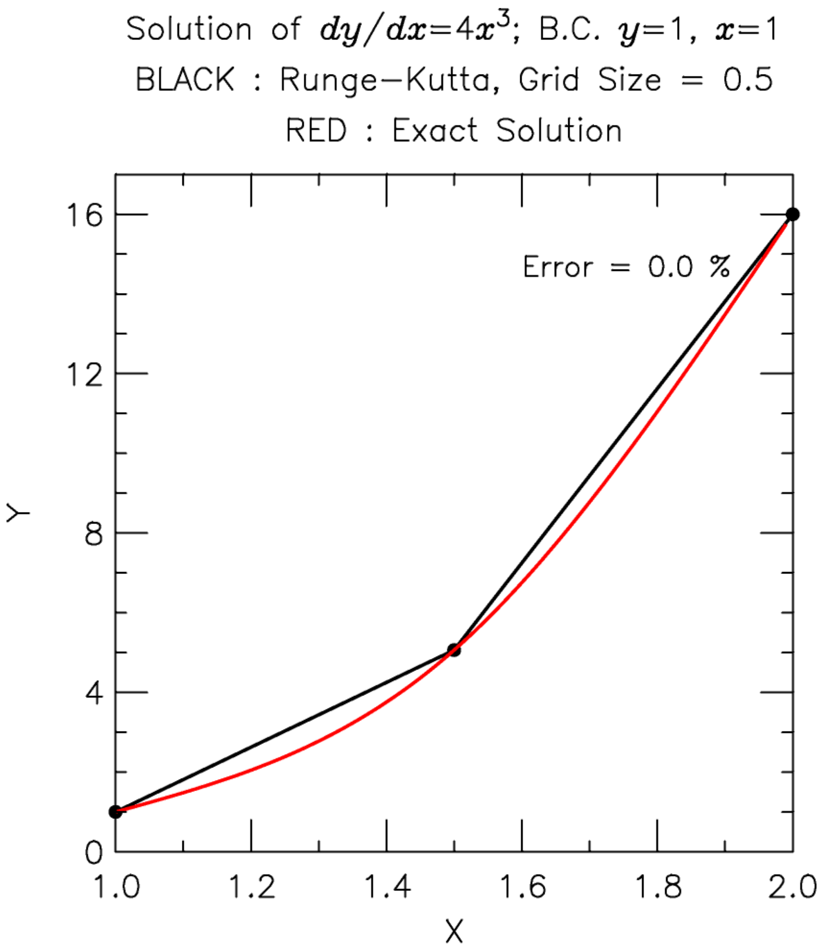


Numerical Solution to Differential Equations

Euler's Method

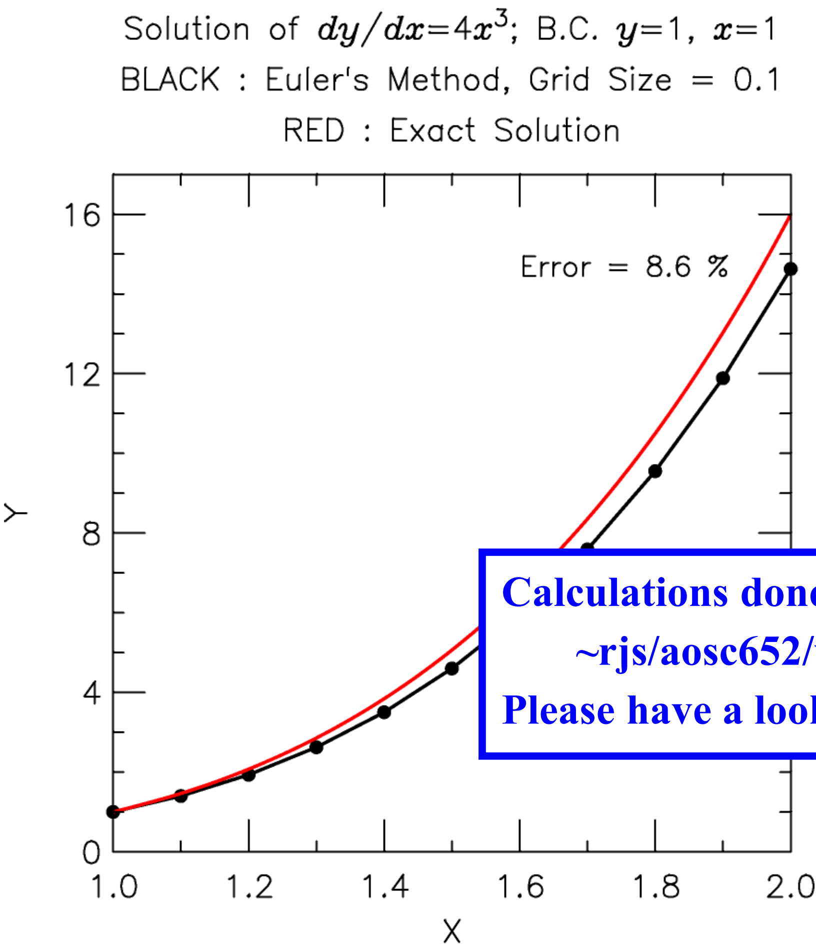


Runge-Kutta Method

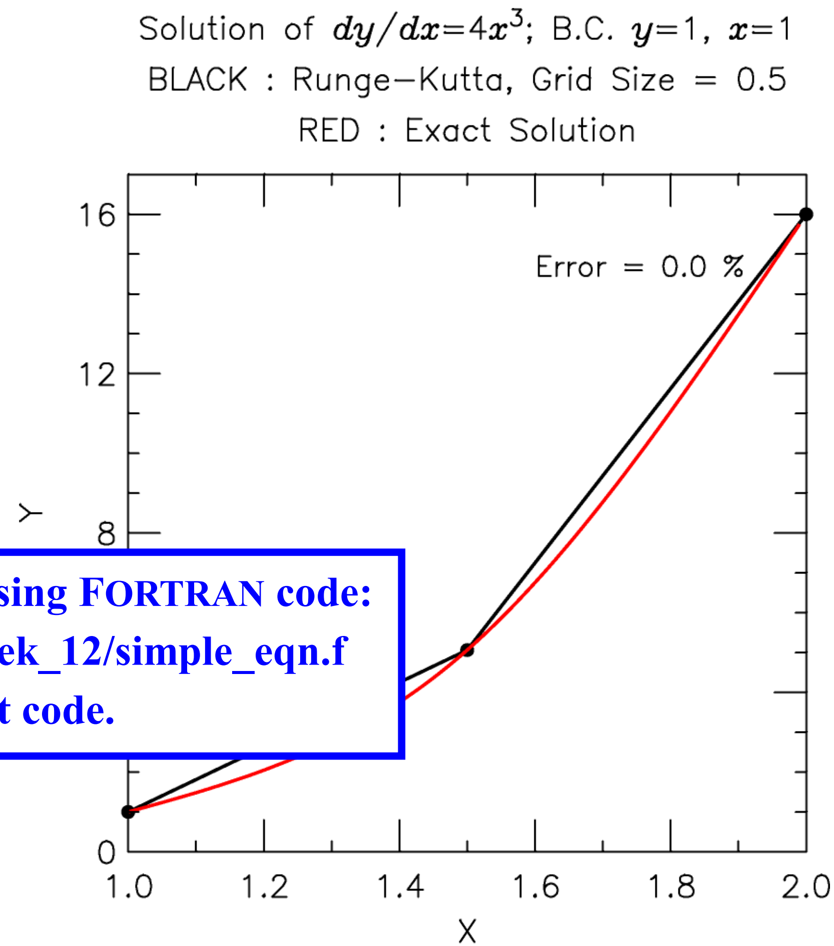


Numerical Solution to Differential Equations

Euler's Method



Runge-Kutta Method



Calculations done using FORTRAN code:

~rjs/aosc652/week_12/simple_eqn.f

Please have a look at code.

Lorenz Equations

The equations Lorenz derived were:

$$\frac{dx}{dt} = 10(y - x) \tag{6.6}$$

$$\frac{dy}{dt} = x(20 - z) - y \tag{6.7}$$

$$\frac{dz}{dt} = xy - \frac{8}{3}z \tag{6.8}$$

We will not discuss the derivation of these equations but they were based on physical arguments relating to atmospheric convection. The variables x, y, z represent physical quantities such as temperatures and flow velocities, while the numbers 10, 20, and 8/3 represent properties of the air.

Lorenz, *Journal of the Atmospheric Sciences*, 1963

AOSC 652: Analysis Methods in AOSC

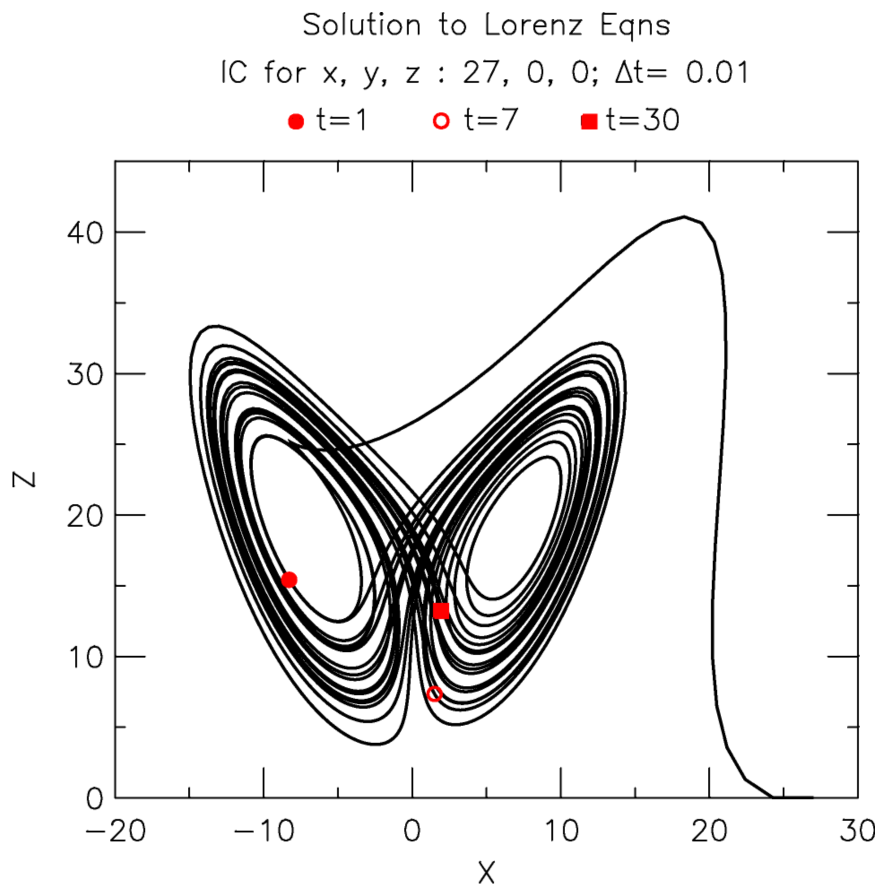
Lorenz Equations

$$\frac{dx}{dt} = 10(y - x)$$

$$\frac{dy}{dt} = x(20 - z) - y$$

$$\frac{dz}{dt} = xy - \frac{8}{3}z$$

Calculations done using FORTRAN code:
~rjs/aosc652/week_12/lorenz_eqns.f
Please have a look at code.



AOSC 652: Analysis Methods in AOSC

Lorenz Equations

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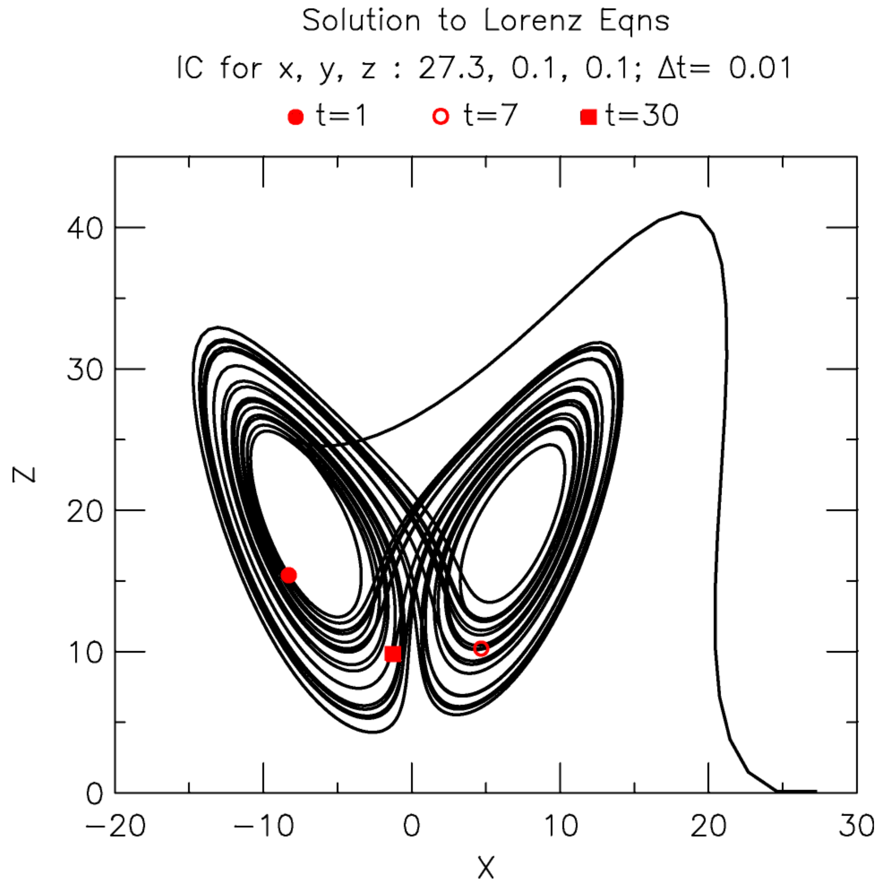

AOSC 652: Analysis Methods in AOSC

Lorenz Equations

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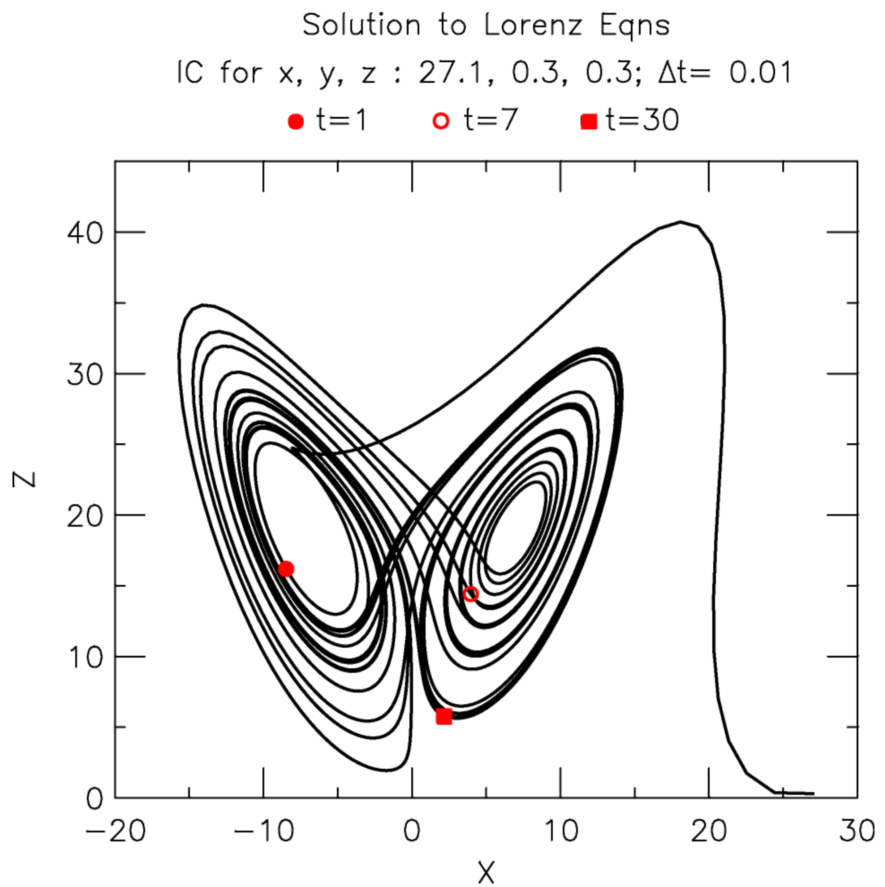

Lorenz Equations

$$\frac{dx}{dt} = 10(y - x)$$

$$\frac{dy}{dt} = x(20 - z) - y$$

$$\frac{dz}{dt} = xy - \frac{8}{3}z$$

If you'd like to see Matlab code that solves these eqns, have a look at
~rjs/aosc652/week_12/lorenz.m