Global Carbon Cycle
AOSC 433/633 & CHEM 433

Ross Salawitch

Class Web Site: **http://www.atmos.umd.edu/~rjs/class/spr2017**

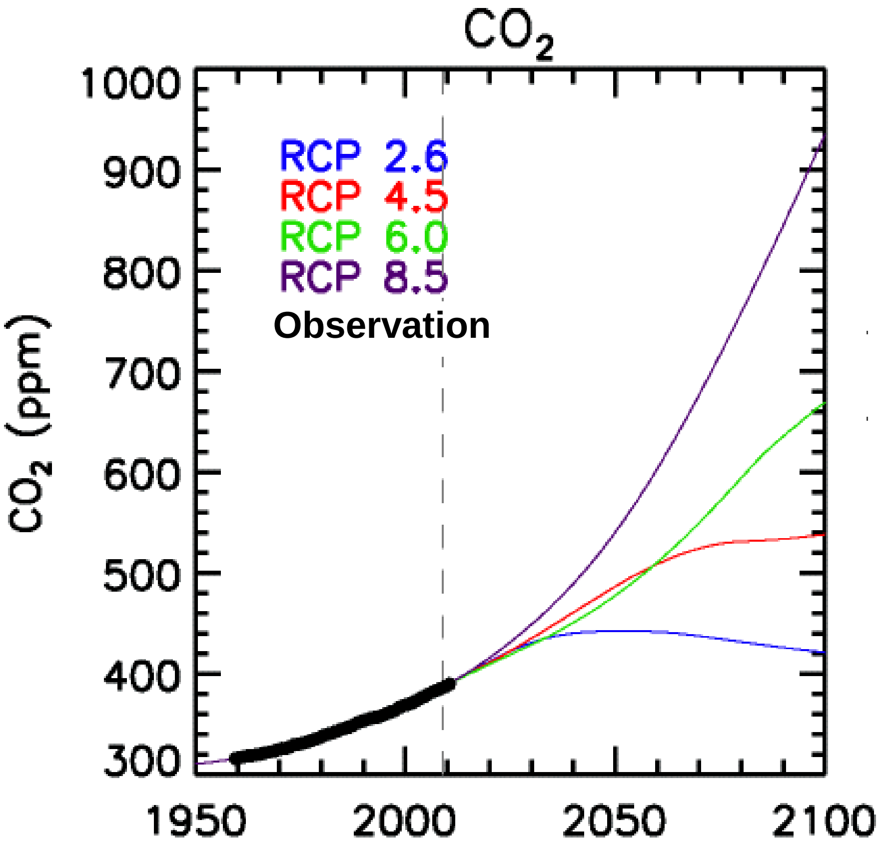
Goals for today:

- **Overview of the Global Carbon Cycle "scratching below the surface"**
- **Ocean and land uptake of CO_2**
- **Connect to prior lecture (glacial CO_2 draw down), policy, and long-term climate change**

Lecture 5

9 February 2017

Motivation 1



- RCP: Representative Concentration Pathway
  Number represents RF of climate, units W m^{-2},
  at the end of this century

- GHG mixing ratio time series for CO_2, CH_4, N_2O, as well as CFCs, HCFCs, and HFCs provided to climate model groups

- What is the utility of "command central" providing GHG scenarios to the climate model groups?

- How do you think these various scenarios are devised?

CO_2 is long lived: society must reduce emissions soon or we will be committed to dramatic, future increases

Curve that levels off at ~560 ppm has emissions peaking ~2030
<u>Less than 20 years from now !</u>

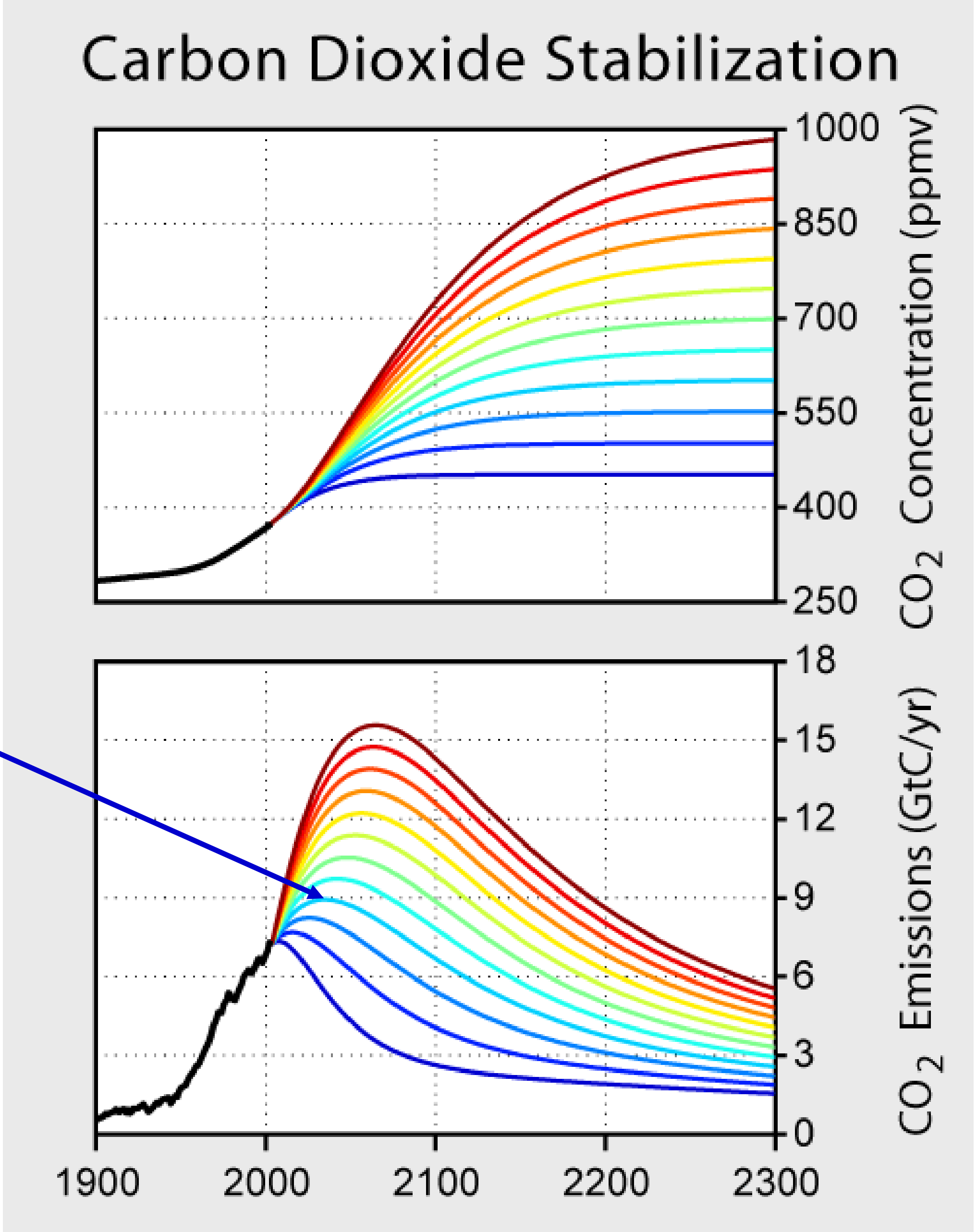
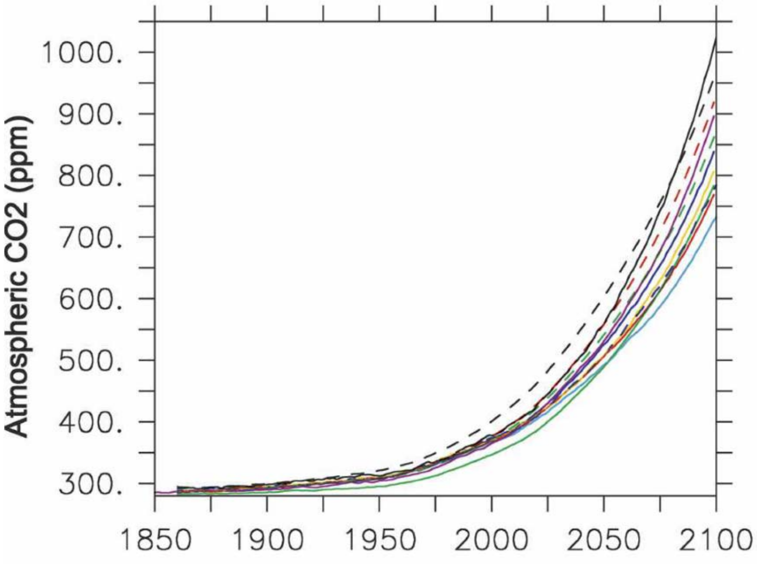
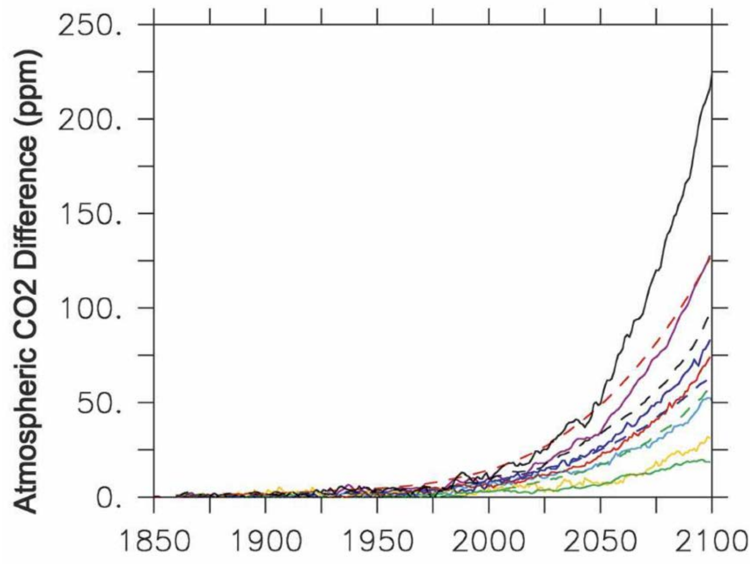


Image: “Global Warming Art” : http://archive.is/JT5rO

Motivation 2





- Prior slide examined atmosphericCO_2 from a single model of the global carbon cycle

- Friedlingstein et al. (2006) compared CO_2 from **11** different coupled climate-carbon cycle models, each constrained by the *same* specified time series of anthropogenic CO_2 emission and found:

    1) future climate change will reduce the efficiency of the *Earth system* to absorb the anthropogenic carbon perturbation

    2) difference in CO_2 between a simulation using an interactive carbon-cycle and another run with a non-interactive carbon-cycle varies from 20 to 200 ppm among these **11** models (yikes!)

Modern CO_2 Record

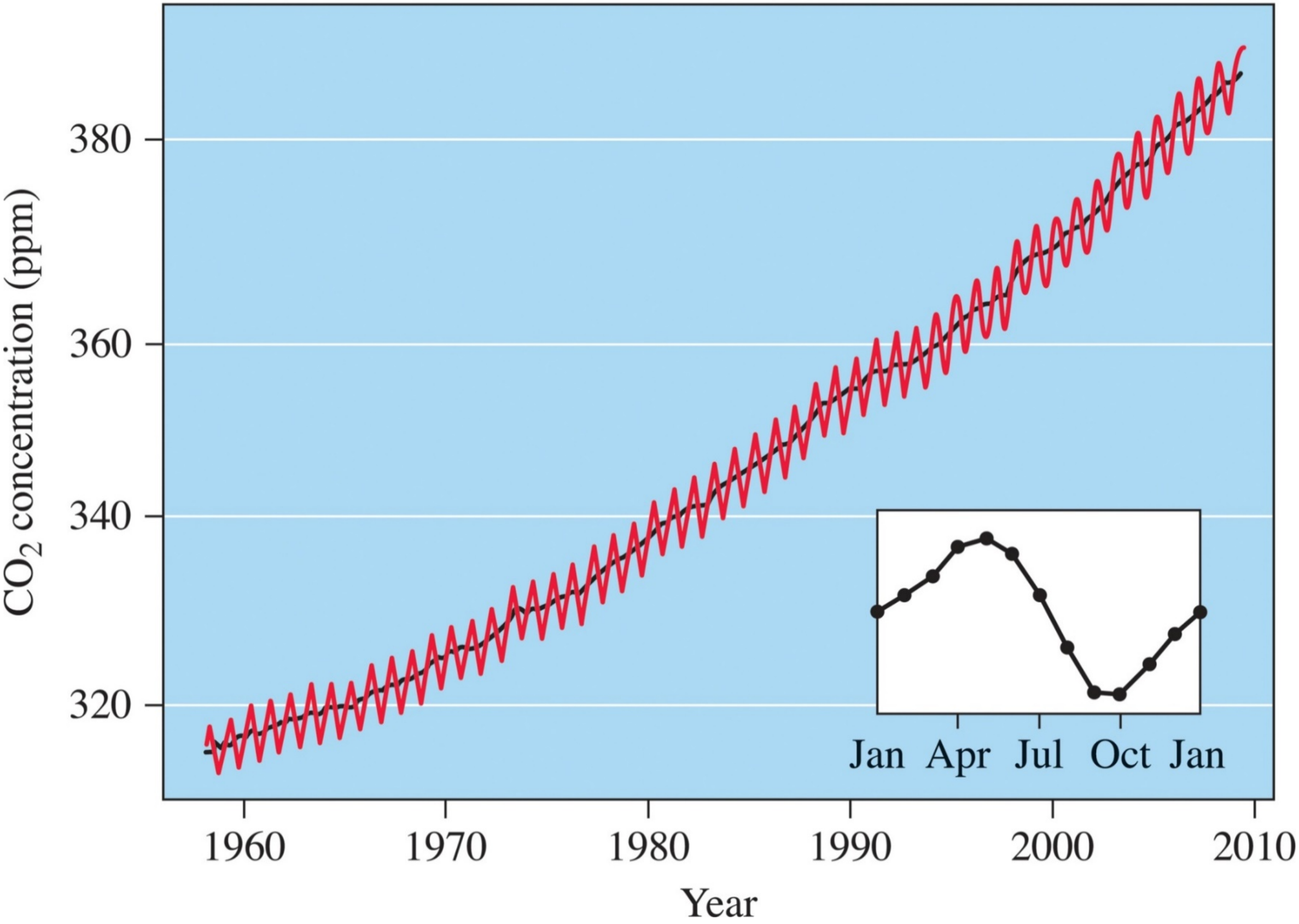
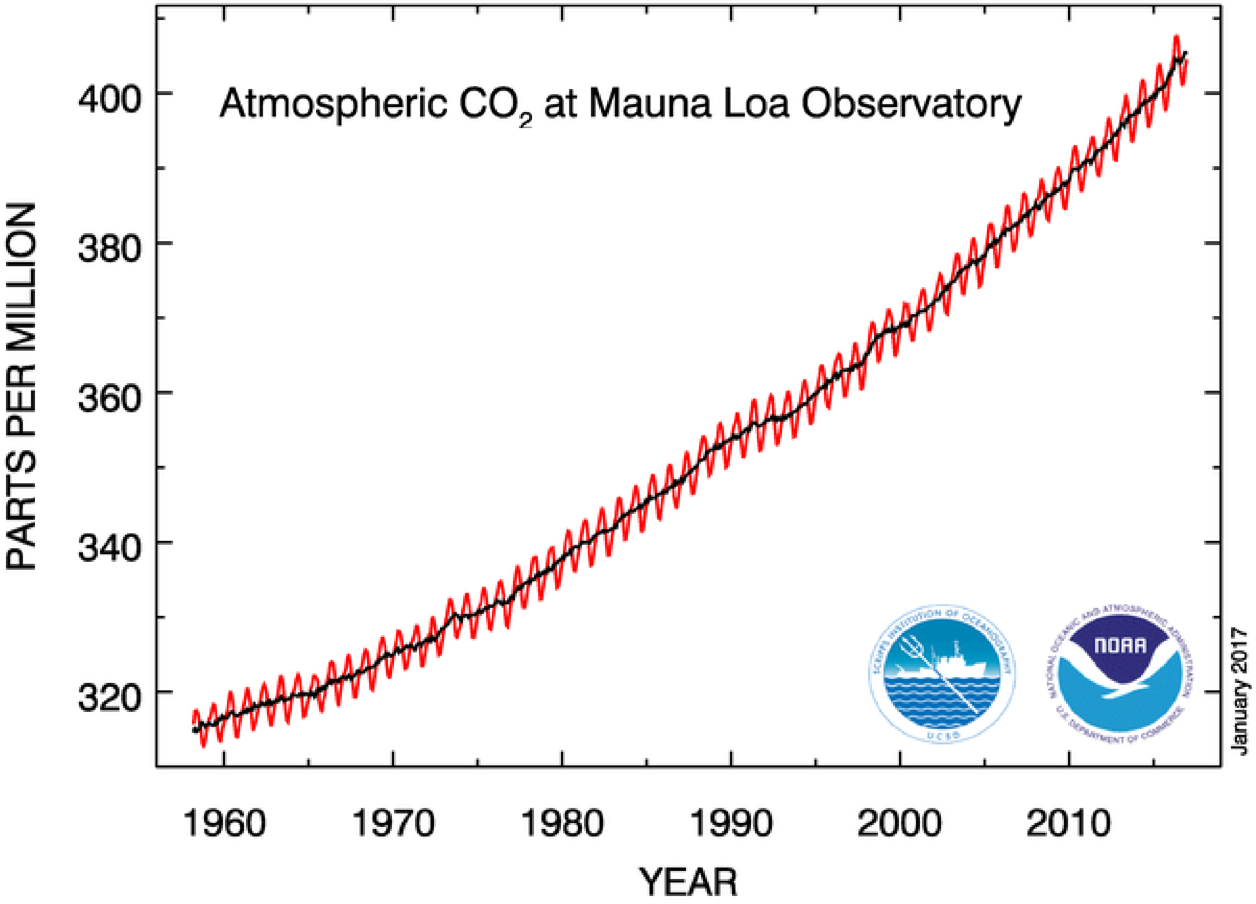


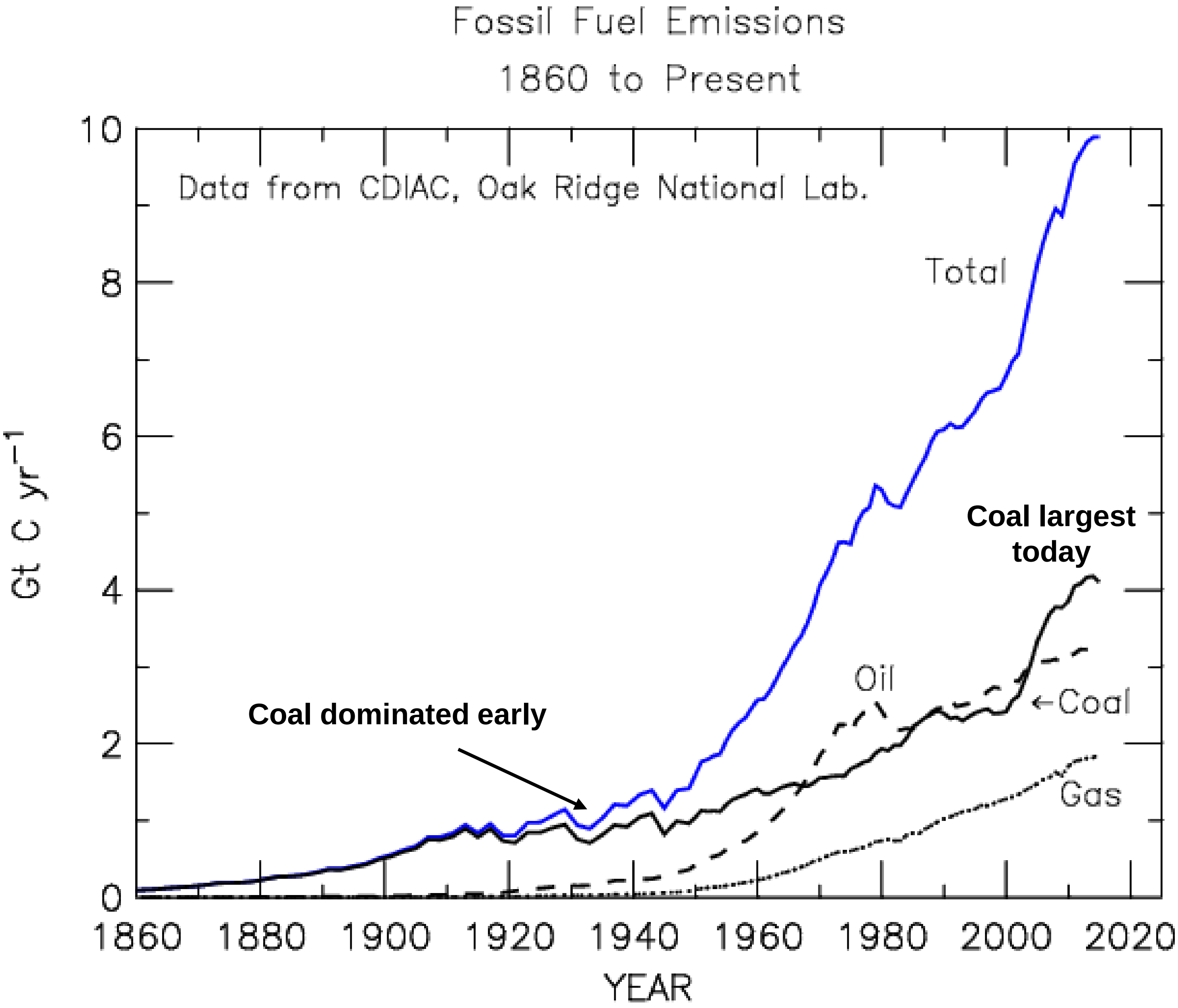
Figure 3.3, Chemistry in Context

Modern CO_2 Record

CO_2 at MLO on 7 Feb 2017: 406.7 parts per million (ppm) and rising !



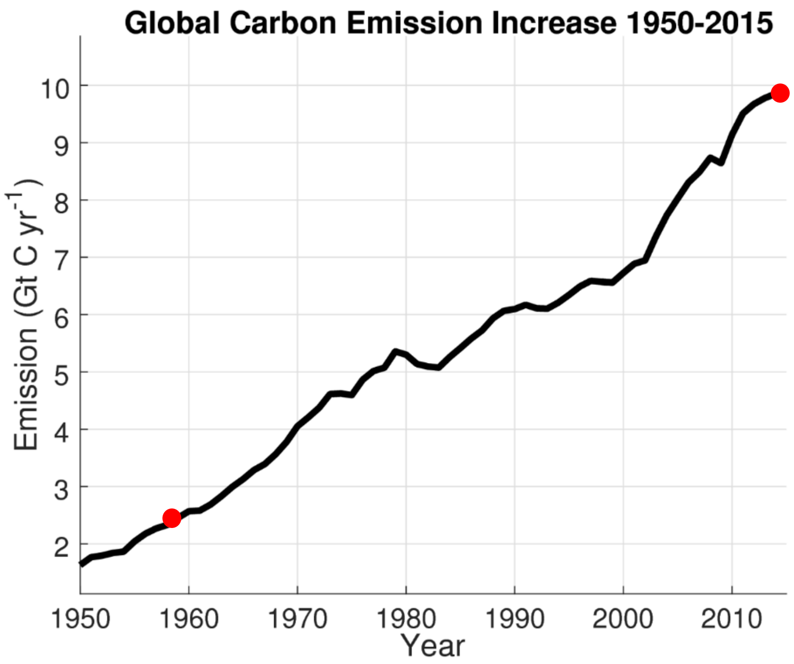
Legacy of Charles Keeling, Scripps Institution of Oceanography, La Jolla, CA
https://www.esrl.noaa.gov/gmd/ccgg/trends/full.html
See also https://www.co2.earth/daily-co2

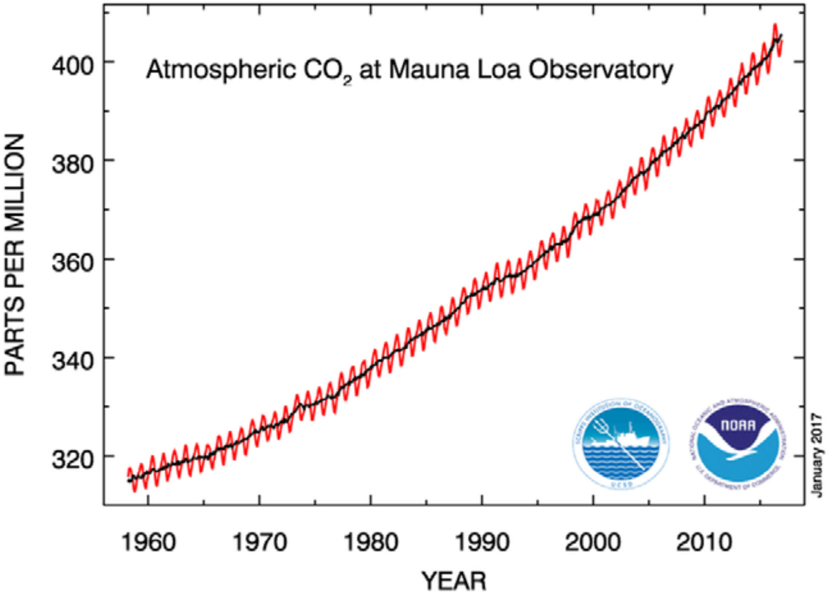


Coal largest today

Coal dominated early

Fossil Fuel Emissions



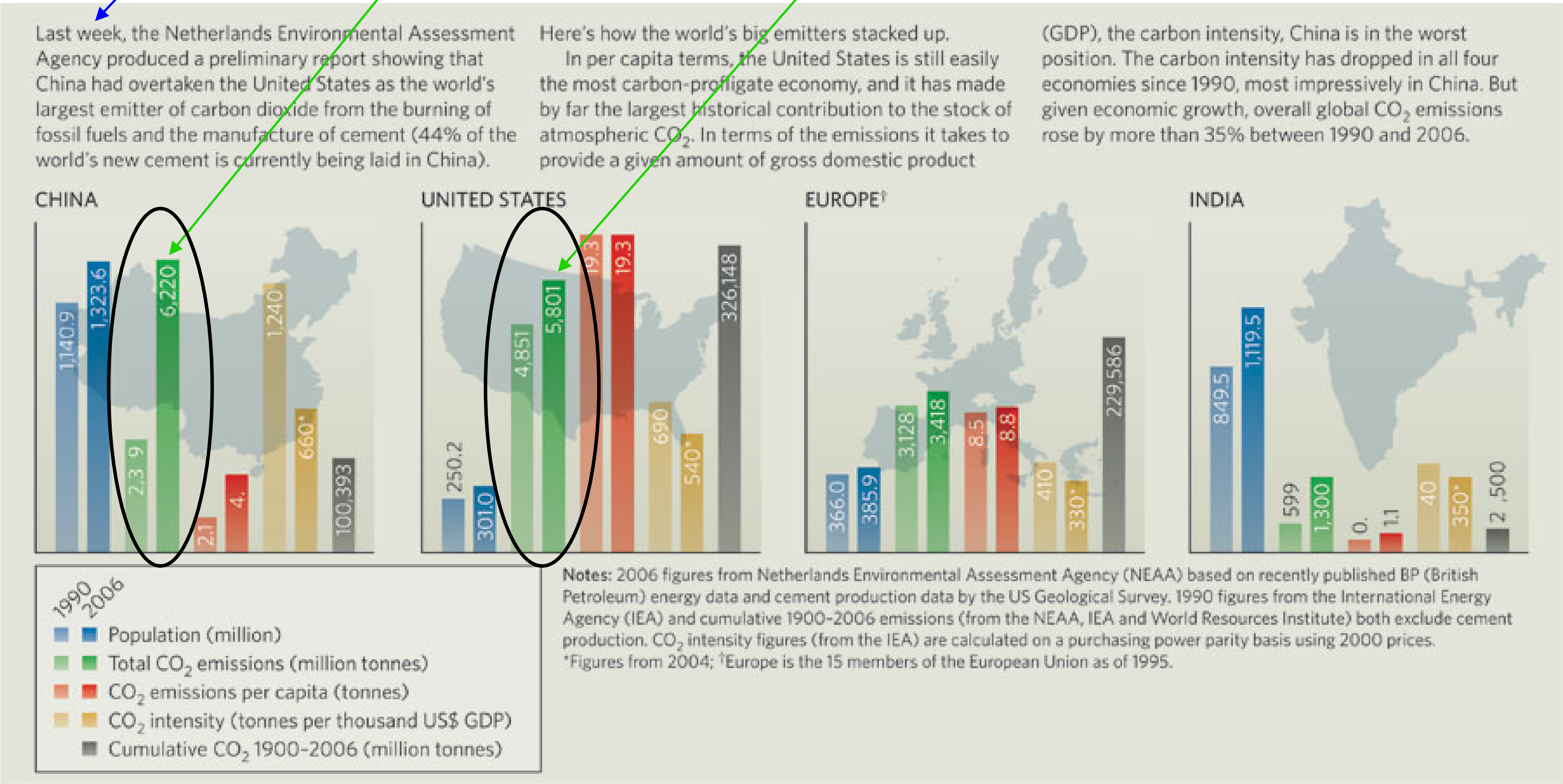


World Carbon Emissions

20 June 2007

China: 1.70 Gt C per year

US: 1.58 Gt C per year



Last week, the Netherlands Environmental Assessment Agency produced a preliminary report showing that China had overtaken the United States as the world's largest emitter of carbon dioxide from the burning of fossil fuels and the manufacture of cement (44% of the world's new cement is currently being laid in China).

Here's how the world's big emitters stacked up.

In per capita terms, the United States is still easily the most carbon-profligate economy, and it has made by far the largest historical contribution to the stock of atmospheric CO_2. In terms of the emissions it takes to provide a given amount of gross domestic product (GDP), the carbon intensity, China is in the worst position. The carbon intensity has dropped in all four economies since 1990, most impressively in China. But given economic growth, overall global CO_2 emissions rose by more than 35% between 1990 and 2006.

Population (million)

Total CO_2 emissions (million tonnes)

CO_2 emissions per capita (tonnes)

CO_2 intensity (tonnes per thousand US$ GDP)

Cumulative CO_2 1900–2006 (million tonnes)

Notes: 2006 figures from Netherlands Environmental Assessment Agency (NEAA) based on recently published BP (British Petroleum) energy data and cement production data by the US Geological Survey. 1990 figures from the International Energy Agency (IEA) and cumulative 1900–2006 emissions (from the NEAA, IEA and World Resources Institute) both exclude cement production. CO_2 intensity figures (from the IEA) are calculated on a purchasing power parity basis using 2000 prices.
*Figures from 2004; †Europe is the 15 members of the European Union as of 1995.

Source: http://www.nature.com/nature/journal/v447/n7148/fig_tab/4471038a_F1.html

US / China Announcement ⇒ Paris Climate Agreement

Nov 2014, Presidents Obama & Xi announced that the **U.S.** would reduce C emissions **27%** below 2005 level **by 2025** & **China** would **peak by 2030** with best effort to peak early

Paris Climate Agreement:

Article 2, Section 1, Part a):

Objective to hold "increase in GMST to well below 2^{o}C above pre-industrial levels and to pursue efforts to limit the temperature increase to 1.5^{o}C above pre-industrial levels"

INDC: Intended **N**ationally **D**etermined **C**ontributions to reduce GHG emissions

- Submitted prior to Dec 2015, COP21-UNFCCC meeting in Paris
- Consist of either unconditional (promise) or conditional (contingent) pledges
- Generally extend from present to year 2030

COP: Conference of the Parties
UNFCCC: United Nations Framework Convention on Climate Change

PARIS2015
UN CLIMATE CHANGE CONFERENCE
COP21·CMP11

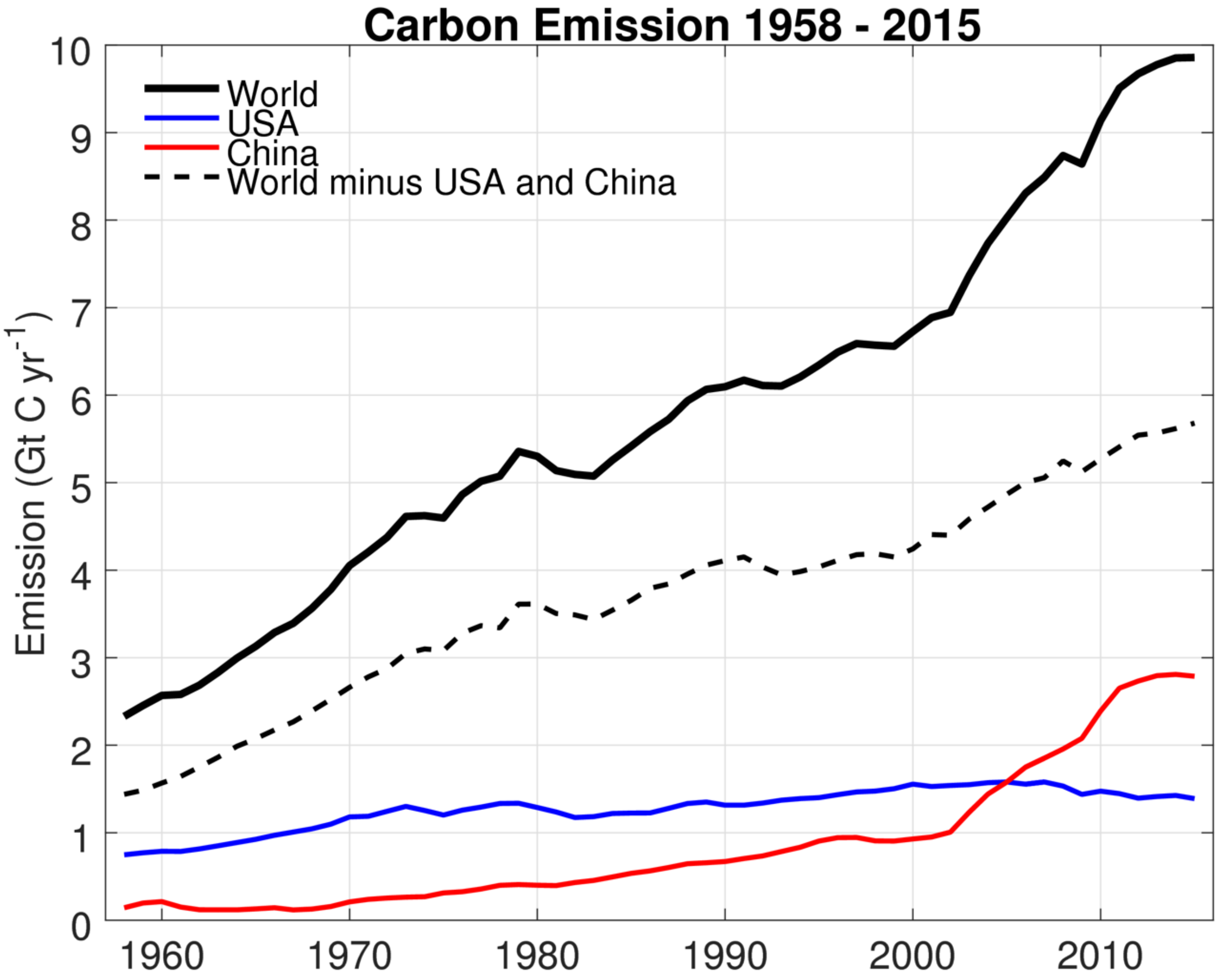


Figure courtesy Walt Tribett

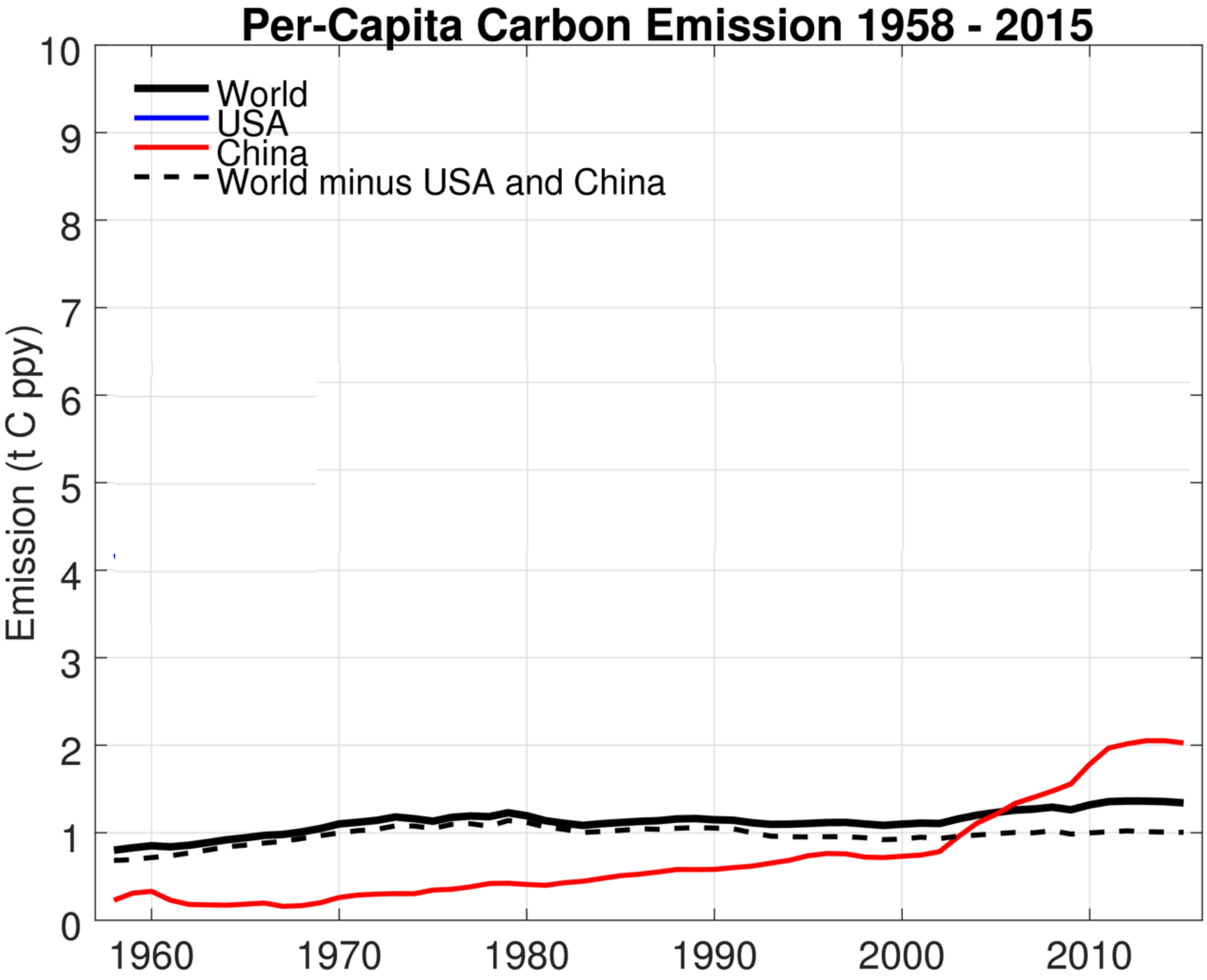


Figure courtesy Walt Tribett

Note, here we use Gt C, whereas in the book, we used GT CO_2

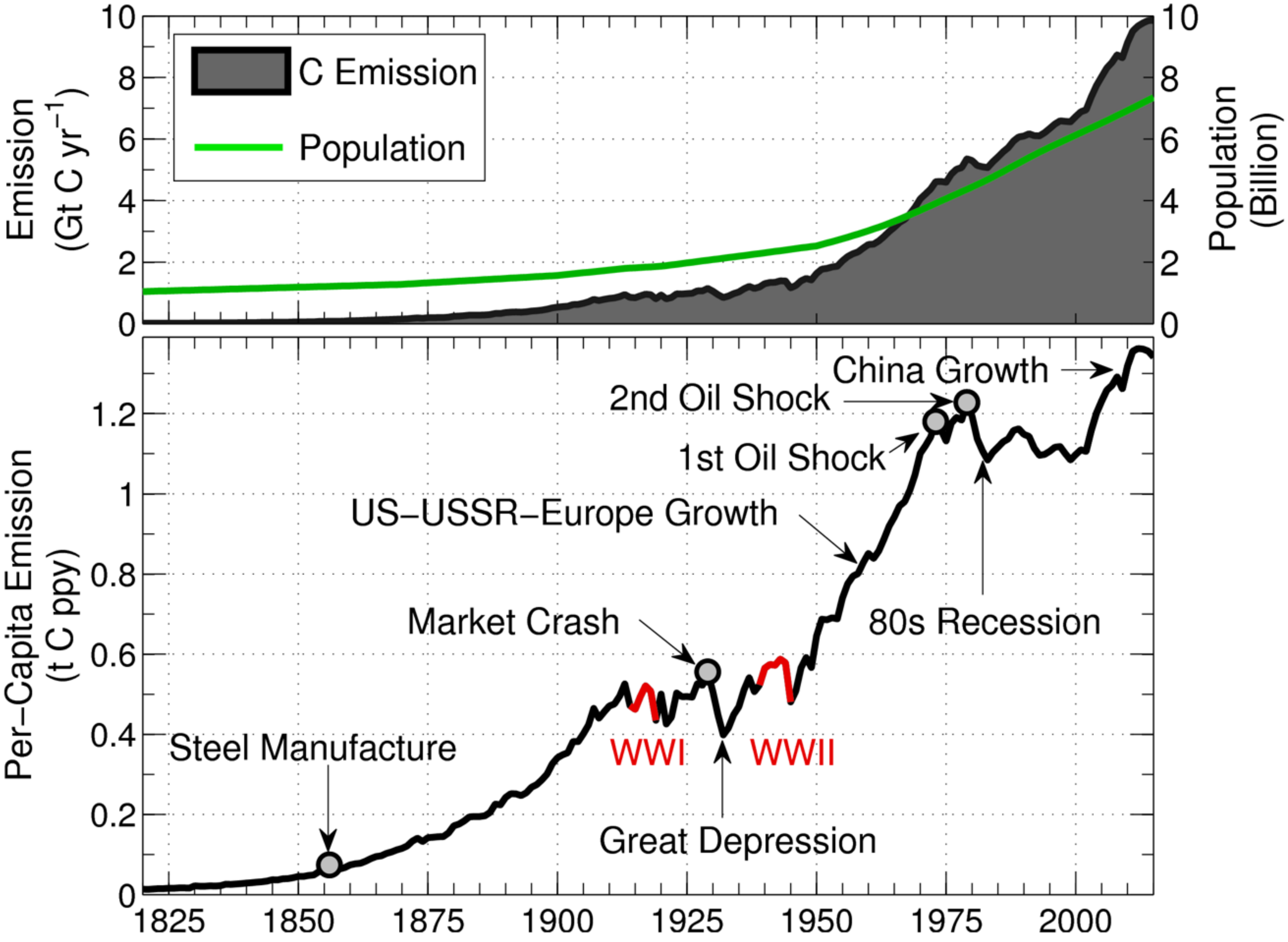
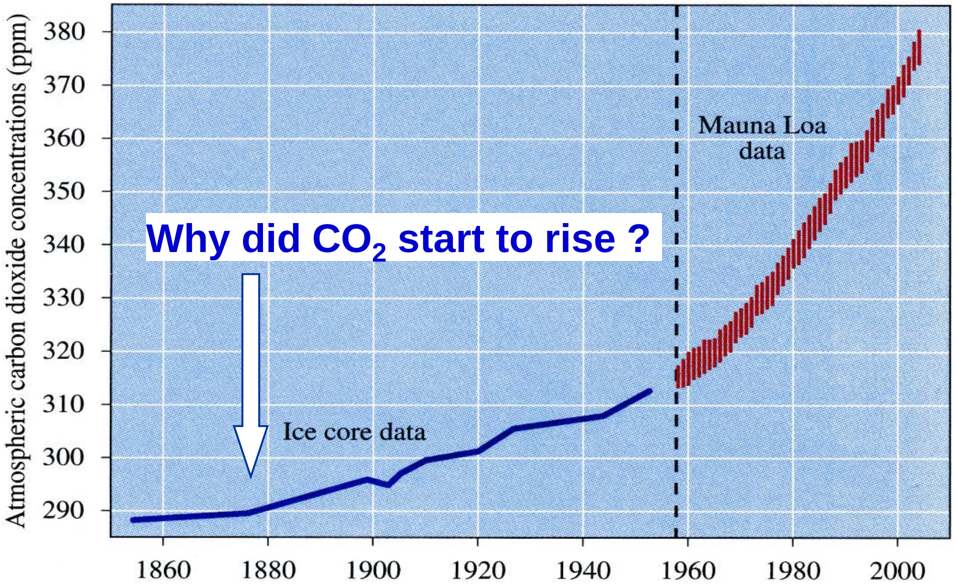


Figure courtesy Walt Tribett

After Fig 3.1 *Paris Beacon of Hope*

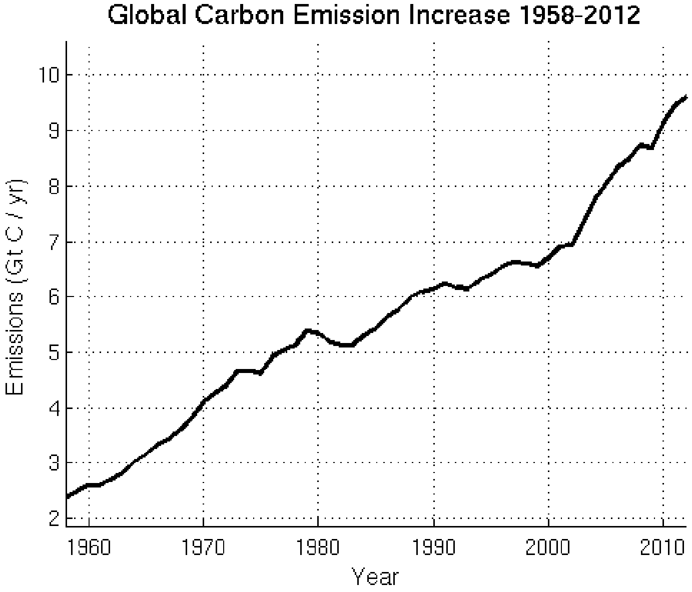
Atmospheric CO_2 since ~1860

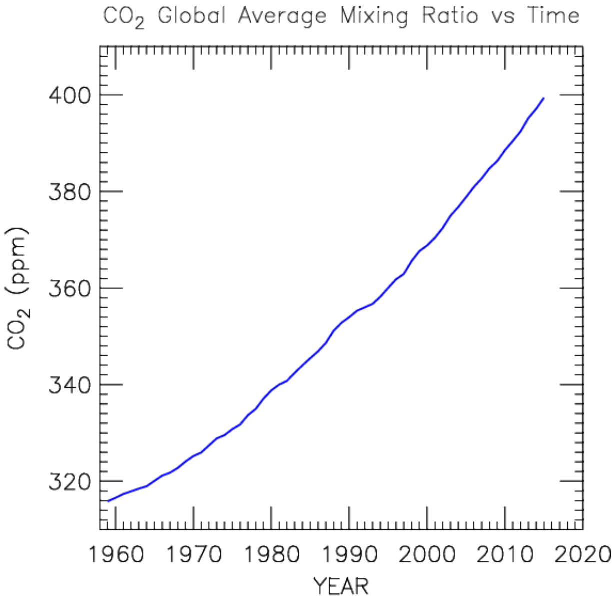


Why did CO_2 start to rise ?

Figure 3.5, *Chemistry in Context 6th Edition*

Fossil Fuel Emissions

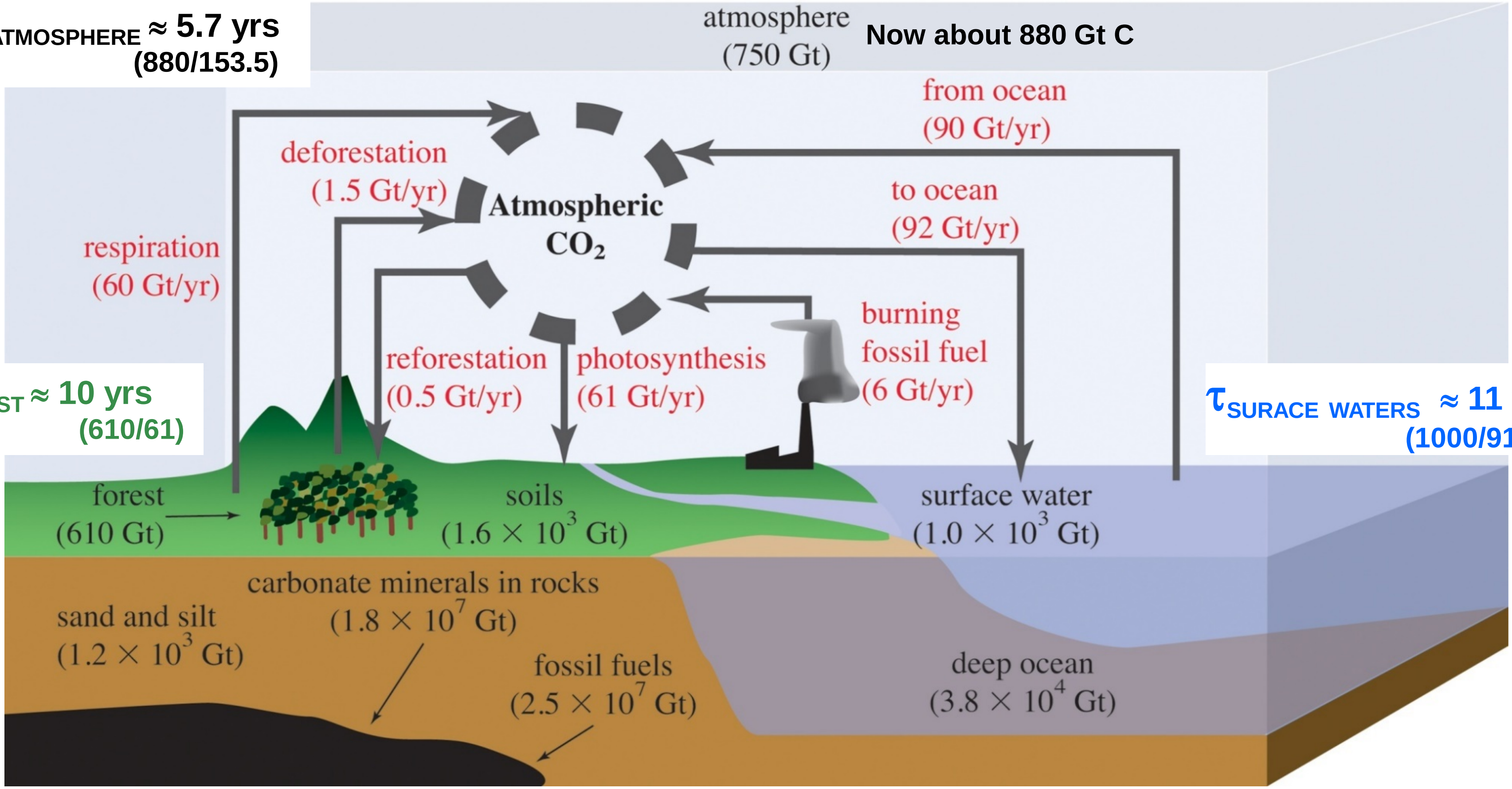




Fossil fuel emissions, 1959 to 2015 = **336 Gt C**

Δ (CO_2) years 1959 to 2015 = **83.6 ppm**

Global Carbon Cycle



$\tau_{\text{ATMOSPHERE}} \approx$ 5.7 yrs
(880/153.5)

Now about 880 Gt C

$\tau_{\text{FOREST}} \approx$ 10 yrs
(610/61)

$\tau_{\text{SURACE WATERS}} \approx$ 11 yrs
(1000/91)

$\tau_{\text{OCEAN}} \approx$ 430 yrs
(39000/91)

$\tau_{\text{SEDIMENT}} \approx 1.6\times10^5$ yrs (RJS estimate)

Fig 3.2, Chemistry in Context

Global Carbon Cycle

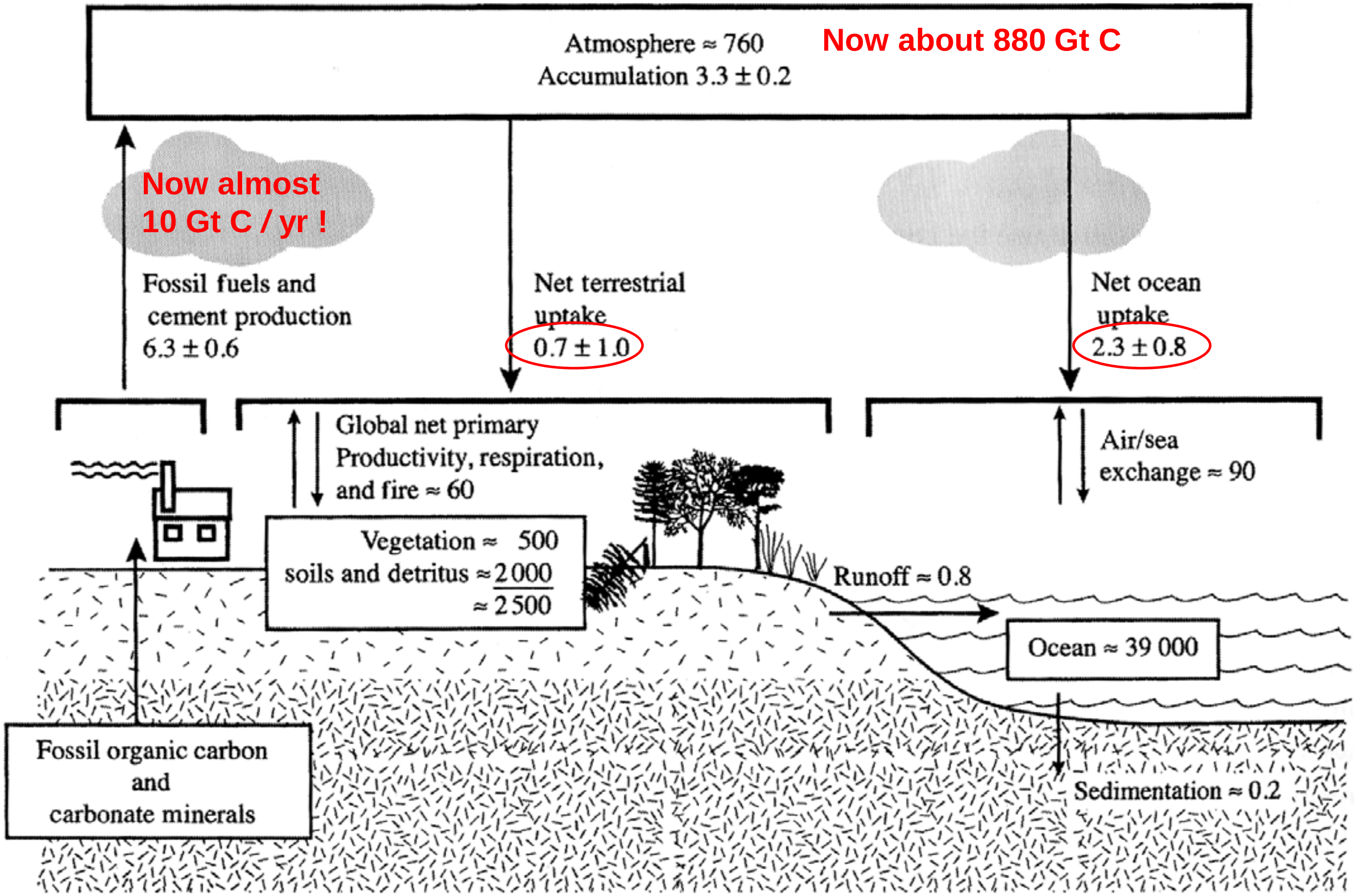


Figure 3.1 The global carbon cycle, showing the carbon stocks in reservoirs (in Gt) and carbon flows (in Gt year^{-1}) relevant to the anthropogenic perturbation as annual averages over the decade from 1989 to 1998. Net ocean uptake of the anthropogenic perturbation equals the net air/sea input plus run-off minus sediment. The units are thousand millions of tonnes or gigatonnes (Gt).

Page 30, Houghton

CO_2 Is Long Lived

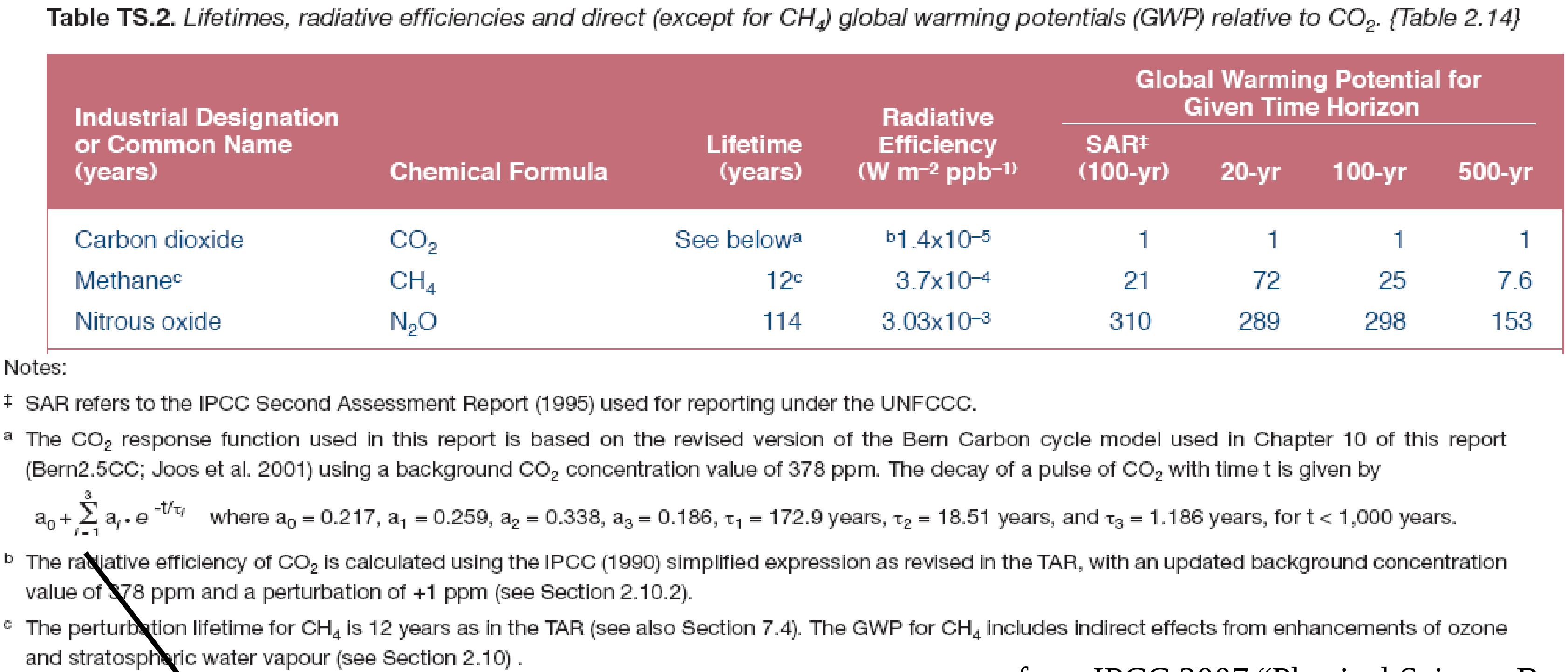


Table TS.2. *Lifetimes, radiative efficiencies and direct (except for CH_4) global warming potentials (GWP) relative to CO_2. {Table 2.14}*

Industrial Designation or Common Name (years)	Chemical Formula	Lifetime (years)	Radiative Efficiency ($W\ m^{-2}\ ppb^{-1}$)	Global Warming Potential for Given Time Horizon: SAR‡ (100-yr)	20-yr	100-yr	500-yr
Carbon dioxide	CO_2	See below[a]	[b]1.4×10^{-5}	1	1	1	1
Methane[c]	CH_4	12[c]	3.7×10^{-4}	21	72	25	7.6
Nitrous oxide	N_2O	114	3.03×10^{-3}	310	289	298	153

Notes:

‡ SAR refers to the IPCC Second Assessment Report (1995) used for reporting under the UNFCCC.

[a] The CO_2 response function used in this report is based on the revised version of the Bern Carbon cycle model used in Chapter 10 of this report (Bern2.5CC; Joos et al. 2001) using a background CO_2 concentration value of 378 ppm. The decay of a pulse of CO_2 with time t is given by

$$a_0 + \sum_{i=1}^{3} a_i \cdot e^{-t/\tau_i}$$ where $a_0 = 0.217$, $a_1 = 0.259$, $a_2 = 0.338$, $a_3 = 0.186$, $\tau_1 = 172.9$ years, $\tau_2 = 18.51$ years, and $\tau_3 = 1.186$ years, for $t < 1{,}000$ years.

[b] The radiative efficiency of CO_2 is calculated using the IPCC (1990) simplified expression as revised in the TAR, with an updated background concentration value of 378 ppm and a perturbation of +1 ppm (see Section 2.10.2).

[c] The perturbation lifetime for CH_4 is 12 years as in the TAR (see also Section 7.4). The GWP for CH_4 includes indirect effects from enhancements of ozone and stratospheric water vapour (see Section 2.10) .

from IPCC 2007 “Physical Science Basis”

CO_2 has multiple time constants

Longest decay of IPCC formula is close to 200 years, which represents time for surface waters to equilibrate with the intermediate ocean

Note: IPCC formula should only be used for t <1000 years

Global Carbon Cycle, 1959 to present

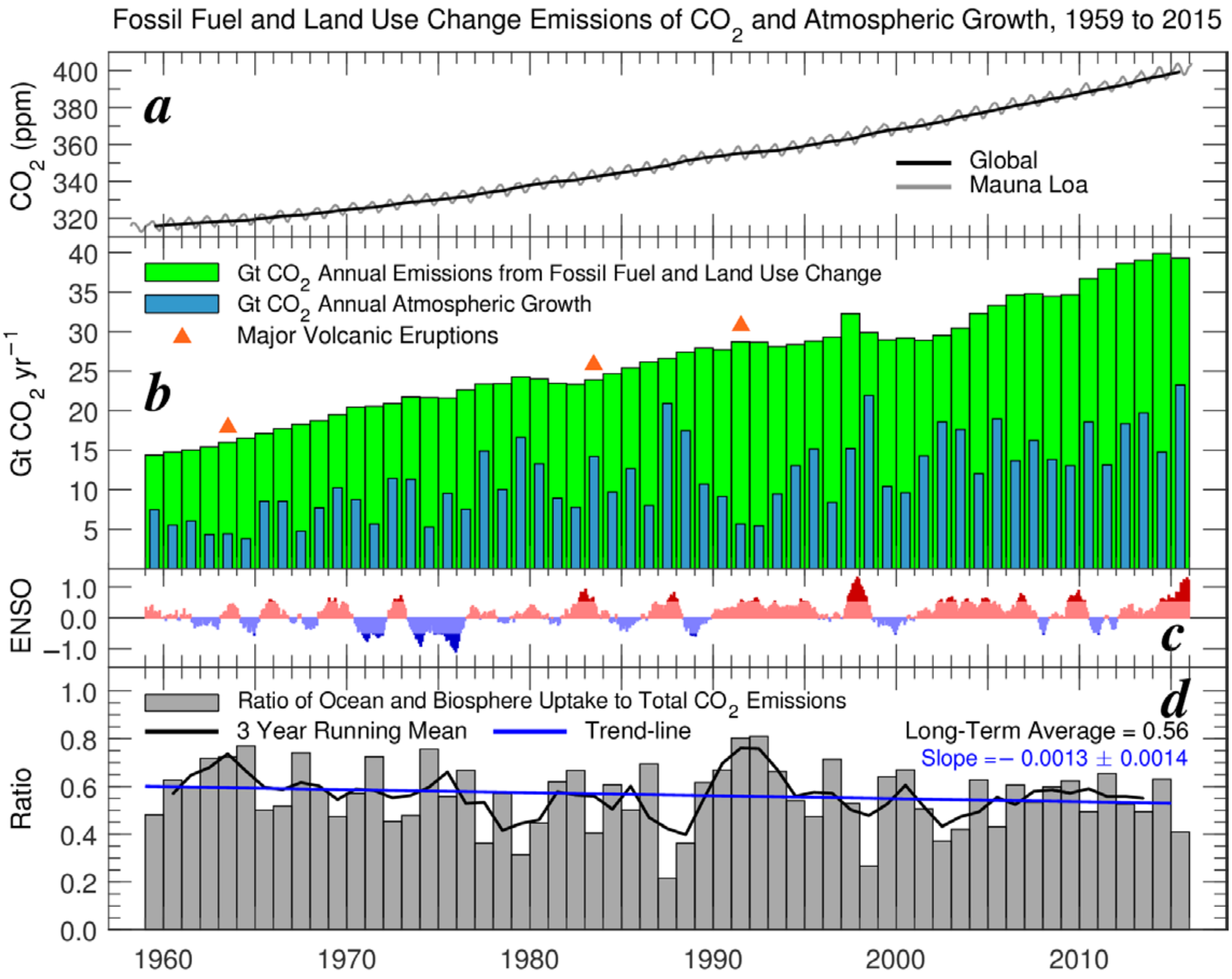


Fig 1.6, *Paris Beacon of Hope*

Human “Fingerprint” on Atmospheric CO_2

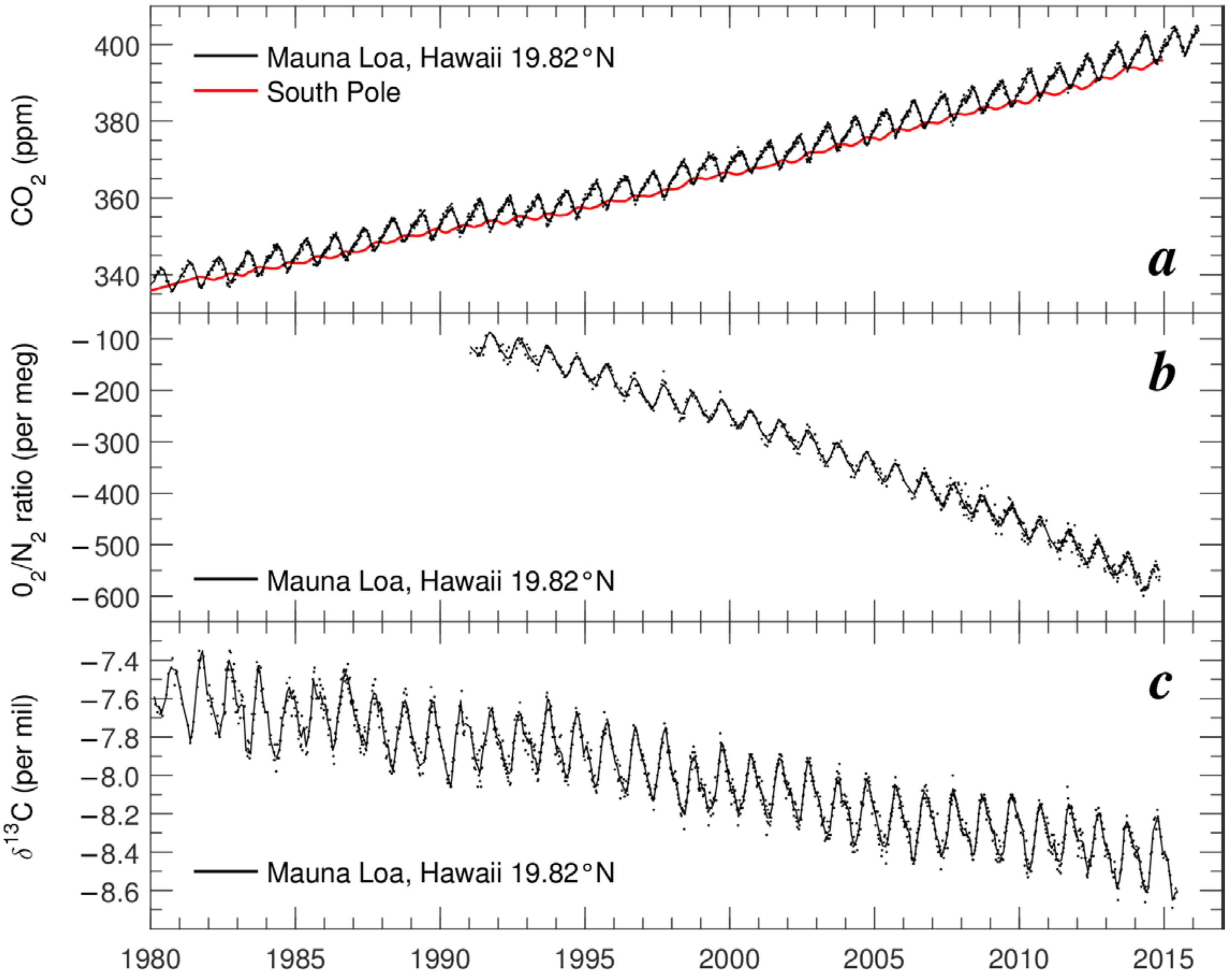


Fig 1.7, *Paris Beacon of Hope*

Human “Fingerprint” on Atmospheric CO_2

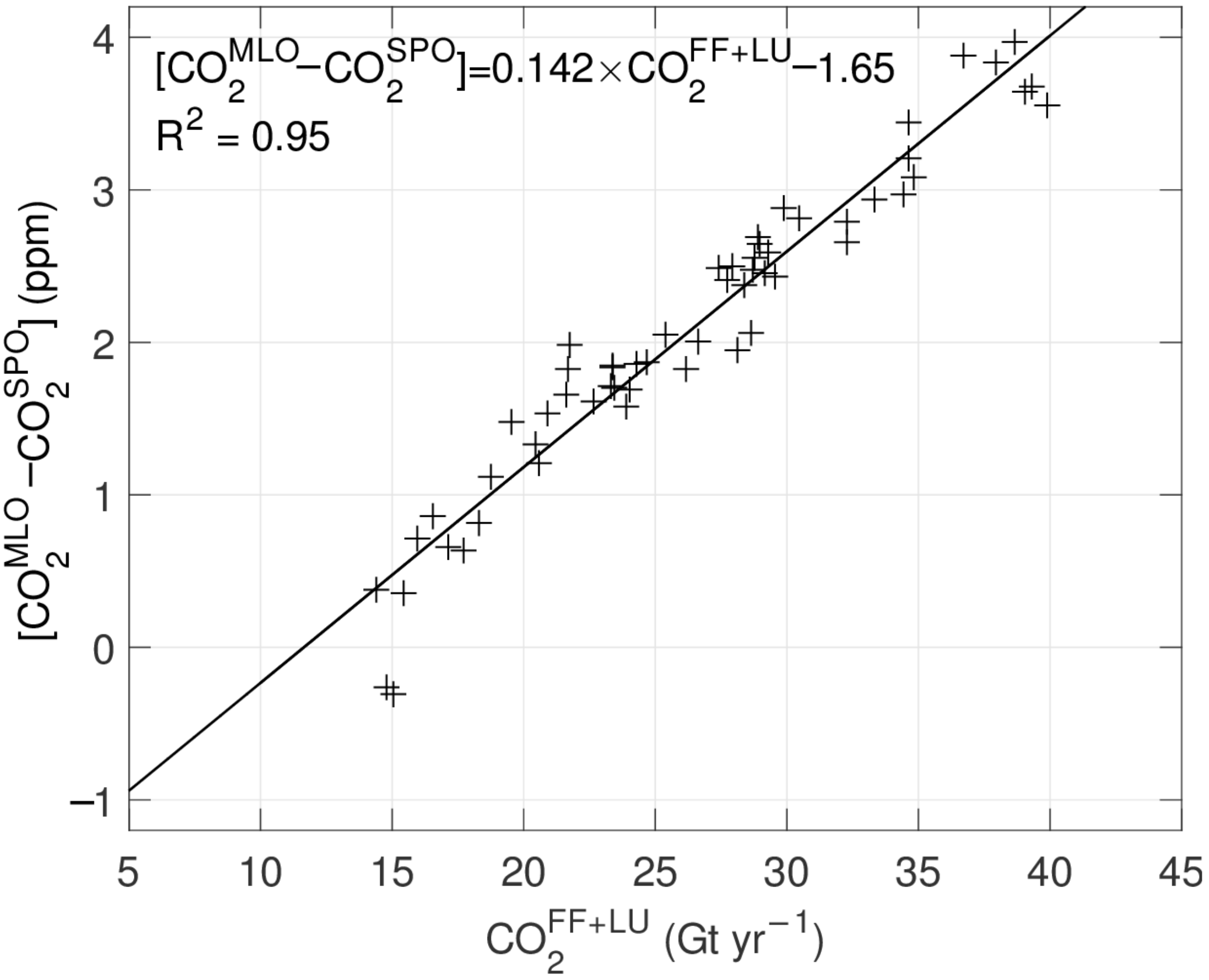
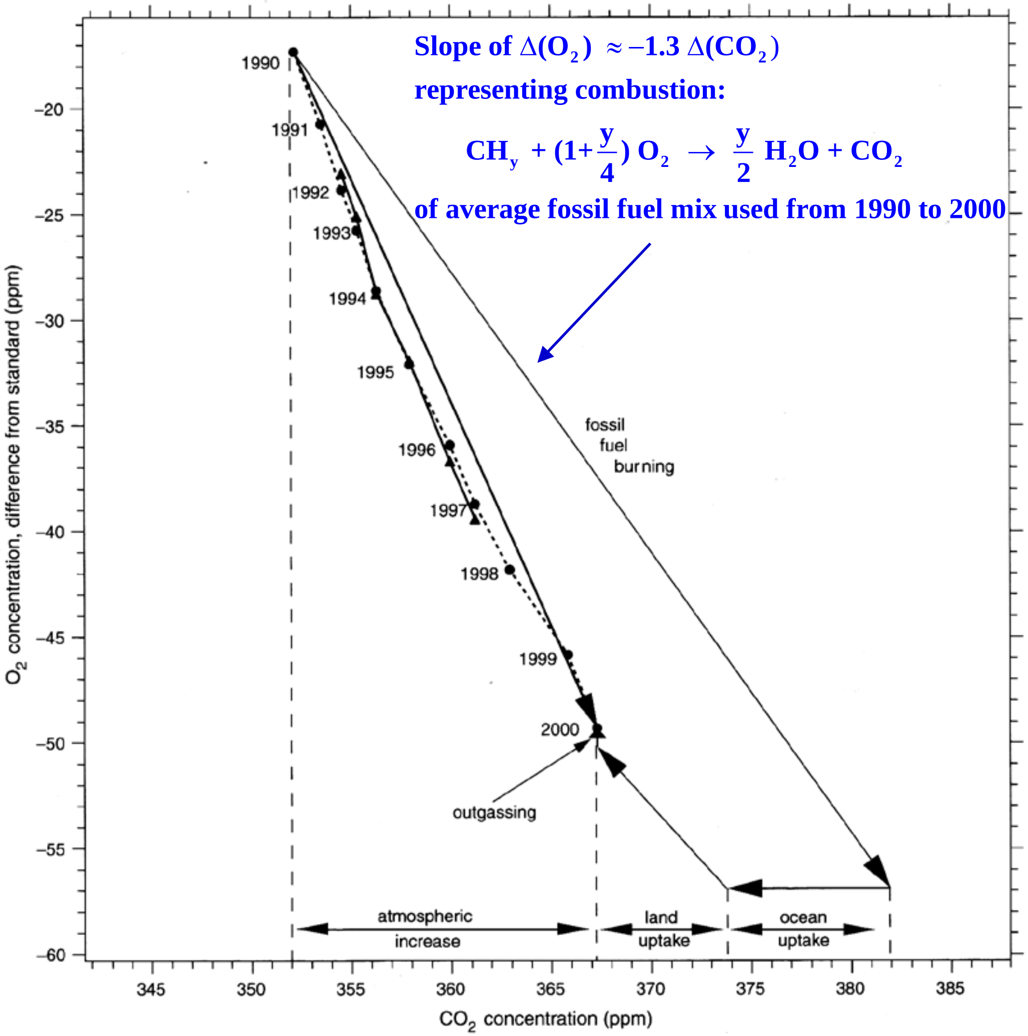


Fig 1.8, *Paris Beacon of Hope*

Inferring CO_2 Uptake Based on $\Delta(O_2)$



Slope of $\Delta(O_2) \approx -1.3\ \Delta(CO_2)$

representing combustion:

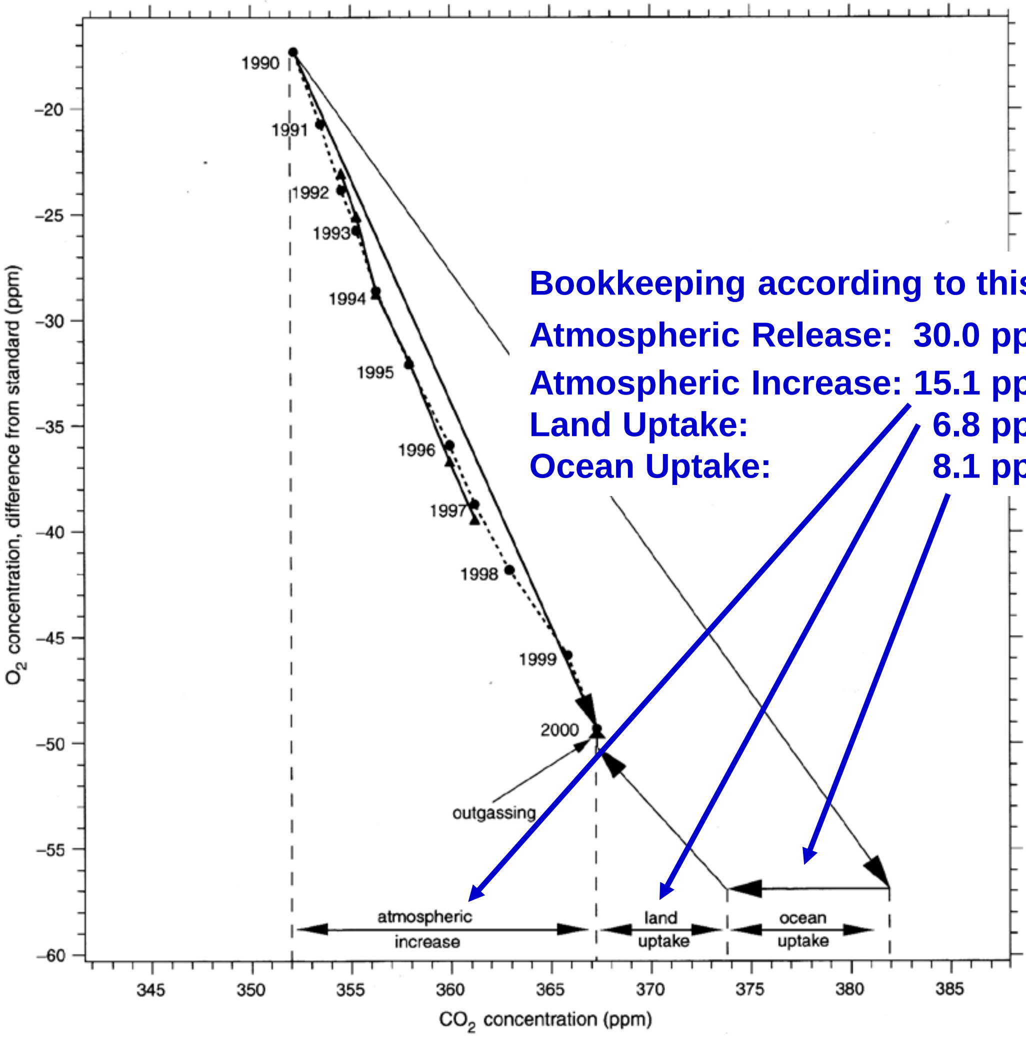
$$CH_y + (1+\frac{y}{4})\ O_2 \rightarrow \frac{y}{2}\ H_2O + CO_2$$

of average fossil fuel mix used from 1990 to 2000

O_2 falls a tiny bit as CO_2 rises

Figure 3.4 Partitioning of fossil fuel carbon dioxide uptake using oxygen measurements. Shown is the relationship between changes in carbon dioxide and oxygen concentrations. Observations are shown by solid circles and triangles. The arrow labelled ‘fossil fuel burning’ denotes the effect of the combustion of fossil fuels based on the $O_2:CO_2$ stoichiometric relation of the different fuel types. Uptake by land and ocean is constrained by the stoichiometric ratio associated with these processes, defining the slopes of the respective arrows.

Inferring CO_2 Uptake Based on $\Delta(O_2)$

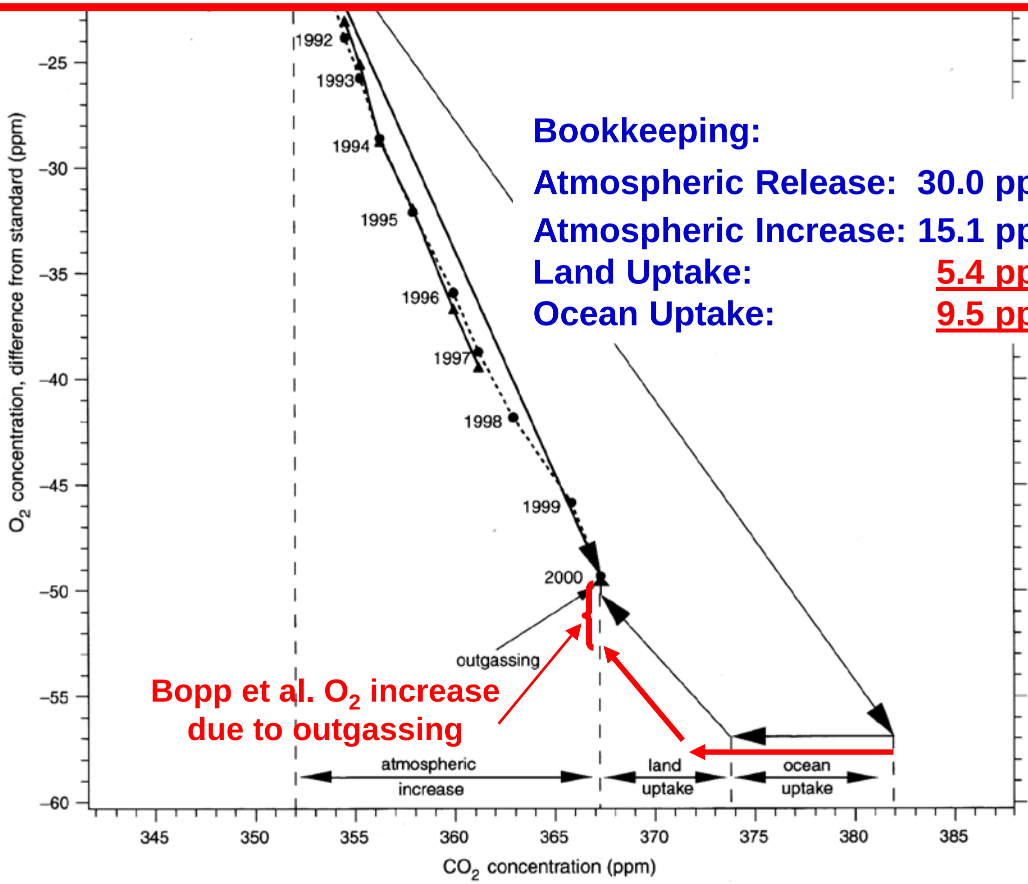


Bookkeeping according to this study:

Atmospheric Release:	**30.0 ppm**	**(66 Gt)**
Atmospheric Increase:	**15.1 ppm**	**(~50% airborne fraction)**
Land Uptake:	**6.8 ppm**	**(46% of uptake)**
Ocean Uptake:	**8.1 ppm**	**(54% of uptake)**

Figure 3.4 Partitioning of fossil fuel carbon dioxide uptake using oxygen measurements. Shown is the relationship between changes in carbon dioxide and oxygen concentrations. Observations are shown by solid circles and triangles. The arrow labelled 'fossil fuel burning' denotes the effect of the combustion of fossil fuels based on the $O_2 : CO_2$ stoichiometric relation of the different fuel types. Uptake by land and ocean is constrained by the stoichiometric ratio associated with these processes, defining the slopes of the respective arrows.

Note: As the ocean warms, O_2 solubility decreases.
In other words, as climate changes, the oceans outgas O_2.
Bopp et al. (GBC, 2002) applied a correction for
ocean outgassing and concluded ____________________



Bookkeeping:

Atmospheric Release:	**30.0 ppm**	**(66 Gt)**
Atmospheric Increase:	**15.1 ppm**	**(~50% airborne fraction)**
Land Uptake:	**5.4 ppm**	**(36% of uptake)**
Ocean Uptake:	**9.5 ppm**	**(64% of uptake)**

Bopp et al. O_2 increase due to outgassing

Figure 3.4 Partitioning of fossil fuel carbon dioxide uptake using oxygen measurements. Shown is the relationship between changes in carbon dioxide and oxygen concentrations. Observations are shown by solid circles and triangles. The arrow labelled 'fossil fuel burning' denotes the effect of the combustion of fossil fuels based on the $O_2 : CO_2$ stoichiometric relation of the different fuel types. Uptake by land and ocean is constrained by the stoichiometric ratio associated with these processes, defining the slopes of the respective arrows.

Uptake of Atmospheric CO_2 by Trees (Land Sink)

Land sink: relatively short lived reservoir

- In this model, future water stress due to climate change eventually limits plant growth
- Feedbacks between climate change & plants could lead to almost 100 ppm additional CO_2 by end of century

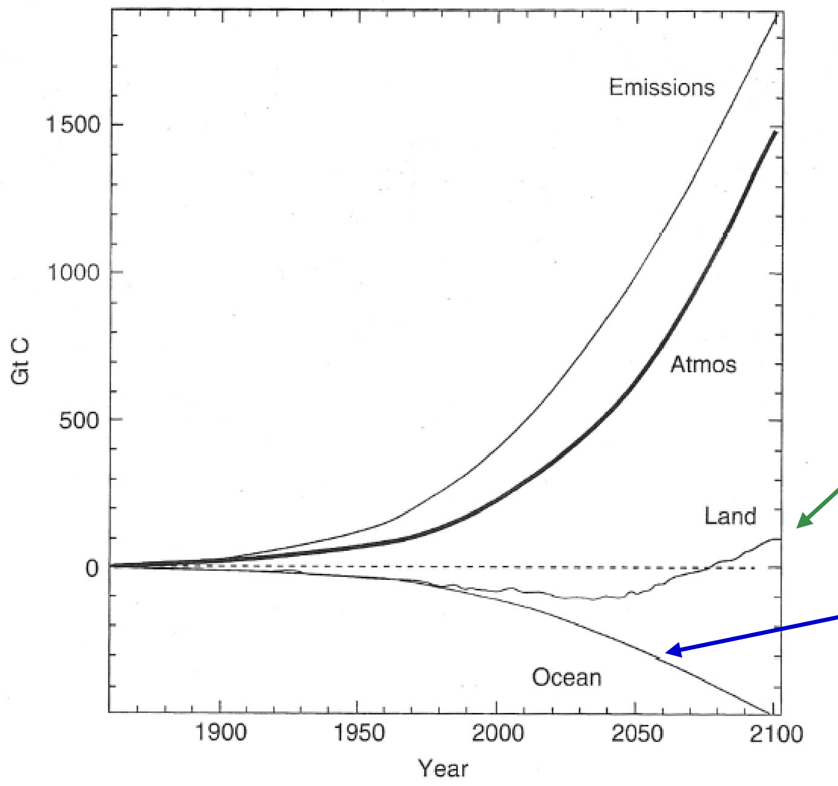


Figure 3.5 Illustrating the possible effects of climate feedbacks on the carbon cycle. Results are shown of the changing budgets of carbon

Page 41, Houghton

Ocean sink: relatively long lived reservoir

In nearly all models, ocean uptake slows relative to rise in atmospheric CO_2

Uptake of Atmospheric CO_2 by Oceans

– Solubility Pump:

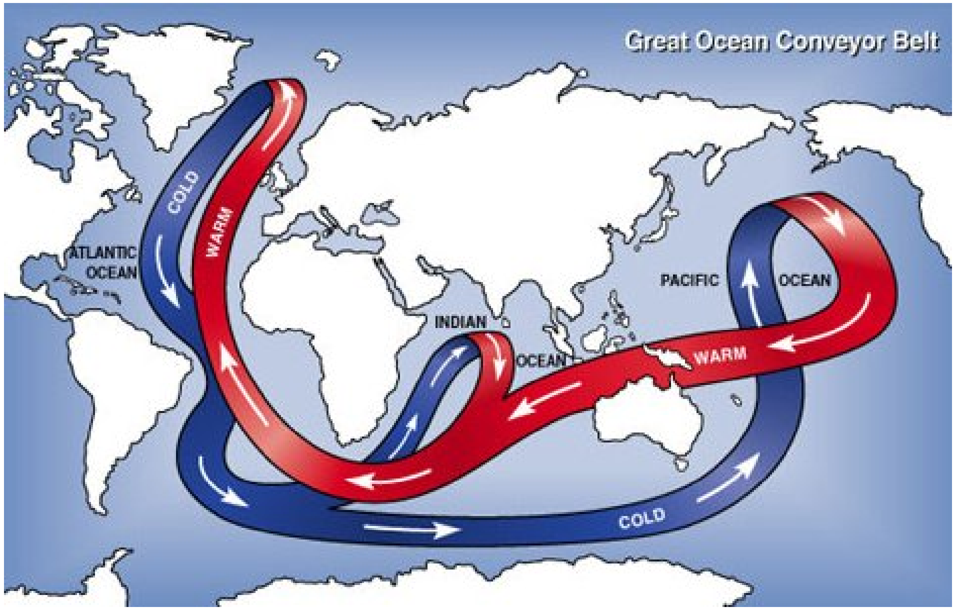
a) More CO_2 can dissolve in cold polar waters than in warm equatorial waters. As major ocean currents (e.g. the Gulf Stream) move waters from tropics to the poles, they are cooled and take up atmospheric CO_2

b) Deep water forms at high latitude. As deep water sinks, ocean carbon (ΣCO_2) accumulated at the surface is moved to the deep ocean interior.

– Biological Pump:

a) Ocean biology limited by availability of nutrients such as NO_3^-, PO_4^-, and Fe^{2+} & Fe^{3+} . Ocean biology is never carbon limited.

b) Detrital material "rains" from surface to deep waters, contributing to higher CO_2 in intermediate and deep waters



http://science.nasa.gov/headlines/y2004/05mar_arctic.htm

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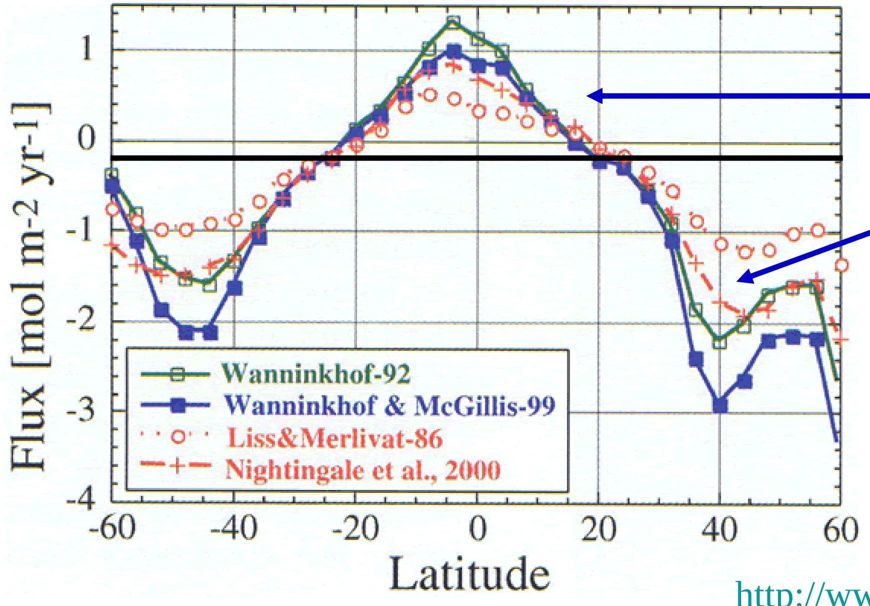

CO_2 comes out of tropical ocean

CO_2 goes into rest of the ocean

http://www.pmel.noaa.gov/pubs/outstand/feel2331/images/fig05.gif

Uptake of Atmospheric CO_2 by Oceans

When CO_2 dissolves:

Net: $\mathbf{CO_2(aq) + CO_3^{2-} + H_2O \rightarrow 2\ HCO_3^-}$

Atmospheric CO_2	280 ppm Pre-Industrial	400 ppm Present Day	560 ppm 2 × Pre-Indus.
Ocean Carbon	2020×10^{-6} M	2075×10^{-6} M	2122×10^{-6} M
$[HCO_3^-]$	1772×10^{-6} M	1875×10^{-6} M	1957×10^{-6} M
$[CO_2\,(aq)]$	9.1×10^{-6} M	13.0×10^{-6} M	18.2×10^{-6} M
$[CO_3^{2-}]$	239×10^{-6} M	188×10^{-6} M	146×10^{-6} M
pH	8.32	8.19	8.06

Ocean Carbon $[\Sigma\ CO_2] = [CO_2(aq)] + [HCO_3^-] + [CO_3^{2-}]$

Notes:

T = 293 K; Alkalinity= 2.25×10^{-3} M

M ≡ mol/liter

Mathematics supporting this calculation on Extra Slide 3

Uptake of Atmospheric CO_2 by Oceans

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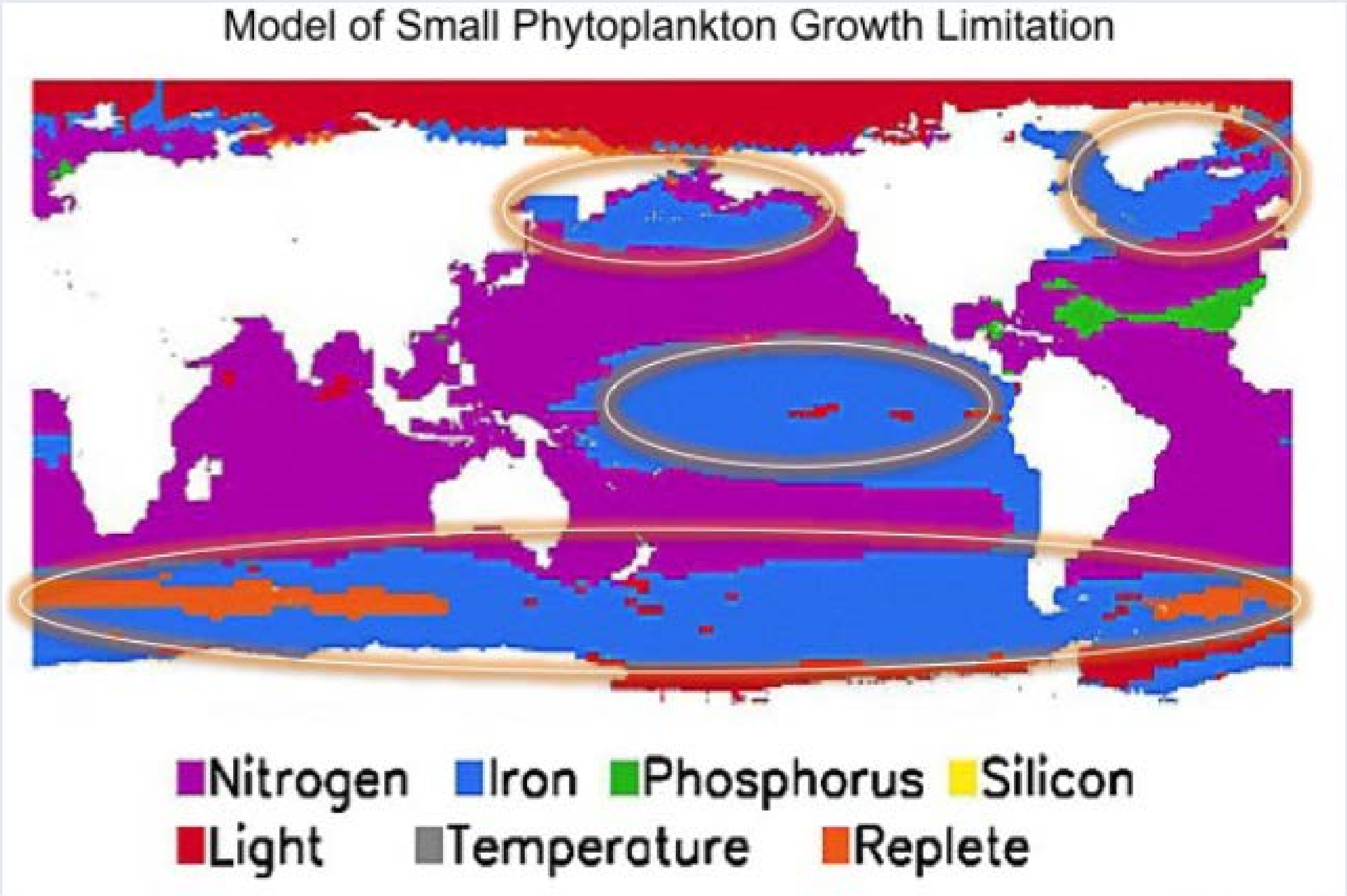
pH	8.32	8.19	8.06

$$\text{Revelle Factor} = \frac{\Delta\text{Ocean Carbon} \big/ \langle \text{Ocean Carbon} \rangle_{\text{AVERAGE}}}{\Delta\text{Atmos}_{\text{CO2}} \big/ \langle \text{Atmos}_{\text{CO2}} \rangle_{\text{AVERAGE}}}$$

$$= \frac{55/2047.5}{120/340} = 0.076 \text{ (from pre-industrial to present-day } CO_2\text{)}$$

$$= \frac{47/2098.5}{160/480} = 0.067 \text{ (from present-day to } 2\times \text{ pre-industrial } CO_2\text{)}$$

Biology in Today’s Ocean

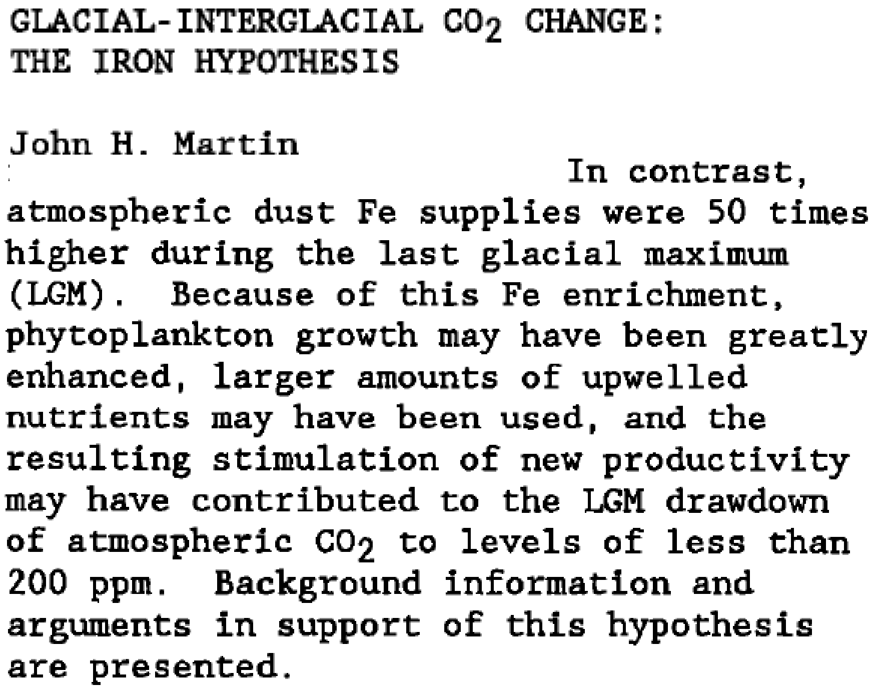


Model of Small Phytoplankton Growth Limitation

Figure 1. Results of a global model of small phytoplankton growth limitation by Moore et al. 2004 (*Global Biogeochemical Cycles*). Blue shaded areas denote regions that are potentially limited by iron availability. Iron is supplied by dust from continents and by upwelling of deep water. However, high iron demand in the euphotic zone quickly drives iron concentrations to nano- and picomolar levels that can be limiting to many phyto- and bacterioplankton.

http://www.whoi.edu/page.do?pid=130796

Connection to Glacial CO_2

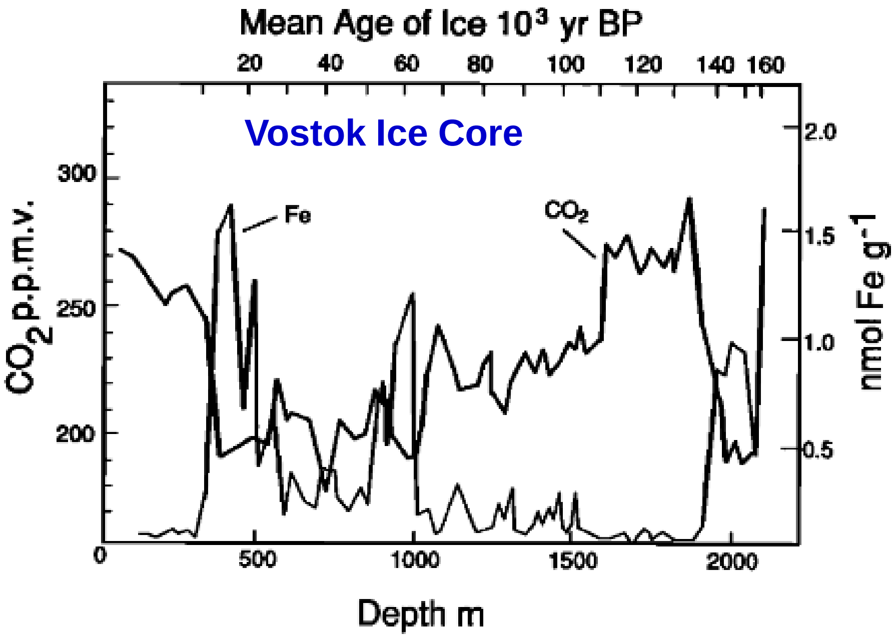


GLACIAL-INTERGLACIAL CO_2 CHANGE:
THE IRON HYPOTHESIS

John H. Martin

In contrast, atmospheric dust Fe supplies were 50 times higher during the last glacial maximum (LGM). Because of this Fe enrichment, phytoplankton growth may have been greatly enhanced, larger amounts of upwelled nutrients may have been used, and the resulting stimulation of new productivity may have contributed to the LGM drawdown of atmospheric CO_2 to levels of less than 200 ppm. Background information and arguments in support of this hypothesis are presented.

PALEOCEANOGRAPHY, VOL.5,
NO.1, PAGES 1-13 1990

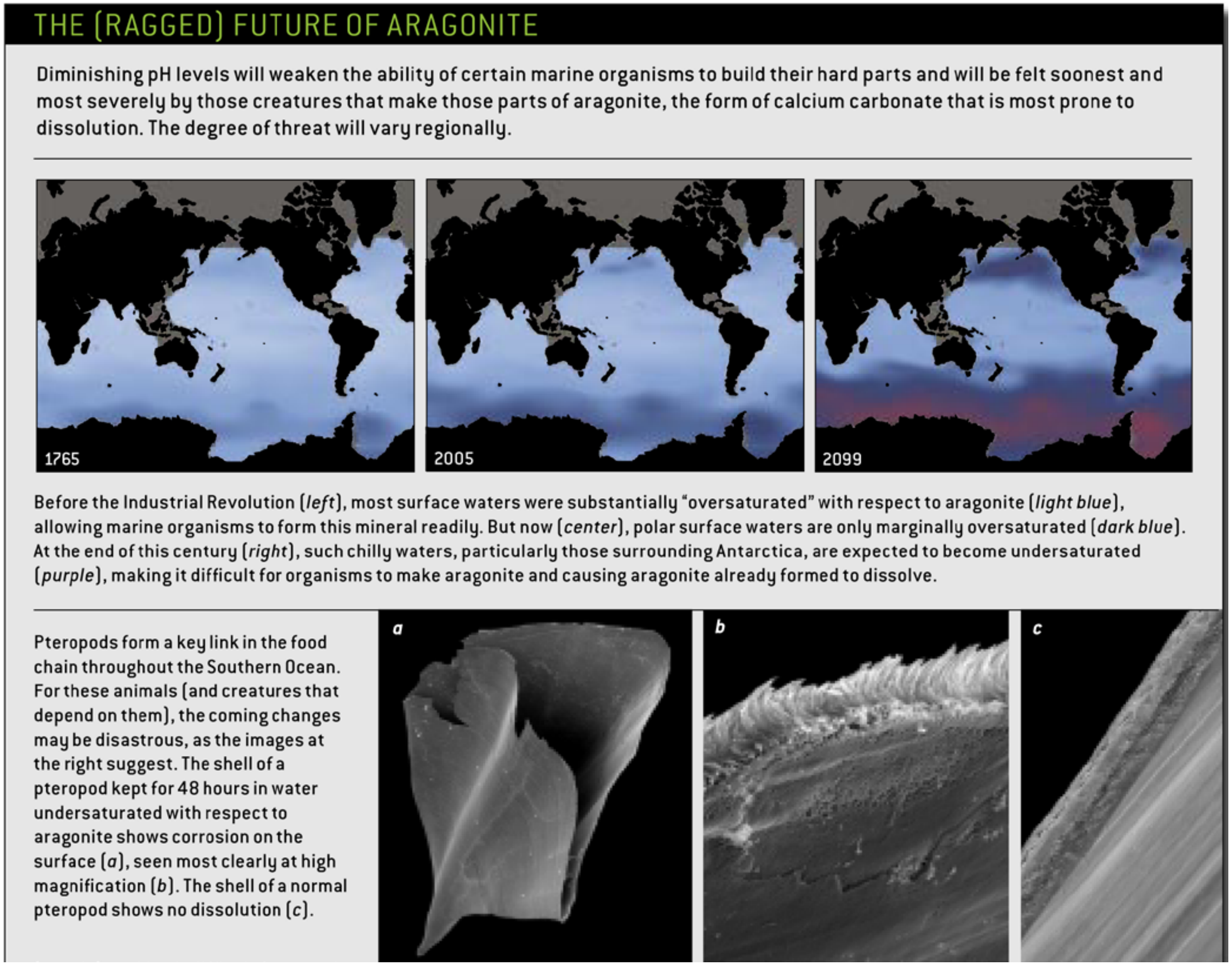


See http://onlinelibrary.wiley.com/doi/10.1029/PA005i001p00001/abstract

Uptake of Atmospheric CO_2 by Oceans

Future ocean uptake of atmospheric CO_2 will lead to ***ocean acidification***

Bad news for ocean dwelling organisms that precipitate shells (basic materials)



THE (RAGGED) FUTURE OF ARAGONITE

Diminishing pH levels will weaken the ability of certain marine organisms to build their hard parts and will be felt soonest and most severely by those creatures that make those parts of aragonite, the form of calcium carbonate that is most prone to dissolution. The degree of threat will vary regionally.

Before the Industrial Revolution (*left*), most surface waters were substantially "oversaturated" with respect to aragonite (*light blue*), allowing marine organisms to form this mineral readily. But now (*center*), polar surface waters are only marginally oversaturated (*dark blue*). At the end of this century (*right*), such chilly waters, particularly those surrounding Antarctica, are expected to become undersaturated (*purple*), making it difficult for organisms to make aragonite and causing aragonite already formed to dissolve.

Pteropods form a key link in the food chain throughout the Southern Ocean. For these animals (and creatures that depend on them), the coming changes may be disastrous, as the images at the right suggest. The shell of a pteropod kept for 48 hours in water undersaturated with respect to aragonite shows corrosion on the surface (*a*), seen most clearly at high magnification (*b*). The shell of a normal pteropod shows no dissolution (*c*).

Doney, The Dangers of Ocean Acidification, *Scientific American*, March, 2006

Uptake of Atmospheric CO_2 by Trees (Land Sink)

Land sink: relatively short lived reservoir

- In this model, future water stress due to climate change eventually limits plant growth
- Feedbacks between climate change & plants lead to almost 100 ppm additional CO_2 by end of century

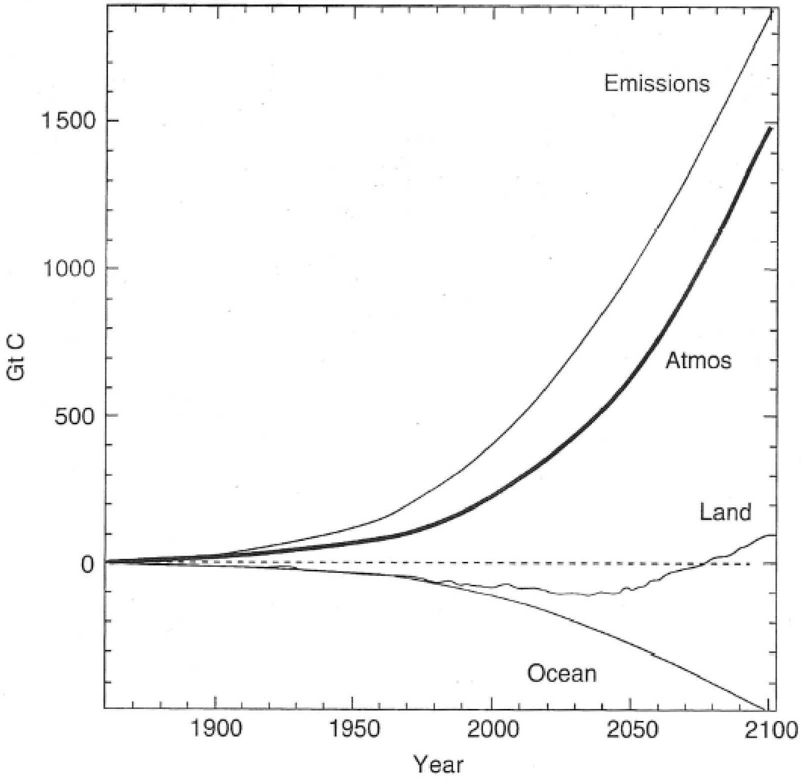


Figure 3.5 Illustrating the possible effects of climate feedbacks on the carbo cycle. Results are shown of the changing budgets of carbon

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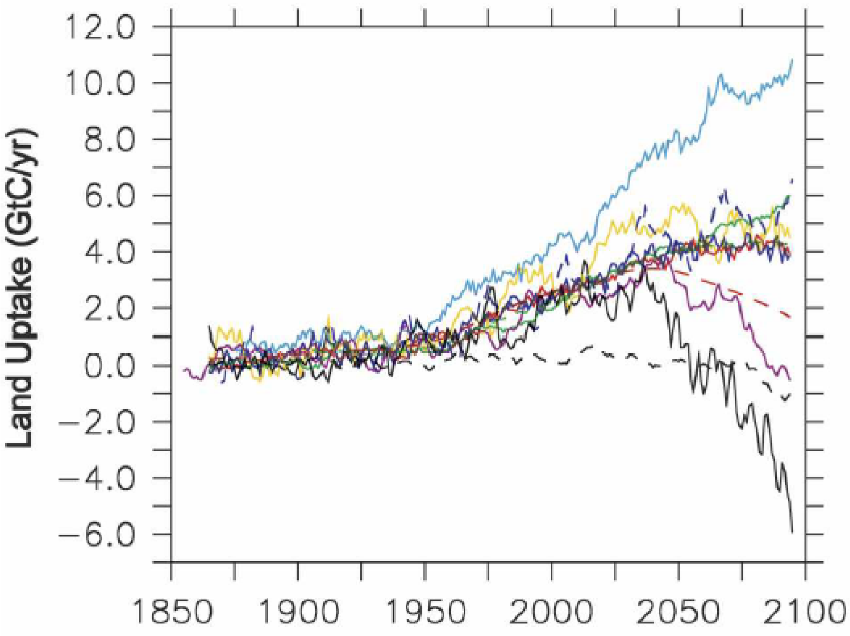
- Future fate of land sink highly uncertain according to **11** coupled climate-carbon cycle models examined by Friedlingstein et al. (2006)



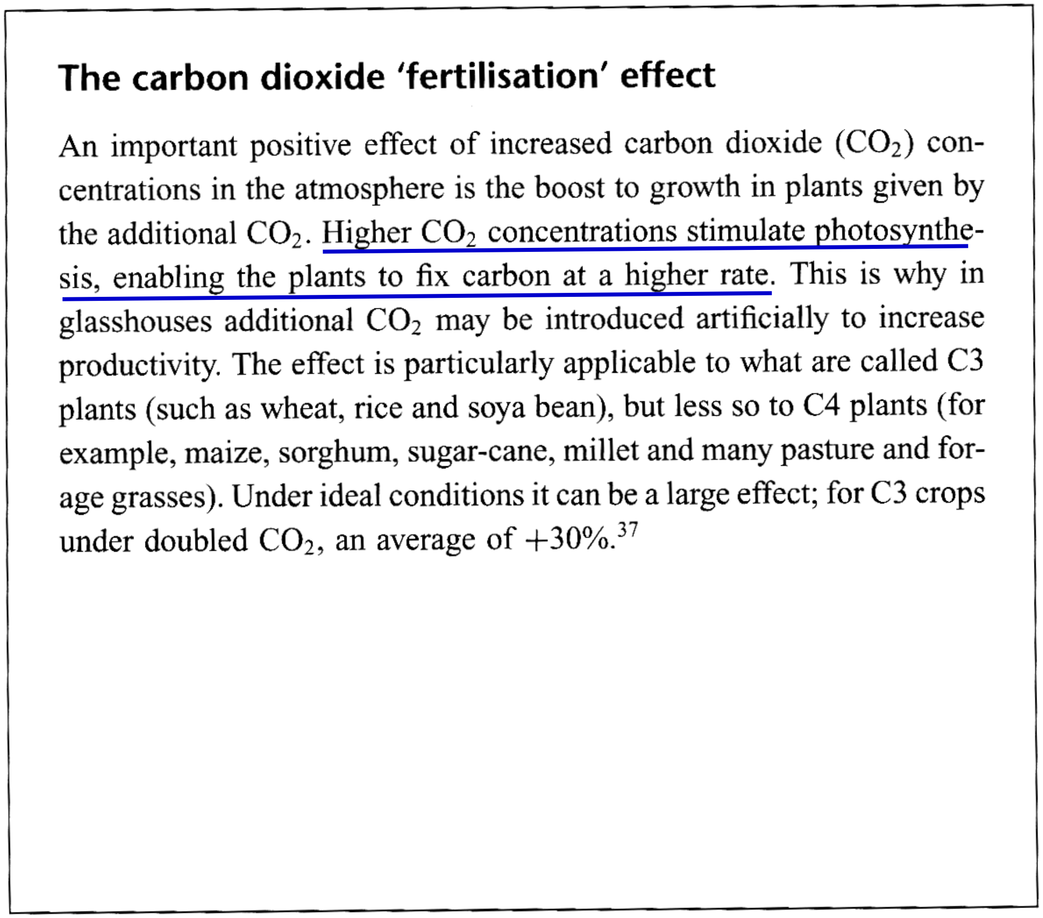
Uptake of Atmospheric CO_2 by Trees (Land Sink)

Land sink

As CO_2 ↑, photosynthesis (all things being equal) will increase.

Known as the **"CO_2 fertilizer" effect**

Difficult to quantify because



The carbon dioxide 'fertilisation' effect

An important positive effect of increased carbon dioxide (CO_2) concentrations in the atmosphere is the boost to growth in plants given by the additional CO_2. Higher CO_2 concentrations stimulate photosynthesis, enabling the plants to fix carbon at a higher rate. This is why in glasshouses additional CO_2 may be introduced artificially to increase productivity. The effect is particularly applicable to what are called C3 plants (such as wheat, rice and soya bean), but less so to C4 plants (for example, maize, sorghum, sugar-cane, millet and many pasture and forage grasses). Under ideal conditions it can be a large effect; for C3 crops under doubled CO_2, an average of +30%.[37]

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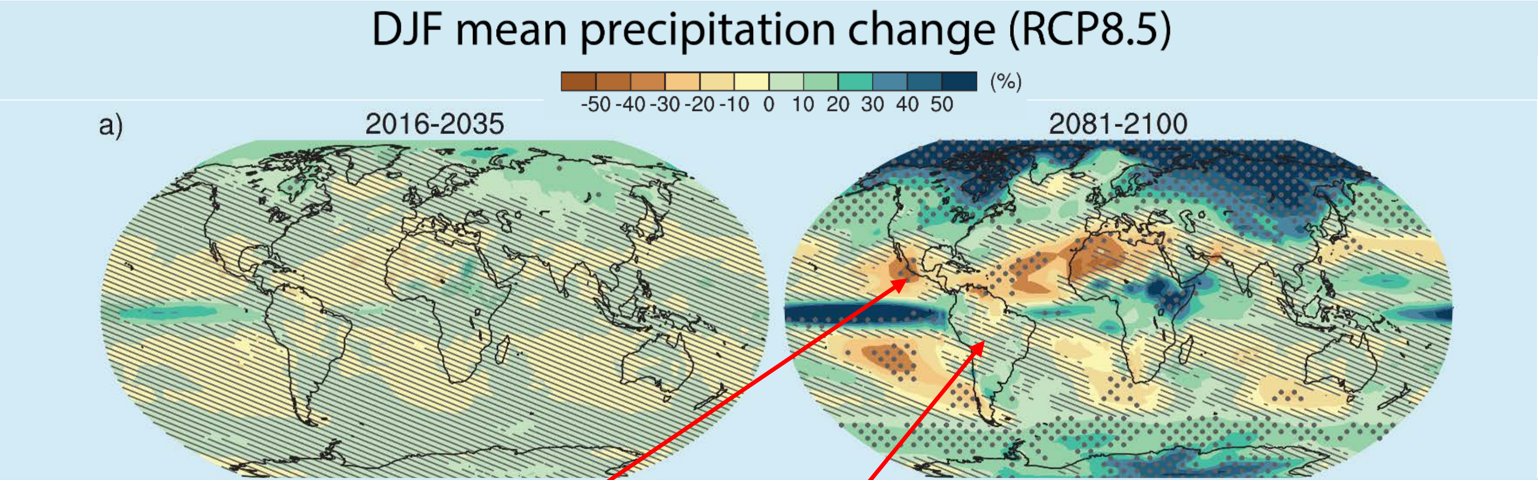
Uptake of Atmospheric CO_2 by Trees (Land Sink)

One more problem: Friedlingstein (2006) changes in land uptake are driven by future drought, and future precipitation is notoriously difficult to predict


DJF mean precipitation change (RCP8.5)

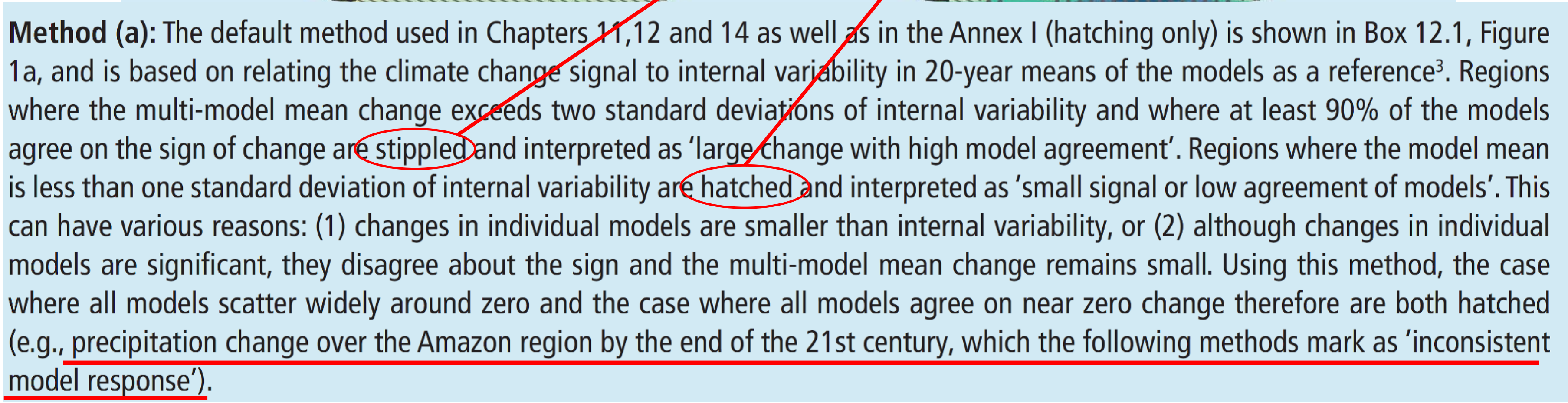

Method (a): The default method used in Chapters 11,12 and 14 as well as in the Annex I (hatching only) is shown in Box 12.1, Figure 1a, and is based on relating the climate change signal to internal variability in 20-year means of the models as a reference[3]. Regions where the multi-model mean change exceeds two standard deviations of internal variability and where at least 90% of the models agree on the sign of change are stippled and interpreted as 'large change with high model agreement'. Regions where the model mean is less than one standard deviation of internal variability are hatched and interpreted as 'small signal or low agreement of models'. This can have various reasons: (1) changes in individual models are smaller than internal variability, or (2) although changes in individual models are significant, they disagree about the sign and the multi-model mean change remains small. Using this method, the case where all models scatter widely around zero and the case where all models agree on near zero change therefore are both hatched (e.g., precipitation change over the Amazon region by the end of the 21st century, which the following methods mark as 'inconsistent model response').

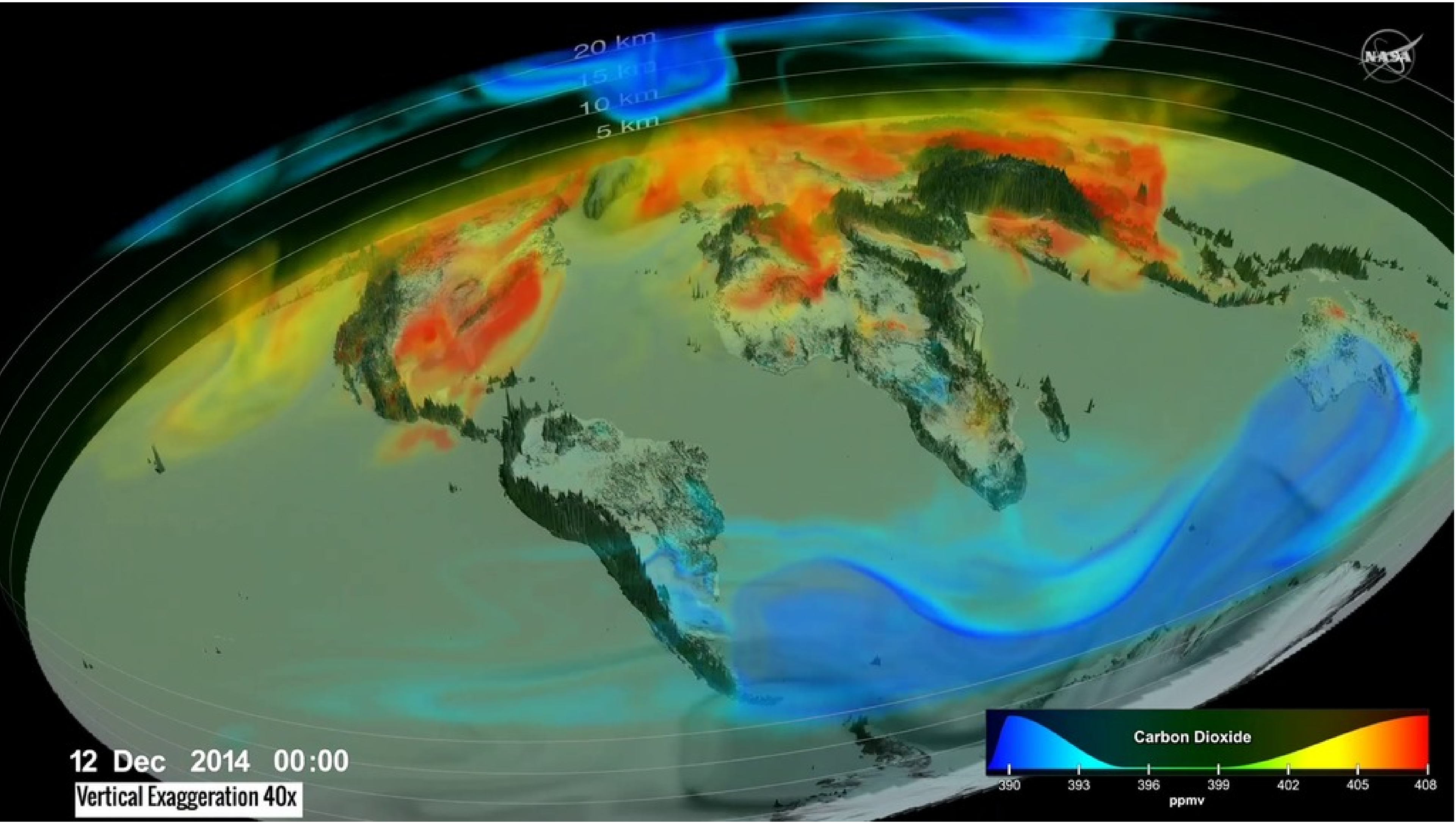
Figure 1, Chapter 12, IPCC (2013)

Exciting New Capability: Launch of Orbiting Carbon Observatory (OCO-2)

Launch of OCO-2 on 2 July 2014, photographed by Stephen Kelly Sullivan from Malibu, California.

http://oco.jpl.nasa.gov & https://co2.jpl.nasa.gov

OCO-2 Data



https://www.nasa.gov/feature/goddard/2016/eye-popping-view-of-co2-critical-step-for-carbon-cycle-science

Extra Slide 1

Carbon Water Chemistry

Acidity of pure water is 7. This means $[H^+] = 10^{-7}$ moles/liter or 10^{-7} M.

What is acidity of water in equilibrium with atmospheric CO_2 ?

$$[CO_2(aq)] = H_{CO2}\ p_{CO2} = 3.4 \times 10^{-2}\ \text{M / atm}\ p_{CO2}$$

For CO_2 = 390 ppm:

$$[CO_2(aq)] = 3.4 \times 10^{-2}\ \text{M / atm}\ 3.9 \times 10^{-4}\ \text{atm} = 1.326 \times 10^{-5}\ \text{M}$$

First equilibrium between CO_2, HCO_3^- (bicarbonate), and H^+

$$CO_2(aq) + H_2O \leftrightarrow HCO_3^- + H^+$$

$$K_1 = \frac{[HCO_3^-]\,[H^+]}{[CO_2(aq)]} = 4.3 \times 10^{-7}\ \text{M (at 298 K)}$$

Second equilibrium between CO_3^{2-} (carbonate), HCO_3^-, and H^+

$$H^+ + CO_3^{2-} \leftrightarrow HCO_3^-$$

$$K_2 = \frac{[CO_3^{2-}]\,[H^+]}{[HCO_3^-]} = 4.7 \times 10^{-11}\ \text{M (at 298 K)}$$

Can solve if we assume charge balance: $[H^+] = [HCO_3^-] + 2\,[CO_3^{2-}]$

- or – by taking a short-cut (see next slide)

Extra Slide 2

Carbon Water Chemistry

Acidity of pure water is 7. What is acidity of water in equilibrium with atmospheric CO_2 ?

Shortcut:

$[CO_2(aq)] = H_{CO2}\ p_{CO2} = 3.4 \times 10^{-2}$ M / atm $p_{CO2} = 1.326 \times 10^{-5}$ M for present atmosphere

$[H^+]\,[HCO_3^-] = K_1\,[CO_2(aq)] = 4.3 \times 10^{-7}\ M \times 1.326 \times 10^{-5}\ M = 5.70 \times 10^{-12}\ M^2$

Assume charge balance is primarily between $[H^+]$ and $[HCO_3^-]$:

i.e., that $[H^+] \approx [HCO_3^-]$ and that both are >> $[CO_3^{2-}]$

$[H^+]\,[H^+] = 5.70 \times 10^{-12}\ M^2 \Rightarrow [H^+] = 2.388 \times 10^{-6}$ M

$pH = -\log_{10}[H+] = 5.6$ (390 ppm, 298 K)

Is the ***assumption*** justified? :

$[CO_3^{2-}] = K_2\,[HCO_3^-] / [H^+] \approx 4.7 \times 10^{-11}$ M

$[H^+]$ & $[HCO_3^-]$ are both ~ 2.4×10^{-6} M which is >> 4.7×10^{-11} M

Extra Slide 3

Ocean Acidity

As noted in class, the actual ocean is basic. The net charge from a series of **cations** (positively charged ions) and minor **anions** (negatively charged ions) is balanced by the total negative charge of the bicarbonate and carbonate ions. We write:

$$[\mathrm{Alk}] = [\mathrm{HCO_3^-}] + 2\,[\mathrm{CO_3^{2-}}] = [\mathrm{Na^+}]+[\mathrm{K^+}]+2[\mathrm{Mg^{2+}}]+2[\mathrm{Ca^{2+}}]-[\mathrm{Cl^-}]-[\mathrm{Br^-}]-2\,[\mathrm{SO_4^{2-}}] + \ldots$$

where Alk stands for Alkalinity

Henry's Law and the equations for the first and second dissociation constants yield:

$$\mathrm{pCO_2(vmr)} = \frac{[\mathrm{CO_2\,(aq)}]}{\alpha} \qquad \mathrm{K_1} = \frac{[\mathrm{HCO_3^-}]\,[\mathrm{H^+}]}{[\mathrm{CO_2(aq)}]} \qquad \mathrm{K_2} = \frac{[\mathrm{CO_3^{2-}}]\,[\mathrm{H^+}]}{[\mathrm{HCO_3^-}]}$$

The three equations above can be re-arranged to yield: $\mathrm{pCO_2(vmr)} = \left(\frac{\mathrm{K_2}}{\alpha \mathrm{K_1}}\right)\frac{[\mathrm{HCO_3^-}]^2}{[\mathrm{CO_3^{2-}}]}$

If we substitute $[\mathrm{HCO_3^-}] = \mathrm{Alk} - 2\,[\mathrm{CO_3^{2-}}]$ into the eqn above, we arrive at a quadratic eqn for $[\mathrm{CO_3^{2-}}]$ as a function of $\mathrm{pCO_2}$ and Alk. Note that α, $\mathrm{K_1}$, and $\mathrm{K_2}$ vary as a function of temperature (T) and ocean salinity (S) (http://en.wikipedia.org/wiki/Salinity)

If T, Alk, & S are specified, it is straightforward to solve for $[\mathrm{CO_3^{2-}}]$ from the quadratic eqn.

Values for $[\mathrm{CO_2(aq)}]$, $[\mathrm{HCO_3^-}]$, and [H+] are then found from Henry's law & the dissoc eqns.

Finally, Ocean Carbon is found from $[\mathrm{CO_2(aq)}]+[\mathrm{HCO_3^-}]+ [\mathrm{CO_3^{2-}}]$.

Numerical values on the slides entitled "Uptake of Atmospheric $\mathrm{CO_2}$ by Oceans" were found in this manner, using Fortran program http://www.atmos.umd.edu/~rjs/class/code/ocean_carbon.f